Available online at http://scik.org

J. Math. Comput. Sci. 6 (2016), No. 6, 1133-1144

ISSN: 1927-5307

CHAOS SYNCHRONIZATION USING BACKSTEPPING CONTROL METHOD OF TWO SYSTEMS

AYUB KHAN[1], NET RAM GARG[2], GEETA JAIN[2,*]

[1]Department of Mathematics, Jamia Millia Islamia University, New Delhi, India

[2]Department of mathematics, Maharshi Dyanand University, Rohtak, India

Abstract. The paper Chaos synchronization using Backstepping control method of two systems studied the performance of synchronization between master and slave system by using single controller. The advantage in many application is that only one controller is used no matter how much the dimensions are there in the system to be synchronized.

Keywords: chaos; synchronization; backstepping; numerical simulation.

2010 AMS Subject Classification: 34H10.

I. INTRODUCTION

After the pioneering work on the chaos control [1-2], synchronization attract the wide attention. Generally, two systems are used in synchronization one master as an input system and other is slave as an output System. Synchronization becomes a very active area of interest in the nonlinear science and in the area of applied mathematics and automation engineering [7-13]. Many more application as secure communication [3-6] the topic of synchronization has various application. Different effective methods are used for the different chaotic systems which are based on the different methods Recently backstepping and active control method gain the popularity in the area of synchronization as these methods are more powerful and effective. For strict feedback systems it is effective in global stabilities, tracking, and transient performance. When the key parameters unknown, transformation of many chaotic system into non-autonomous form including Duffing oscillator, Rossler system, Chen system and Chua's circuit,

*Corresponding author

Received December 14, 2015

has been studied and to control these chaotic systems the backstepping control schemes have been employed. In recent year this method has importance in hydraulic servo system [15], backstepping Decision and fractional derivative equation [14]. Among other applications recursive procedure has importance in to demonstrate to control a third-order phase-locked loops.With the design of the controller the backstepping method is effective to choice of lyapunov exponent. Through the transmission of the signal the trajectory of slave system approaches asymptotically to the trajectory of master system which is input system so that the error dynamics converges to zero. When several single oscillators are coupled together then a complicated system is obtained. For the study of these types of oscillators complex variables are used which are more convenient. Based on Lyapunov function for determination of the controllers the backsteeping technique is used and also for synchronize two identical chaotic system. In this paper, between two chaotic systems for achieving the global synchronization we design backstepping control method. This presentation is divided in sections: In Section II, formulation of the problem is introduced.in III. Design for chaos synchronization and methodology is presented. SectionIV, deals with numerical simulation results. Section V, presented finally the simulation results.

II. PROBLEM FORMULATION

Consider the system of the form

$$\dot{x_1} = f_1\,(x_1, x_2)$$
$$\dot{x_2} = f_2\,(x_1, x_2, x_3)$$
$$\dot{x_n} = f_n\,(x_1, x_2, x_3, \ldots\ldots x_n) \qquad (1.1)$$

In the system (1.1) the function f_1 is the linear function and the functions used ($f_2 \ldots\ldots f_n$) are the nonlinear function . The input system (1.1) is considered as an master system.

Now consider the another system which is taken as output system as that is the slave system

$$\dot{y_1} = f_1\,(y_1, y_2)$$
$$\dot{y_2} = f_2\,(y_1, y_2, y_3)$$
$$\dot{y_n} = f_n\,(y_1, y_2, y_3, \ldots\ldots y_n) + u \qquad (1.2)$$

System (1.2) is the output that is slave system of system (1.1) and u is an controller by assuming the appropriate value of the controller between two system that is master and slave synchronization is obtained.

For the two systems (1.1) and (1.2) considering the error dynamical system as

$$e_i = y_i - x_i \qquad (1.3)$$

After subtracting (1.2) and (1.1) the error system becomes

$$\dot{e_1} = h_1 (e_1, e_2)$$

$$\dot{e_2} = h_2 (e_1, e_2, e_3, x_1, x_2, x_3)$$

$$\dot{e_n} = h_n \left(e_1, e_2 , e_3, \dots \dots . e_{n,} x_1, x_2 , x_3, \dots \dots . x_n \right) + u \qquad (1.4)$$

the function h_1 is the linear function and the functions used ($h_2 \dots \dots h_n$) are the nonlinear function with inputs of system (1.1) that is $(x_1, x_2 , x_3, \dots \dots . x_n)$. Now choosing of appropriate value of the controller u is the problem so that drive and response system are synchronized and error vector becomes zero when time is increased. In this paper our objective is achieved by using backsteeping method.

