Available online at http://scik.org
J. Math. Comput. Sci. 6 (2016), No. 3, 347-359
ISSN: 1927-5307

SOFT L-TOPOLOGIES AND SOFT L-NEIGHBORHOOD SYSTEMS

JUNG MI KO, YONG CHAN KIM*

Department of Mathematics, Gangneung-Wonju National University, Gangwondo 210-702, Korea

Abstract. In this paper, we introduce soft L-neighborhood systems and soft topologies in a complete residuated lattice. We study the relations between soft L-neighborhood systems and soft topologies. In particular, we study some functorial relationships between previous spaces. We give their examples.

Keywords: Complete residuated lattices; L-neighborhood systems; L-topologies; Continuous soft maps

2000 AMS Subject Classification: 54A40, 03E72, 03G10, 06A15

1. Introduction

Molodtsov [15] introduced the soft set as a mathematical tool for dealing information as the uncertainty of data in engineering, physics, computer sciences and many other diverse field. Presently, the soft set theory is making progress rapidly [1,3,5,11,12-14,25,27]. Pawlak's rough set [17,18] can be viewed as a special case of soft rough sets [5]. The topological structures of soft sets have been developed by many researchers [3,9,10,22,23,27,28].

On the other hand, Hájek [7] introduced a complete residuated lattice which is an algebraic structure for many valued logic. It is an important mathematical tool for algebraic structure of fuzzy contexts [2,6,7,9,10,19,20,21,24].

*Corresponding author.

Received October 25, 2015

Kim [9,10] introduced a fuzzy soft $F : A \to L^U$ as an extension as the soft $F : A \to P(U)$ where L is a complete residuated lattice. He introduced soft L-fuzzy interior and closure operators, quasi-uniformities and soft L-fuzzy topogenous orders in complete residuated lattices.

In this paper, we introduce soft L-neighborhood systems and soft topologies in a complete residuated lattice. We study the relations between soft L-neighborhood systems and soft topologies. In particular, we study some functorial relationships between previous spaces. We give their examples.

2. Preliminaries

Definition 2.1. [2,6,7,24] An algebra $(L,\wedge,\vee,\odot,\to,0,1)$ is called a complete residuated lattice if it satisfies the following conditions:

(C1) $L = (L,\leq,\vee,\wedge,1,0)$ is a complete lattice with the greatest element 1 and the least element 0;

(C2) $(L,\odot,1)$ is a commutative monoid;

(C3) $x \odot y \leq z$ iff $x \leq y \to z$ for $x,y,z \in L$.

In this paper, we assume that $(L,\leq,\odot,\to,\oplus,^*)$ is a complete residuated lattice with an order reversing involution $x^* = x \to 0$ which is defined by $x \oplus y = (x^* \odot y^*)^*$ unless otherwise specified and we denote $L_0 = L - \{0\}$.

Lemma 2.1. [2,6,7,24] For each $x,y,z,x_i,y_i,w \in L$, we have the following properties.

(1) $1 \to x = x$, $0 \odot x = 0$,

(2) If $y \leq z$, then $x \odot y \leq x \odot z$, $x \oplus y \leq x \oplus z$, $x \to y \leq x \to z$ and $z \to x \leq y \to x$,

(3) $x \odot y \leq x \wedge y \leq x \vee y \leq x \oplus y$,

(4) $(\bigwedge_i y_i)^* = \bigvee_i y_i^*$, $(\bigvee_i y_i)^* = \bigwedge_i y_i^*$,

(5) $x \odot (\bigvee_i y_i) = \bigvee_i (x \odot y_i)$,

(6) $x \oplus (\bigwedge_i y_i) = \bigwedge_i (x \oplus y_i)$,

(7) $x \to (\bigwedge_i y_i) = \bigwedge_i (x \to y_i)$,

(8) $(\bigvee_i x_i) \to y = \bigwedge_i (x_i \to y)$,

(9) $x \to (\bigvee_i y_i) \geq \bigvee_i (x \to y_i)$,

(10) $(\bigwedge_i x_i) \to y \geq \bigvee_i (x_i \to y)$,

(11) $(x \odot y) \to z = x \to (y \to z) = y \to (x \to z)$,

(12) $x \odot (x \to y) \leq y$ and $x \to y \leq (y \to z) \to (x \to z)$,

(13) $(x \to y) \odot (z \to w) \leq (x \odot z) \to (y \odot w)$,

(14) $(x \to y) \odot (z \to w) \leq (x \oplus z) \to (y \oplus w)$,

(15) $x \to y \leq (x \odot z) \to (y \odot z)$ and $(x \to y) \odot (y \to z) \leq x \to z$,

(16) $(x \oplus z) \odot y \leq x \oplus (y \odot z)$,

(17) $x \to y = y^* \to x^*$.

