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A NEW STRUCTURE AND CONSTRUCTION OF L – FUZZY M – COSETS OF M – HX GROUPS

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Abstract: In this paper, we introduce some definitions and results of L – fuzzy M – cosets of M – HX groups. Some properties of L – fuzzy M – cosets and its types are also established.

Key Words: L –fuzzy subset; HX groups; M – HX groups; L – fuzzy M – HX subgroups.

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1. INTRODUCTION

The fuzzy sets theory which was introduced by Zadeh [10] is applied to many mathematical branches. Subramanians. S at al [9] discussed some properties of M - fuzzy groups. This concept studied by many researchers [5, 6, 7, 8]. Li Hongxing [1] introduced the concept of HX group and the authors Luo Chengzhong at al [2] introduced the concepts of fuzzy HX group, this concept discussed by Muthuraj. R at al [3, 4]. In this paper, we introduce a new algebraic structure of L – fuzzy M – cosets and its types of a M – HX group. Some of their related properties also study.

2. PRELIMINARIES

2.1 Definition: Let X be a non empty set and L = (L, $\leq$) be a lattice with least element 0 and greater element 1. A L – fuzzy subset λ of X is a function $\lambda: X \rightarrow L$.

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2.2 Definition: [1] In $2^G - \{ \phi \}$, a non empty set $\upsilon \subset 2^G - \{ \phi \}$ is called a HX group on G, if υ is a group with respect to algebraic operation defined by AB = {ab; a $\in$ A & b $\in$ B}, which its unit element is denoted by E.

2.3 Definition: A HX group with operators is an algebraic system consisting of a HX group υ, a set M and a function defined in the product set $M \times \upsilon$ and having values in υ such that, if m(AB) denotes the element in υ determined by element AB of υ and the element m of M, then υ is called M – HX group with operators.

2.4 Definition: A L – fuzzy set λ is called a L – fuzzy M- HX subgroup of a M- HX group υ if for A, B $\in \upsilon$ and m$\in$ M.

1. $\lambda(m(AB)) \geq \min \{\lambda(mA), \lambda(mB)\}$
2. $\lambda(A^{-1}) \geq \lambda(A)$.

2.5 Definition: Let υ be a M- HX group. A L – fuzzy M - HX subgroup λ of υ is said to be normal if for all A,B $\in \upsilon$ and m $\in$ M, $\lambda(m(ABA^{-1})) = \lambda(mB)$ or $\lambda(m(AB)) = \lambda(m(BA))$.

2.6 Definition: Let λ be a L – fuzzy M – HX subgroup of υ and U = {A $\in \upsilon$; $\lambda(mA) = \lambda(mE)$}, then O($\lambda$) order of λ is defined as O(λ) = O(U).

2.7 Definition: Let λ be a L – fuzzy subset of X, for $\alpha \in$ L the level subset of λ is the set λ_α = {x $\in$ X; $\lambda(x) \leq \alpha$}, this is called a L – fuzzy level subset of λ.

2.8 Definition: A L – fuzzy M – HX subgroup λ of υ is said to be a generalized characteristic L – fuzzy M – HX subgroup if for all A, B $\in \upsilon$. O(A) = O(B) implies $\lambda(mA) = \lambda(mB)$.

3.PROPERTIES OF L – FUZZY M – COSETS OF M – HX GROUPS

3.1 Definition: Let λ be a L – fuzzy M – HX subgroup of υ, for any A $\in \upsilon$ and m $\in$ M , then Aλ defined by $(A\lambda)(mX) = \lambda(A^{-1}(mX))$ for every X $\in \upsilon$ is called the L – fuzzy M – cosets of υ.

3.2 Proposition: If λ is a L – fuzzy M – HX subgroup of υ and if A = E, then the L – fuzzy M - cosets Aλ is also L – fuzzy M – HX subgroup of υ.

Proof: Straight forward.

3.3 Theorem: Let λ be a L – fuzzy M – HX subgroup of υ. Then Aλ = Bλ for any A, B $\in \upsilon$ and m $\in$ M iff $\lambda(m(A^{-1}B)) = \lambda(m(B^{-1}A)) = \lambda(mE)$.