Theorem 1(a):- LaSalle-Yoshizawa theorem

Let $x = 0$ be an equilibrium point of

$$\dot{x} = f (x, u)$$

Let $v(x)$ be a continuous differentiable positive definite and radially unbounded function such that

$$v = \frac{\partial v}{\partial x} f(x, t) \leq -w(x) \leq 0$$

Where w is a continuous function. Then all solution of $\dot{x} = f (x, u)$ are globally uniformly bounded and satisfy $\lim_{t \to \infty} w\left(x(t)\right) = 0$

III. Backstepping design for synchronization of two systems

In this section for chaos synchronization Backstepping method is designed which produces Reliable performance of the control method for chaos synchronization.

$$\dot{x_1} = \propto (x_2 - x_1)$$

$$\dot{x_2} = (\Upsilon - \propto) x_1 - x_1 x_3 + \Upsilon x_2$$

$$\dot{x_3} = -\beta x_3 - \delta x_4 + x_1 x_2$$

$$\dot{x}_4 = -d x_4 + f x_3 + x_1 x_2 \qquad (1.5)$$

And the response system is:-

$$\dot{y_1} = \propto (y_2 - y_1)$$

$$\dot{y_2} = b y_1 - c y_2 - y_1 y_3$$

$$\dot{y_3} = y_1^2 - dy_3 \tag{1.6}$$

$$\dot{y_4} = -y_1 y_3 - \delta y_4 + u$$

Where u is the controller . The given system has the error dynamics as

$$e_i = y_i - x_i$$

$$e_1 = y_1 - x_1$$

$$e_2 = y_2 - x_2$$

$$e_3 = y_3 - x_3$$

$$e_4 = y_4 - x_4 \tag{1.7}$$

And the system

$$\dot{e_1} = \dot{y_1} - \dot{x_1}$$

$$\dot{e_2} = \dot{y_2} - \dot{x_2}$$

$$\dot{e_3} = \dot{y_3} - \dot{x_3}$$

$$\dot{e_4} = \dot{y_4} - \dot{x_4} \tag{1.8}$$

Thus

$$\dot{e_1} = -\propto(x_2 - x_1) + \propto(y_2 - y_1)$$

$$\dot{e_1} = \propto(e_2 - e_1)$$

$$\dot{e_2} = by_1 - cy_2 - y_1 y_3 - (\Upsilon - \propto)x_1 + x_1 x_3 - \Upsilon x_2$$

$$\dot{e_2} = be_1 + x_1(b - \Upsilon + \propto) - ce_2 - x_2(c + \Upsilon) - e_3(e_1 + x_1) - e_1 x_3$$

$$\dot{e_3} = y_1^2 - dy_3 + \beta x_3 + \delta x_4 - x_1 x_2$$

$$\dot{e_3} = e_1^2 + x_1^2 + 2e_1 x_1 - de_3 + x_3(\beta - d) + \delta x_4 - x_1 x_2$$

$$\dot{e_4} = -y_1 y_3 - \delta y_4 + u + dx_4 - fx_3 - x_1 x_2$$

$$\dot{e_4} = -e_3(e_1 + x_1) - x_3(e_1 + f + x_1) - \delta e_4 - x_4(\delta - d) + u - x_1 x_2 \tag{1.9}$$

System (1.9) exist an equilibrium (0,0,0,0) when there is no controller,then the synchronization problem of the drive (input)-response (output) system would be reduced to that of asymptotic stability of system (1.9). Thus, the main aim is to find a controller u such that system (1.9) is stabilized at the origin. The stability of system considered as :

$$\dot{e_1} = \propto(e_2 - e_1)$$

And assuming that $\propto(e_2 - e_1)$ as a virtual control function , for the virtual control $\propto(e_2 - e_1)$ function designed an estimate stabilizing function $\propto_1 e_1$.

Now Lyapunov function is chosen such that

$v_1(e_1)= \frac{1}{2}(e_1{}^2)$

The derivative is

$\dot{v_1}(e_1) = \dot{e_1}e_1$

For $v_1(e_1)$ to be negative definite, then, $\dot{e_1} = -e_1$, so that

$\dot{v_1}(e_1)= -e_1{}^2 < 0$

Thus. $\propto_1 (e_1) = -e_1$, when $\propto(e_2 - e_1)$ is considered as a controller then function $\propto_1 e_1$ is an estimate control function . Let us consider

$$w_2 = e_2 + e_1$$

$$w_2 = e_2 - \propto_1 (e_1)$$

$$e_2 = w_2 - e_1 \tag{1.10}$$

and consider the subspace (e_1, w_2) given by

$$\dot{e_1} = \propto(e_2 - e_1)$$

$$\dot{e_2} = be_1 + x_1(b - \Upsilon + \propto) - ce_2 - x_2(c + \Upsilon) - e_3(e_1 + x_1) - e_1x_3$$

$$\dot{e_2} = e_1(b + c) + x_1 (b - \Upsilon + \alpha) - cw_2 - x_2(c + \Upsilon) - e_3(e_1 + x_1) - e_1x_3 \tag{1.11}$$