Definition 2.2. [9,10] Let X be an initial universe of objects and E the set of parameters (attributes) in X. A pair (F,A) is called a *fuzzy soft set* over X, where $A \subset E$ and $F : A \to L^X$ is a mapping. We denote $S(X,A)$ as the family of all fuzzy soft sets under the parameter A.

Definition 2.3.[9,10] Let (F,A) and (G,A) be two fuzzy soft sets over a common universe X.

(1) (F,A) is a fuzzy soft subset of (G,A), denoted by $(F,A) \leq (G,A)$ if $F(a) \leq G(a)$, for each $a \in A$.

(2) $(F,A) \wedge (G,A) = (F \wedge G,A)$ if $(F \wedge G)(a) = F(a) \wedge G(a)$ for each $a \in A$.

(3) $(F,A) \vee (G,A) = (F \vee G,A)$ if $(F \vee G)(a) = F(a) \vee G(a)$ for each $a \in A$.

(4) $(F,A) \odot (G,A) = (F \odot G,A)$ if $(F \odot G)(a) = F(a) \odot G(a)$ for each $a \in A$.

(5) $(F,A)^* = (F^*,A)$ if $F^*(a) = (F(a))^*$ for each $a \in A$.

(6) $(F,A) \oplus (G,A) = (F \oplus G,A)$ if $(F \oplus G)(a) = (F^*(a) \odot G^*(a))^*$ for each $a \in A$.

(7) $\alpha \odot (F,A) = (\alpha \odot F,A)$ for each $\alpha \in L$.

Definition 2.4. [9,10] Let $S(X,A)$ and $S(Y,B)$ be the families of all fuzzy soft sets over X and Y, respectively. The mapping $f_\phi : S(X,A) \to S(Y,B)$ is a soft mapping where $f : X \to Y$ and $\phi : A \to B$ are mappings.

(1) The image of $(F,A) \in S(X,A)$ under the mapping f_ϕ is denoted by $f_\phi((F,A)) = (f_\phi(F),B)$ where

$$f_\phi(F)(b)(y) = \begin{cases} \bigvee_{a \in \phi^{-1}(\{b\})} f^{\to}(F(a))(y), & \text{if } \phi^{-1}(\{b\}) \neq \emptyset, \\ 0, & \text{otherwise.} \end{cases}$$

(2) The inverse image of $(G,B) \in S(Y,B)$ under the mapping f_ϕ is denoted by $f_\phi^{-1}((G,B)) = (f_\phi^{-1}(G),A)$ where

$$f_\phi^{-1}(G)(a)(x) = f^{\leftarrow}(G(\phi(a)))(x),\ \forall a \in A, x \in X.$$

(3) The soft mapping $f_\phi : S(X,A) \to S(Y,B)$ is called injective (resp. surjective, bijective) if f and ϕ are both injective (resp. surjective, bijective).

Lemma 2.5. [9,10] Let $f_\phi : S(X,A) \to S(Y,B)$ be a soft mapping. Then we have the following properties. For $(F,A),(F_i,A) \in S(X,A)$ and $(G,B),(G_i,B) \in S(Y,B)$,

(1) $(G,B) \geq f_\phi(f_\phi^{-1}((G,B)))$ with equality if f is surjective,

(2) $(F,A) \leq f_\phi^{-1}(f_\phi((F,A)))$ with equality if f is injective,

(3) $f_\phi^{-1}((G,B)^*) = (f_\phi^{-1}((G,B)))^*$,

(4) $f_\phi^{-1}(\bigvee_{i\in I}(G_i,B)) = \bigvee_{i\in I} f_\phi^{-1}((G_i,B))$,

(5) $f_\phi^{-1}(\bigwedge_{i\in I}(G_i,B)) = \bigwedge_{i\in I} f_\phi^{-1}((G_i,B))$,

(6) $f_\phi(\bigvee_{i\in I}(F_i,A)) = \bigvee_{i\in I} f_\phi((F_i,A))$,

(7) $f_\phi(\bigwedge_{i\in I}(F_i,A)) \leq \bigwedge_{i\in I} f_\phi((F_i,A))$ with equality if f is injective,

(8) $f_\phi^{-1}((G_1,B)\odot(G_2,B)) = f_\phi^{-1}((G_1,B)) \odot f_\phi^{-1}((G_2,B))$,

(9) $f_\phi^{-1}((G_1,B)\oplus(G_2,B)) = f_\phi^{-1}((G_1,B)) \oplus f_\phi^{-1}((G_2,B))$,

(10) $f_\phi((F_1,A)\odot(F_2,A)) \leq f_\phi((F_1,A)) \odot f_\phi((F_2,A))$ with equality if f is injective,

(11) $f_\phi((F_1,A)\oplus(F_2,A)) \leq f_\phi((F_1,A)) \oplus f_\phi((F_2,A))$ with equality if f is injective.