Proof:

Since λ is a L – fuzzy M – HX subgroup of υ. If $A\lambda = B\lambda$ for any $A, B \in \upsilon$ and $m \in M$. Then $A\lambda(mA) = B\lambda(mA)$ and $A\lambda(mB) = B\lambda(mB)$

thus $\lambda(m(A^{-1}A)) = \lambda(m(B^{-1}A))$ and $\lambda(m(A^{-1}B)) = \lambda(m(B^{-1}B))$ also $\lambda(m(A^{-1}B)) = \lambda(mE)$ and $\lambda(m(B^{-1}A)) = \lambda(mE)$. Therefore $\lambda(m(A^{-1}B)) = \lambda(m(B^{-1}A)) = \lambda(mE)$.

Now, if $\lambda(m(A^{-1}B)) = \lambda(m(B^{-1}A)) = \lambda(mE)$ for $A, B \in \upsilon$. For every $X \in \upsilon$ and $m \in M$ we have $A\lambda(mX) = \lambda(m(A^{-1}X)) = \lambda(m(A^{-1}BB^{-1}X)) \geq \min\{\lambda(m(A^{-1}B)), \lambda(m(B^{-1}X))\} = \min\{\lambda(mE), \lambda(m(B^{-1}X))\} = \lambda(m(B^{-1}X)) = B\lambda(mX)$ then $A\lambda(mX) \geq B\lambda(mX)$. By the same method $B\lambda(mX) \geq A\lambda(mX)$ hence $A\lambda(mX) = B\lambda(mX)$ and $A\lambda = B\lambda$.

3.4 Theorem: Let λ be a L – fuzzy M – HX subgroup of υ and $A\lambda = B\lambda$ for $A, B \in \upsilon$ and $m \in M$ then $\lambda(mA) = \lambda(mB)$.

Proof:

Since λ be a L – fuzzy M – HX subgroup of υ and $A\lambda = B\lambda$ for $A, B \in \upsilon$

$\lambda(mA) = \lambda(m(BB^{-1}A))$

$\geq \min\{\lambda(mB), \lambda(m(B^{-1}A))\}$

$\geq \min\{\lambda(mB), \lambda(m(E))\}$

$= \lambda(mB)$

$\lambda(mB) = \lambda(m(AA^{-1}B))$

$\geq \min\{\lambda(mA), \lambda(m(A^{-1}B))\}$

$\geq \min\{\lambda(mA), \lambda(m(E))\}$

$= \lambda(mA)$

Thus $\lambda(mA) = \lambda(mB)$.

3.5 Theorem: Let λ be a L – fuzzy M – HX subgroup of υ. Then $A\lambda_\alpha = (A\lambda)_\alpha$ for every $A \in \upsilon$ and $t \in L$.

Proof:

Let $B \in (A\lambda)_\alpha \Leftrightarrow A\lambda(mB) \leq \alpha$

$\Leftrightarrow \lambda(m(A^{-1}B)) \leq \alpha$

$\Leftrightarrow A^{-1}B \in \lambda_\alpha$

$\Leftrightarrow B \in A\lambda_\alpha$

Then $A\lambda_\alpha = (A\lambda)_\alpha$ for every $B \in \upsilon$.

3.6 Theorem: Let λ be a L – fuzzy M – HX subgroup of υ and $A\lambda_\alpha = B\lambda_\alpha$ for A, B $\in \upsilon - \lambda_\alpha$ and $\alpha \in L$. Then $\lambda(mA) = \lambda(mB)$

Proof:

Since $m(B^{-1}A), m(A^{-1}B) \in \lambda_\alpha$

$\lambda(mA) = \lambda(m(BB^{-1}A))$

$\geq \min\{\lambda(mB), \lambda(m(B^{-1}A))\}$

$\geq \min\{\lambda(mB),\lambda(m(E))\}$

$= \lambda(mB)$

$\lambda(mB) = \lambda(m(AA^{-1}B))$

$\geq \min\{\lambda(mA), \lambda(m(A^{-1}B))\}$

$\geq \min\{\lambda(mA),\lambda(m(E))\}$

$= \lambda(mA)$

Thus $\lambda(mA) = \lambda(mB)$.

3.7 Definition: Let λ be a L – fuzzy M – HX subgroup of υ then for any A, B $\in \upsilon$ and $m \in M$, a L – fuzzy M –middle cosets $A\lambda B$ of υ is defined by $(A\lambda B)(mX) = \lambda(A^{-1}(mX)B^{-1})$ for every X $\in \upsilon$.