As virtual controller in system is $\propto(e_2 - e_1)$ and assume that when $\propto(e_2 - e_1) = \propto_1(e_{1,}w_2)$ system (1.11) is made asymptotically stable. Choose the Lyapunov function

$$v_2(e_1, w_2) = v_1 (e_1) + \frac{1}{2}(w_2{}^2) \tag{1.12}$$

for subspace above. The derivative of (1.12) is given by

$\dot{v_2}(\dot{e_1}, \dot{w_2}) = \dot{v_1}(\dot{e_1}) + w_2\dot{w_2}$

$\dot{v_2}(\dot{e_1}, \dot{w_2}) = -e_1{}^2 - w_2{}^2 + w_2[e_1(b + c) + x_1 (b - \Upsilon + \alpha) - cw_2 - x_2(c + \Upsilon) - e_3(e_1 + x_1)$
$- e_1x_3 + \propto(e_2 - e_1)]$

If $\propto_1(e_{1,}w_2) = -[e_1(b + c) + x_1 (b - \Upsilon + \alpha) - cw_2 - x_2(c + \Upsilon) - e_3(e_1 + x_1) - e_1x_3]$

Then

$\propto(e_2 - e_1) = -[e_1(b + c) + x_1 (b - \Upsilon + \alpha) - cw_2 - x_2(c + \Upsilon) - e_3(e_1 + x_1) - e_1x_3]$

And the subspace

$$\dot{v_2}(e_1, w_2) = -e_1{}^2 - w_2{}^2 < 0 \tag{1.13}$$

This is negative definite. Consider the error dynamics w_3 as

$w_3 = -\propto_2(e_{1,}w_2) + e_3$

Now discuss the full dimension space (e_1, w_2, w_3)

$\dot{e_1} = -[e_1(b + c) + x_1 (b - \Upsilon + \alpha) - cw_2 - x_2(c + \Upsilon) - e_3(e_1 + x_1) - e_1x_3]$

$\dot{e_1} = \propto (e_2 - e_1)$

$\dot{w}_2 = be_1 + x_1\,(b - \Upsilon + \alpha) - cw_2 - x_2(c + \Upsilon) - e_3(e_1 + x_1) - e_1x_3$

$\dot{w}_3 = {e_1}^2 + {x_1}^2 + 2e_1x_1 - de_3 + x_3(\beta - d) + \delta x_4 - x_1x_2$

$\dot{w}_3 = {e_1}^2 + {x_1}^2 + 2e_1x_1 - dw_3 + x_3(\beta - d) + \delta x_4 - x_1x_2$

$\dot{w}_4 = -e_3(e_1 + x_1) - x_3(e_1 + f + x_1) - \delta e_4 - x_4(\delta - d) + u - x_1x_2$

$\dot{w}_4 = -e_3(e_1 + x_1) - x_3(e_1 + f + x_1) - \delta w_4 - x_4(\delta - d) + u - x_1x_2$

Define the error dynamics $w_4$as

$w_4 = w_4 - \propto_3\,(e_1w_3)$

$w_4 = w_4 - (-w_3)$

$\dot{w}_4 = -e_3(e_1 + x_1) - x_3(e_1 + f + x_1) - \delta[{e_1}^2 + {x_1}^2 + 2e_1x_1 - de_3 + x_3(\beta - d) + \delta x_4 + e_3(e_1 + x_1) + x_3(e_1 + f + x_1) + \delta e_4 + x_4(\delta - d) - u] - x_4(\delta - d) + u - x_1x_2$

Choose a Lyapunov function

$v_4(e_1, w_3) = v_3(e_1, w_3) + \frac{1}{2}{w_4}^2$

If

$u = -\frac{1}{(\delta+1)}[e_3(e_1 + x_1) + x_3(e_1 + f + x_1) + \delta[{e_1}^2 + {x_1}^2 + 2e_1x_1 - de_3 + x_3(\beta - d) + \delta x_4 + e_3(e_1 + x_1) + x_3(e_1 + f + x_1) + \delta e_4 + \delta x_4 - \mathrm{d}x_4] + x_4(\delta - d) + x_1x_2]$

Then

$$v_4(e_1, w_3) = -{e_1}^2 - {w_3}^2 - {w_4}^2 < 0 \qquad (1.14)$$

Is a negative definite . and according to LaSalle-Yoshizawa theorem 1(a), the equilibrium (0,0,0,0) remains asymptotically stable and the error dynamics(e_1, e_2, e_3, e_4) will converge to zero as t→ ∞. Thus, the two system are in the synchronized state.