Definition 2.6. [9,10] A map $\tau \subset S(X,A)$ is called a soft L topology on X if it satisfies the following conditions.

(ST1) $(0_X,A),(1_X,A) \in \tau$, where $0_X(a)(x) = 0, 1_X(a)(x) = 1$ for all $a \in A, x \in X$,

(ST2) If $(F,A),(G,A) \in \tau$, then $(F,A)\odot(G,A)) \in \tau$,

(ST3) If $(F_i,A) \in \tau$ for each $i \in I$, $\bigvee_{i\in I}(F_i,A) \in \tau$.

The triple (X,A,τ) is called a soft L-topological space.

A soft L-topology is called enriched if $\alpha \odot (F,A) \in \tau$ for each $(F,A) \in \tau$ and $\alpha \in L$.

Let (X,A,τ_1) and (X,A,τ_2) be soft L-topological spaces. Then τ_1 is finer than τ_2 if $(F,A) \in \tau_1$, for all $(F,A) \in \tau_2$.

Let (X,A,τ_X) and (Y,B,τ_Y) be soft L-topological spaces and $f_\phi : S(X,A) \to S(Y,B)$ be a soft map. Then f_ϕ is called a continuous soft map if

$$f_\phi^{-1}((G,B)) \in \tau_X,\ \forall (G,B) \in \tau_Y.$$

Definition 2.7. [9,10] Let X be a set. A function $e_X : X \times X \to L$ is called:

(E1) reflexive if $e_X(x,x) = 1$ for all $x \in X$,

(E2) transitive if $e_X(x,y) \odot e_X(y,z) \le e_X(x,z)$, for all $x,y,z \in X$,

(E3) if $e_X(x,y) = e_X(y,x) = 1$, then $x = y$.

If e_X satisfies (E1) and (E2), e_X is a fuzzy preorder on X. If e_X satisfies (E1), (E2) and (E3), e_X is a fuzzy partially order on X.

Lemma 2.8. [9,10] For a given set X, define a binary mapping $e_X : S(X,A) \times S(X,A) \to L$ by

$$e_X((F,A),(G,A)) = \bigwedge_{a \in A} \bigwedge_{x \in X} (F(a)(x) \to G(a)(x)).$$

Then, for each $(F,A),(G,A),(H,A),(K,A) \in S(X,A)$ and $\alpha \in L$ the following properties hold.

(1) $(F,A) \le (G,A)$ iff $e_X((F,A),(G,A)) = 1$.

(2) e_X is a fuzzy partially order on $S(X,A)$,

(3) If $(F,A) \le (G,A)$, then

$$e_X((H,A),(F,A)) \le e_X((H,A),(G,A)),$$
$$e_X((F,A),(H,A)) \ge e_X((G,A),(H,A)).$$

(4) $e_X((F,A),(G,A)) \odot e_X((K,A),(H,A)) \le e_X((F,A) \odot (K,A),(G,A) \odot (H,A))$.

(5) $e_X((F,A),(G,A)) \odot e_X((K,A),(H,A)) \le e_X((F,A) \oplus (K,A),(G,A) \oplus (H,A))$.

(6) $e_X((F,A),\alpha \to (G,A)) = e_X(\alpha \odot (F,A),(G,A)) = \alpha \to e_X((F,A),(G,A))$ and $\alpha \odot e_X((F,A),(G,A)) \le e_X((F,A),\alpha \odot (G,A))$.

(7) $(G,A) \odot e_X((G,A),(F,A)) \le (F,A)$ and $(G,A) \le e_X((G,A),(F,A)) \to (F,A)$.

(8) $e_X((G,A),(H,A)) \le e_X((F,A),(G,A)) \to e_X((F,A),(H,A))$.

(9) $e_X((F,A),(G,A)) \le e_X((G,A),(H,A)) \to e_X((F,A),(H,A))$.

(10) if $x^* = x \to 0$, then $e_X((F,A),(G,A)) = e_X((G,A)^*,(F,A)^*)$.