3.8 Proposition: Let λ be a L – fuzzy M – HX subgroup of υ then the L – fuzzy M – middle cosets $A\lambda B$ is also L – fuzzy M – HX subgroup of υ if $B = A^{-1}$.

Proof: Straight forward.

3.9 Definition: Let λ be a L – fuzzy M – HX subgroup of υ and $A \in \upsilon$. Then a L – fuzzy M – pseudo cosets $(A\lambda)^p$ is defined by $(A\lambda)^p(mX) = p(A)\lambda(mX)$ for every $X \in \upsilon$, $p \in P$ and $m \in M$.

3.10 Proposition: If λ is a L – fuzzy M – HX subgroup of υ and $A \in \upsilon$. Then a L – fuzzy M – pseudo cosets $(A\lambda)^p$ is a L – fuzzy M – HX subgroup of υ if $p(mA) \leq p(mE)$ for every $A \in \upsilon$, $p \in P$ and $m \in M$.

Proof: Straight forward.

3.11 Definition: If λ, μ are any two L – fuzzy M – HX subgroup of υ,then a L – fuzzy M – pseudo double cosets $(\lambda A\mu)^p$ is defined by $(\lambda A\mu)^p = \min\{(A\lambda)^p, (A\mu)^p\}$ for every $A \in \upsilon$and $p \in P$.

3.12 Theorem: Let λ, μ be any L – fuzzy M – HX subgroup of υ,then a L – fuzzy M – pseudo double cosets $(\lambda A\mu)^p$ also L – fuzzy M – HX subgroup of υ.

Proof:

For all $X, Y \in \upsilon$, $m \in M$.

$(\lambda A\mu)^p (m(XY)) = \min\{(A\lambda)^p(m(XY)), (A\mu)^p(m(XY))\}$

$= \min\{ p(A)\lambda(m(XY)), p(A)\mu(m(XY))\}$

$= p(A)\min\{ \lambda(m(XY)), \mu(m(XY))\}$

$\geq p(A)\min\{ \min \{\lambda(mX), \lambda(mY)\}, \min \{\mu(mX), \mu(mY)\}\}$

$\geq p(A)\min\{ \min \{\lambda(mX), \mu(mX) \}, \min \{\lambda(mY), \mu(mY)\}\}$

$\geq \min\{\min\{p(A)\lambda(mX),p(A)\mu(mX)\},\min\{ p(A)\lambda(mY), p(A)\mu(mY)\}\}$

$\geq \min\{\min\{(A\lambda)^p(mX), (A\mu)^p(mX)\},\min\{(A\lambda)^p(mY),(A\mu)^p (mY)\}\}$

$= \min\{(\lambda A\mu)^p (mX), (\lambda A\mu)^p (mY)$

$(\lambda A\mu)^p (mX^{-1}) = \min\{(A\lambda)^p(mX^{-1}), (A\mu)^p(mX^{-1}) \}$

$= \min\{ p(A)\lambda(mX^{-1}), p(A)\mu(mX^{-1})\}$

$\geq \min\{ p(A)\lambda(mX), p(A)\mu(mX)\}$

$= \min\{(A\lambda)^p(mX), (A\mu)^p(mX) \}$

$= (\lambda A\mu)^p (mX)$

Therefore $(\lambda A\mu)^p$ also L – fuzzy M – HX subgroup of υ.

3.13 Corollary: Let λ, μ be any L – fuzzy normal M – HX subgroup of υ,then a L – fuzzy M – pseudo double cosets $(\lambda A\mu)^p$ also L – fuzzy normal M – HX subgroup of υ.

Proof: Straight forward.

4. CONJUGATE AND ORDER AND OF L – FUZZY M – HX SUBGROUP OF υ

4.1 Definition: Let λ, μ be two L – fuzzy M – HX subgroup of υ, then λ and μ are said to be conjugate L – fuzzy M – HX subgroup of υ if for some $C \in \upsilon$. $\lambda(mA) = \mu(C^{-1}(mA)C)$ for every $A \in \upsilon$ and $m \in M$.

4.2 Theorem: Let λ, μ be two L – fuzzy subset of υ, then λ and μ are conjugate L – fuzzy subset of υ iff $\lambda = \mu$.