(11) Let $f_\phi : (X,A) \to (Y,B)$ be a soft map. Then for $(F,A),(G,A) \in S(X,A)$ and $(H,A),(K,A) \in S(Y,B)$,

$$e_X((F,A),(G,A)) \leq e_Y(f_\phi((F,A)), f_\phi((G,A))),$$

$$e_Y((H,A),(K,A)) \leq e_X(f_\phi^{-1}((H,A)), f_\phi^{-1}((K,A))),$$

and the equalities hold if f_ϕ is bijective.

3. Soft L-topologies and soft L-neighborhood systems

Definition 3.1. A map $N : X \to (L^A)^{S(X,A)}$ is called a soft L-neighborhood system on X if $N = \{N_x = N(x) \mid x \in X\}$ satisfies the following conditions

(SN1) $N_x((1_X,A)) = 1_A$ and $N_x(0_X) = 0_A$,

(SN2) $N_x((F,A) \odot (G,A)) \geq N_x((F,A)) \odot N_x((G,A))$ for each $(F,A),(G,A) \in S(X,A)$,

(SN3) If $(F,A) \leq (G,A)$, then $N_x((F,A)) \leq N_x((G,A))$,

(SN4) $N_x((F,A)) \leq (F,A)(x)$ for all $(F,A) \in S(X,A)$ where $(F,A)(x) = F(-)(x)$ and $N_x((F,A))(a) \leq F(a)(x)$.

(SN5) $N_x((F,A)) \leq \bigvee\{N_x(((G,A))) \mid ((G,A))(y) \leq N_y(((F,A))),\ \forall\, y \in X\}$.

A soft L-neighborhood system is called stratified if

(S) $N_x(\alpha \odot (F,A)) \geq \alpha \odot N_x((F,A))$ for all $(F,A) \in S(X,A)$ and $\alpha \in L$.

The triple (X,A,N) is called a soft L-neighborhood space.

Let (X,A,N) and (Y,B,M) be soft L-neighborhood spaces. A mapping $f_\phi : (X,A,N) \to (Y,B,M)$ is said to be a continuous soft map iff $M_{f(x)}((G,B))(\phi(a)) \leq N_x(f_\phi^{-1}((G,B)))(a)$ for each $x \in X, (G,B) \in S(Y,B)$.

Theorem 3.2. Let (X,A,τ) be a soft L-topological space. Define a map $N^\tau : X \to (L^A)^{S(X,A)}$ by

$$N_x^\tau((F,A)) = \bigvee\{(G,A)(x) \mid (G,A) \leq (F,A),\ (G,A) \in \tau\},$$

where $(G,A)(x) = G(-)(x)$. Then the following properties hold.

(1) (X,A,N^τ) is a soft L-neighborhood space.

(2) If τ is enriched, then N^τ is stratified and

$$N_x^\tau((F,A)) = \bigvee_{(G,A)\in\tau} ((G,A)(x) \odot e_X((G,A),(F,A)).$$

Proof. (1) (SN1) Since $(1_X,A),(0_X,A) \in \tau$, $N_x^\tau((1_X,A)) = 1_X,A)(x) = 1_A$ and $N_x^\tau((0_X,A)) = (0_X,A)(x) = 0_A$.

(SN2) For each $a \in A$,

$$\begin{aligned}
&N_x^\tau((F,A)) \odot N_x^\tau((G,A)) \\
&= \big(\bigvee\{(F_1,A)(x) \mid (F_1,A) \le (F,A),\ (F_1,A) \in \tau\}\big) \\
&\odot\big(\bigvee\{(F_2,A)(x) \mid (F_2,A) \le (G,A),\ (F_2,A) \in \tau\}\big) \\
&\le \bigvee\{((F_1,A) \odot (F_2,A))(x) \mid (F_1,A) \odot (F_2,A) \le (F,A) \odot (G,A),\ (F_1,A) \odot (F_2,A) \in \tau\} \\
&\le N_x^\tau((F,A) \odot (G,A)).
\end{aligned}$$

(SN3-4) follow from the definition of N^τ.

(SN5) Put $N_-^\tau((F,A)) = \bigvee\{(G,A) \mid (G,A) \le (F,A),\ (G,A) \in \tau\}$ with $N_-^\tau(x) = N_x^\tau$. Then $N_-^\tau((F,A)) \in \tau$. By the definition of N_x^τ, since $N_-^\tau((F,A)) \le N_-^\tau((F,A))$, we have $N_x^\tau(N_-^\tau((F,A))) \ge N_x^\tau((F,A))$. By (4), $N_x^\tau(N_-^\tau((F,A))) \le N_x^\tau((F,A))$. Hence $N_x^\tau(N_-^\tau((F,A))) = N_x^\tau((F,A))$. Thus,

$$\begin{aligned}
N_x^\tau((F,A)) &= N_x^\tau(N_-^\tau((F,A))) \\
&\le \bigvee\{N_x^\tau((G,A)) \mid (G,A)(y) \le N_y^\tau((F,A))\}.
\end{aligned}$$

So, (X,A,N^τ) is a soft L-neighborhood space.

(2)

$$\begin{aligned}
&\alpha \odot N_x^\tau((F,A)) = \alpha \odot \bigvee\{(G,A) \mid (G,A) \le (F,A),\ (G,A) \in \tau\} \\
&\le \bigvee\{\alpha \odot (G,A) \mid \alpha \odot (G,A) \le \alpha \odot (F,A),\ \alpha \odot (G,A) \in \tau\} \le N_x^\tau(\alpha \odot (F,A)).
\end{aligned}$$

Hence N^τ is stratified.

Put $\gamma(x) = \bigvee_{(G,A)\in\tau}((G,A)(x) \odot e_X((G,A),(F,A)))$. Let (G,A) with $(G,A) \le (F,A)$ and $(G,A) \in \tau$. Then $(G,A)(x) \odot e_X((G,A),(F,A)) = (G,A)(x) \odot 1 = (G,A)(x)$. Thus $(G,A)(x) \le \gamma(x)$. Therefore $N_x^\tau((F,A)) \le \gamma(x)$.

Let $(G,A)(x) \odot e_X((G,A),(F,A))$ with $(G,A) \in \tau$. Since τ is enriched, $(G,A) \odot e_X((G,A),(F,A)) \in \tau$ and $(G,A)(x) \odot e_X((G,A),(F,A)) \le (G,A)(x) \odot ((G,A)(x) \to F(a)(x)) \le F(a)(x)$. Then $\gamma(x) \le N_x^\tau((F,A))$.

Theorem 3.3. Let (X,N) be a soft L-neighborhood space. Define $\tau_N \subset S(X,A)$ as follows

$$\tau_N = \{(F,A) \in S(X,A) \mid (F,A)(x) = N_x((F,A)), \forall x \in X\}.$$

Then,

(1) τ_N is a soft L-topology on X,

(2) If N is stratified, then τ_N is an enriched soft L-topology.

(3) $N = N^{\tau_N}$.

(4) If (X,τ) is a soft L-topological space, then $\tau = \tau_{N^\tau}$.

Proof. (1) (ST1) Since $N_x((1_X,A)) = (1_X,A) = 1_A$ and $N_x((0_X,A)) = (0_X,A) = 0_A$, we have $(1_X,A),(0_X,A) \in \tau_N$.

(ST2) Let $(F,A),(G,A) \in \tau_N$. Since $N_x((F,A)\odot(G,A)) \geq N_x((F,A))\odot N_x((G,A)) = ((F,A)\odot(G,A))(x)$ and (SN4), then $(F,A)\odot(G,A) \in \tau_N$.

(ST3) Let $(F_i,A) \in \tau_N$ for all $i \in \Gamma$. Since $N_x(\bigvee_{i\in\Gamma}(F_i,A)) \geq \bigvee_{i\in\Gamma} N_x((F_i,A)) = \bigvee_{i\in\Gamma}(F_i,A)$ and (SN4), then $\bigvee_{i\in\Gamma}(F_i,A) \in \tau_N$.

(2) Let $(F,A) \in \tau_N$. Since $N_x(\alpha\odot(F,A)) \geq \alpha\odot N_x((F,A)) = \alpha\odot(F,A)(x)$ and (SN4), then $\alpha\odot(F,A) \in \tau_N$.

(3) Since $N_x((F,A)) \leq N_x(N_-((F,A))) \leq N_x((F,A))$ from (SN3) and (SN5), $N_x((F,A)) = N_x(N_-((F,A)))$ for all $x \in X$. Since $N_-((F,A)) \in \tau_N$, by the definition of N^{τ_N}, $N_x((F,A)) \leq N_x^{\tau_N}((F,A))$.