Proof:

Suppose that λ and μ conjugate L – fuzzy subset of υ, then for some $C \in \upsilon$ we have $\lambda(mA) = \mu(C^{-1}(mA)C)$ for every $A \in \upsilon$ and $m \in M$.

$\lambda(mA) = \mu(C^{-1}(mA)C) = \mu(C^{-1}C(mA)) = \mu(mEA) = \mu(mA)$. Then $\lambda(mA) = \mu(mA)$, hence $\lambda = \mu$.

Now, suppose that $\lambda = \mu$, since $E \in \upsilon$ we have $\lambda(mA) = \mu(E^{-1}(mA)E)$ for every $A \in \upsilon$ and $m \in M$. Thus λ, μ are conjugate L – fuzzy subset of υ.

4.3 Definition: Let λ be L – fuzzy M – HX subgroup of υ, $U = \{A \in \upsilon; \lambda(mX) = \lambda(mE)\}$ then $O(\lambda)$ order of λ is defined by $O(\lambda) = O(U)$.

4.4 Theorem: {Generalized Lagrange Theorem}

Let λ be a L – fuzzy M – HX subgroup of a finite M – HX group υ, then $O(\lambda) \mid O(\upsilon)$.

Proof:

Suppose λ is L – fuzzy M – HX subgroup of a finite M – HX group υ with E as its identity element, since $U = \{A \in \upsilon; \lambda(mX) = \lambda(mE)\}$is a M – HX subgroup of υ for U is α - level subset of υ where $\alpha = \lambda(mE)$. By usual Lagrange Theorem $O(U) \mid O(\upsilon)$ therefore $O(\lambda) \mid O(\upsilon)$.

4.5 Theorem: If λ and μ are conjugate L – fuzzy M – HX subgroup of υ then $O(\lambda) = O(\mu)$.

Proof:

Since λ and μ are conjugate L – fuzzy M – HX subgroup of υ

$O(\lambda) = \text{order}\{ A \in \upsilon; \lambda(mA) = \lambda(mE)\}$

$= \text{order}\{ A \in \upsilon; \mu(B^{-1} (mA)B) = \mu(B^{-1} (mE)B)\}$

$= \text{order}\{ A \in \upsilon; \mu(mA) = \mu(mE)\}$

$= O(\mu)$.

4.6 Theorem: If λ is a L – fuzzy M – HX subgroup of υ and $A\lambda A^{-1}$ is a L – fuzzy M – middle cosets of υ, then $O(A\lambda A^{-1}) = O(\lambda)$ for every $A \in \upsilon$.

Proof:

By Proposition 3.8 $A\lambda A^{-1}$ is a L – fuzzy M – HX subgroup of υ also

$(A\lambda A^{-1})(mX) = \lambda(A^{-1}(mX)A)$ for every $A \in \upsilon$, also for any $A \in \upsilon$ λ and $A\lambda A^{-1}$ are conjugate L – fuzzy M – HX subgroup of υ, as there exists $A \in \upsilon$ such that $(A\lambda A^{-1})(mX) = \lambda(A^{-1}(mX)A)$ for every $X \in \upsilon$. Then by Theorem 4.5 $O(A\lambda A^{-1}) = O(\lambda)$.

4.7 Theorem: Let λ be a L – fuzzy M – HX subgroup of υ and μ be a. L – fuzzy subset of υ. If λ and μ are conjugate L – fuzzy subset then μ is a L – fuzzy M – HX subgroup of υ.

Proof:

Let $A, B \in \upsilon$ and $m \in M$, then $mAB \in \upsilon$

$\mu(m(AB)) = \lambda(m(X^{-1}ABX))$ for every $X \in \upsilon$

$= \lambda(m(X^{-1}AXX^{-1}BX))$

$= \lambda(m((X^{-1}AX)(X^{-1}BX)))$

$\geq \min\{\lambda(m (X^{-1}AX)) , \lambda(m(X^{-1}BX))\}$

$\geq \min\{\mu(m A) , \lambda(mB)\}$

$\mu(m(AB)) = \lambda(m(X^{-1}A^{-1}X))$ for every $X \in \upsilon$

$= \lambda(m\,(X^{-1}A\,X)^{-1})$

$= \lambda(m\,(X^{-1}A\,X))$

$= \mu(m\,A)$

Then μ is a L – fuzzy M – HX subgroup of υ.

Conflict of Interests

The authors declare that there is no conflict of interests.

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