Since $N_x^{\tau_N}((F,A)) = \bigvee\{(G_i,A)(x) \mid (G_i,A) \leq (F,A),\ (G_i,A) \in \tau_N\}$ and $(G_i,A)(x) = N_x((G_i,A))$, then

$$\bigvee_i (G_i,A)(x) = \bigvee_i N_x((G_i,A)) \leq N_x(N_-^{\tau_N}((F,A))) = N_x(\bigvee_i (G_i,A)) \leq \bigvee_i (G_i,A)(x).$$

Hence $N_x(N_-^{\tau_N}((F,A))) = N_x^{\tau_N}((F,A))$. Since $N_-^{\tau_N}((F,A)) \leq (F,A)$, by (SN3),

$$N_x^{\tau_N}((F,A)) = N_x(N_-^{\tau_N}((F,A))) \leq N_x((F,A)).$$

Thus $N_x^{\tau_N} = N_x$ for all $x \in X$.

(4) Let $(F,A) \in \tau_{N^\tau}$. Then $(F,A) = N_-^\tau((F,A)) \in \tau$.

Let $(G,A) \in \tau$. Then $(G,A)(x) = N_x^\tau((G,A))$ for all $x \in X$. Then $(G,A) \in \tau_{N^\tau}$.

Theorem 3.4. $f_\phi : (X,A,\tau_X) \to (Y,B,\tau_Y)$ is a continuous soft map iff $f_\phi : (X,A,N^{\tau_X}) \to (Y,B,N^{\tau_Y})$ is a continuous soft map.

Proof. ($\Rightarrow$) Since $f_\phi^{-1}((G,A)) \in \tau_X$ for each $(G,A) \in \tau_Y$, we have

$$\begin{aligned}
&N^{\tau_Y}_{\phi(x)}((H,B))(\phi(a)) = \bigvee\{G(\phi(a))(\phi(x)) \mid (G,B) \leq (H,B), (G,B) \in \tau_Y\} \\
&(f_\phi^{\leftarrow}((G,B))(a)(x) = f^{\leftarrow}(G(\phi(a)))(x)) \\
&\leq \bigvee\{f_\phi^{\leftarrow}((G,B))(a)(x) \mid f_\phi^{-1}((G,A)) \leq f_\phi^{-1}((F,A)), f_\phi^{-1}((G,A)) \in \tau_X\} \\
&\leq N^{\tau_X}_x(f_\phi^{-1}((G,B))).
\end{aligned}$$

($\Leftarrow$) Let $(G,B) \in \tau_Y$. Since $\tau_Y = \tau_{N^{\tau_Y}}$ from Theorem 3.3(4),

$$\begin{aligned}
&f_\phi^{\leftarrow}((G,B))(a)(x) = f^{\leftarrow}(G(\phi(a)))(x)) \\
&= G(\phi(a))(\phi(x)) = N^{\tau_Y}_{f(x)}((G,B))(\phi(a)) \\
&\leq N^{\tau_X}_x(f_\phi^{-1}((G,B))(x)(a).
\end{aligned}$$

Hence $f_\phi^{\leftarrow}((G,B))(x) \leq N^{\tau_X}_x(f_\phi^{-1}((G,B))$, that is, $f_\phi^{-1}((G,B)) \in \tau_X$.

We similarly prove the following corollary as Theorem 3.4.

Corollary 3.5. $f_\phi : (X,A,N_X) \to (Y,B,N_Y)$ is a continuous soft map iff $f_\phi : (X,B,\tau_{N_X}) \to (Y,B,\tau_{N_Y})$ is a continuous soft map.

Example 3.6. Let $Y = \{h_i \mid i = \{1,...,4\}\}$ with h_i=house and $E_Y = \{e,b,w,c,i\}$ with e=expensive,b= beautiful, w=wooden, c= creative, i=in the green surroundings.

Define operations $\odot$, $\to$ and * on [0,1](called Lukasierwicz structure) by

$$x \odot y = \max\{0, x+y-1\}, x^* = 1-x$$

$$x \to y = \min\{1-x+y, 1\}.$$

Then $([0,1], \odot, \to, ^* 0, 1)$ is a complete residuated lattice (ref.[2,8,28]).

(1) Let $A=\{w,c,i\}\subset E_Y$ and $X=\{h_1,h_2,h_4\}$, (F_1,A) and (F_2,A) be fuzzy soft sets as follows:

(F_1,A)	h_1	h_2	h_4	(F_2,A)	h_1	h_2	h_4
w	0.6	0.3	0.5	e	0.3	0.4	0.2
c	0.6	0.3	0.5	b	0.4	0.3	0.1
i	0.3	0.6	0.5	b	0.2	0.3	0.2

$(F_1\odot F_1,A)$	h_1	h_2	h_4
w	0.2	0	0
c	0.2	0	0
i	0	0.2	0

Define a soft $[0,1]$-topology as follows:

$$\tau_X=\{(0_X,A),(1_X,A),(F_1,A),(F_2,A),(F_1\odot F_1,A)\}.$$

By Theorem 3.2, we can obtain a soft L-neighborhood system $N^{\tau_X}:X\to(L^A)^{S(X\times X,A)}$ as follows:

$$N^{\tau_X}_{h_i}((F,A))=\begin{cases}(1_X,A)(h_i), & \text{if } (F,A)=(1_X,A)\\ (F_1,A)(h_i), & \text{if}(F,A)\geq(F_1,A),\\ (F_2,A)(h_i), & \text{if}(F,A)\geq(F_2,A),(F,A)\not\geq(F_1,A)\\ (F_1\odot F_1,A)(h_i), & \text{if}(F,A)\geq(F_1\odot F_1,A),(F,A)\not\geq(F_2,A),\\ (0_X,A)(h_i), & \text{otherwise.}\end{cases}$$

Put $(F,A)(h_i)=(F(w)(h_i),F(c)(h_i),F(i)(h_i))$. Then

$$(F_1,A)(h_1)=(0.6,0.6.0.3),\quad (F_1,A)(h_2)=(0.3,0.3,0.6),\quad (F_1,A)(h_4)=(0.5,0.5,0.5)$$
$$(F_2,A)(h_1)=(0.3,0.4.0.2),\quad (F_1,A)(h_2)=(0.4,0.3,0.3),\quad (F_1,A)(h_4)=(0.2,0.1,0.2)$$

$$(F_1\odot F_1,A)(h_1)=(0.2,0.2,0),\quad (F_1\odot F_1,A)(h_2)=(0,0,0.2),$$
$$(F_1\odot F_1,A)(h_1)=(0,0,0),$$

(2) Let $B=\{e,b\}\subset E_Y$, (G_1,B) and (G_2,B) be fuzzy soft sets as follows:

(G,B)	h_1	h_2	h_3	h_4
e	0.3	0.6	0.5	0.0
b	0.6	0.3	0.5	0.3

$(G\odot G,B)$	h_1	h_2	h_3	h_4
e	0.0	0.2	0.0	0.0
b	0.2	0.0	0.0	0.0

Define a soft $[0,1]$-topology as follows:

$$\tau_Y=\{(0_X,B),(0_X,B),(G,B),(G\odot G,B)\}.$$

By Theorem 3.2, we can obtain a soft L-neighborhood system $N^{\tau_Y}:Y\to(L^A)^{S(X\times X,B)}$ as follows:

$$N_{h_i}^{\tau_Y}((H,B))=\begin{cases}(1_Y,B)(h_i), & \text{if } (H,B)=(1_Y,B)\\ (G,B)(h_i), & \text{if} (H,B)\geq(G,B),\\ (G\odot G,B)(h_i), & \text{if} (H,B)\geq(G\odot G,B),(H,B)\ngeq(G,B),\\ (0_Y,B)(h_i), & \text{otherwise.}\end{cases}$$

Put $(G,B)(h_i)=(G(e)(h_i),G(b)(h_i))$. Then

$$(G,B)(h_1)=(0.3,0.6),\qquad (G,B)(h_2)=(0.6,0.3),$$
$$(G,B)(h_3)=(0.5,0.5),\qquad (G,B)(h_4)=(0.0,0.3),$$
$$(G\odot G,B)(h_1)=(0.0,0.2),\quad (G\odot G,B)(h_2)=(0.2,0.0),$$
$$(G\odot G,B)(h_3)=(0.0,0.0),\quad (G\odot G,B)(h_4)=(0.0,0.0),$$

(3) Define functions $f:X\to Y$ and $\phi:A\to B$ as follows:

$$f(h_1)=h_2,f(h_2)=h_1,f(h_4)=h_3$$

$$\phi(w)=\phi(c)=e,\phi(i)=b.$$

Then $f_\phi^{-1}((G,B))=(F_1,A)\in\tau$ and $f_\phi^{-1}((G\odot G,B))=(F_1\odot F_1,A)\in\tau$. Hence $f_\phi:(X,A,\tau_X)\to(Y,B,\tau_Y)$ is a continuous soft map. Since

$$M_{f(h_i)}((G,B))(\phi(a))=N_{h_i}(f_\phi^{-1}((G,B)))(a),h_i\in X,a\in A$$

$$M_{f(h_i)}((G\odot G,B))(\phi(a))=N_{h_i}(f_\phi^{-1}((G\odot G,B)))(a),h_i\in X,a\in A$$

Hence $N_{f(h_i)}^{\tau_Y}((H,B))(\phi(a))=N_{h_i}^{\tau_X}(f_\phi^{-1}((H,B)))(a),h_i\in X,a\in A,(H,B)\in S(Y,B)$, that is, $f_\phi:(X,A,N_{\tau_X})\to(Y,B,N_{\tau_Y})$ is a continuous soft map.

REFERENCES

[1] K.V. Babitha, J.J. Sunil, Soft set relations and functions, Comput. Math. Appl. 60 (2010), 1840-1849.

[2] R. Bělohlávek, Fuzzy Relational Systems, Kluwer Academic Publishers, New York, 2002.

[3] N. Cağman, S. Karatas and S. Enginoglu, Soft topology, Comput. Math. Appl. 62 (2011), 351-358.

[4] C.L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1968), 182-190.

[5] F. Feng, X. Liu, V.L. Fotea, Y.B. Jun, Soft sets and soft rough sets, Information Sci. 181 (2011), 1125-1137.

[6] P. Hájek, Metamathematices of Fuzzy Logic, Kluwer Academic Publishers, Dordrecht, 1998.

[7] U. Höhle, S.E. Rodabaugh, Mathematics of Fuzzy Sets: Logic, Topology, and Measure Theory, The Handbooks of Fuzzy Sets Series 3, Kluwer Academic Publishers, Boston, 1999.

[8] A. Kharal, B. Ahmad, Mappings on soft classes, New Math. Nat. Comput. 7 (2011), 471-481.

[9] Y.C. Kim and J.M. Ko, Soft *L*-fuzzy quasi-uniformities and soft *L*-fuzzy topogenous orders, (submit to) J. Intelligent and Fuzzy Systems.

[10] Y.C. Kim and J.M. Ko, Soft implicative *L*-fuzzy interior and closure operators, (submit to) J. Intelligent and Fuzzy Systems.

[11] P.K. Maji, R. Biswas, R.Roy, Soft set theory, Comput. Math. Appl. 45 (2003), 555-562.

[12] P.K. Maji, R. Biswas, R.Roy, An application of soft sets in decision making problems, Comput. Math. Appl. 44 (2002), 1077-1083.

[13] P.K. Maji, R.Roy, A fuzzy set theoretic approach to decision making problems, J. Comput. Appl. 203 (2007), 412-418.

[14] P. Majumdar, S.K. Samanta, Generalised fuzzy soft sets, Comput. Math. Appl. 59 (2010), 1425-1432.

[15] D. Molodtsov, Soft set theory, Comput. Math. Appl. 37 (1999), 19-31.

[16] R. Lowen, Fuzzy neighborhood spaces, Fuzzy Sets and Systems 7 (1982), 165-189.

[17] Z. Pawlak, Rough sets, Int. J. Comput. Inf. Sci., 11 (1982), 341-356.

[18] Z. Pawlak, Rough probability, Bull. Pol. Acad. Sci. Math. 32 (1984), 607-615.

[19] A.A. Ramadan, E.H. Elkordy, Y.C. Kim, Relationships between *L*-fuzzy quasi-uniform structures and *L*-fuzzy topologies, J. Intelligent Fuzzy Sys. 28 (2015), 2319-2327.

[20] A.A. Ramadan, E.H. Elkordy, Y.C. Kim, Perfect *L*-fuzzy topogenous space, *L*-fuzzy quasi-proximities and *L*-fuzzy quasi-uniform spaces, J. Intelligent Fuzzy Sys. 28 (2015), 2591-2604.

[21] S.E. Rodabaugh, E.P. Klement, Topological and Algebraic Structures In Fuzzy Sets, The Handbook of Recent Developments in the Mathematics of Fuzzy Sets, Kluwer Academic Publishers, Boston, Dordrecht, London, 2003.

[22] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl. 61 (2011), 1786-1799.

[23] B. Tanay and M. B. Kandemir, Topological structure of fuzzy soft sets, Comput. Math. Appl. 61 (2011), 2952-2957.

[24] E. Turunen, Mathematics Behind Fuzzy Logic, A Springer-Verlag Co., Heidelberg, 1999.

[25] J. M. Zhan and Y. B. Jun, Soft BL-algebras based on fuzzy sets, Comput. Math. Appl., 59 (2010), 2037-2046.

[26] W.Z. Wu and W.X. Zhang, Neighborhood operator systems and approximations , Information Sciences, 144 (2002), 201-217.

[27] Hu Zhao and Sheng-Gang Li, L-fuzzifying soft topological spaces and L-fuzzifying soft interior operators, Ann. Fuzzy Math. Inform. 5 (2013), 493-503.

[28] Í. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform. 3 (2012), 171-185.