IV. NUMERICAL RESULTS

The values of initials conditions $x(0) = (0.1,0.1,0.1,0)$ and by choosing the values of $(\alpha, \beta, \gamma, \delta, d, f, b, c)$ as (0.5,0,1,1,0.1,0.5,10) two equations (1.5) and (1.6) are solved by using the MATLAB numerically . The error behaviour is shown by Fig 2(a) to 2(b) with time t, shows that the two system are synchronized as error system converges to zero . Between x_i and y_i where $i = 1,2, \dots .4$ the time series of signals is shown by Fig3 (a) to 3(d) . For the system (1.5) and (1.6) the chaotic behaviour is shown by Fig 1.1(a) to 1.1(d) , by choosing the values of $(\alpha, \beta, \gamma, \delta, d, f, b, c)$ as (10,10,$\frac{8}{3}$,10,10,10)

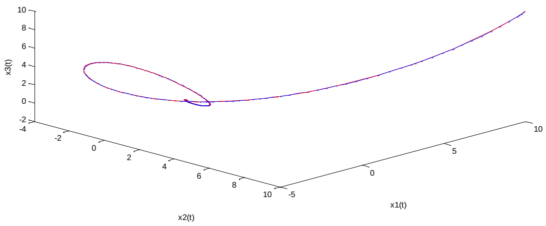

Fig 1.1(a). Chaotic behaviour of master system between x_1, x_2, x_3

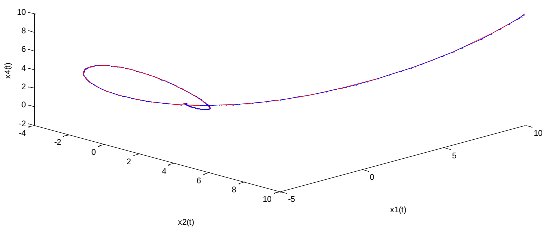

Fig 1.1(b). Chaotic behaviour of master system between x_1, x_2, x_4

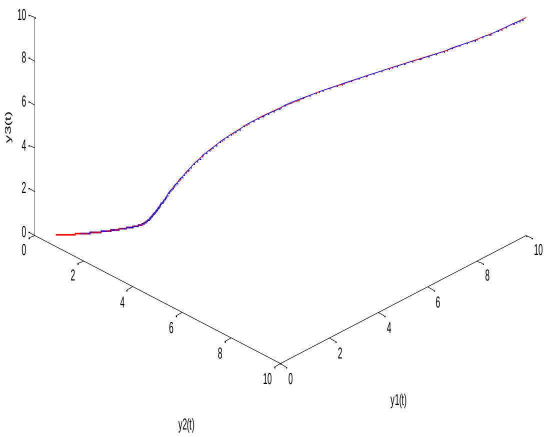

Fig 1.1(c). Chaotic behaviour of slave system between y_1, y_2, y_3

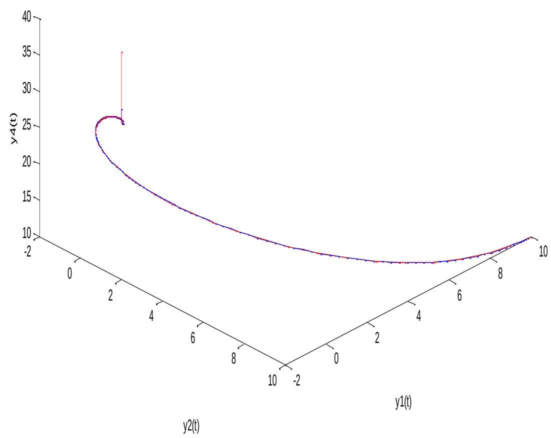

Fig 1.1(d). Chaotic behaviour of slave system between y_1, y_2, y_4

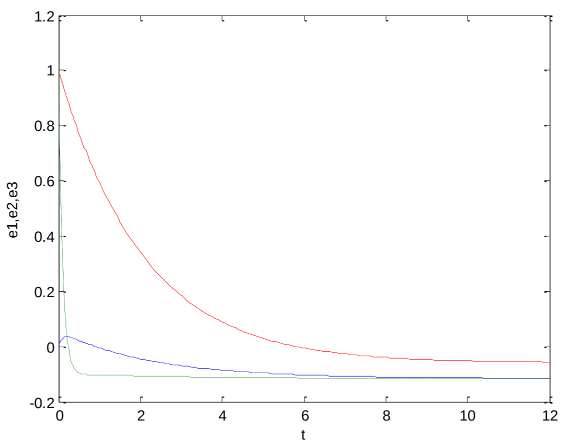

Fig. 2(a). Synchronization between e_1 and e_2, e_3 with time t

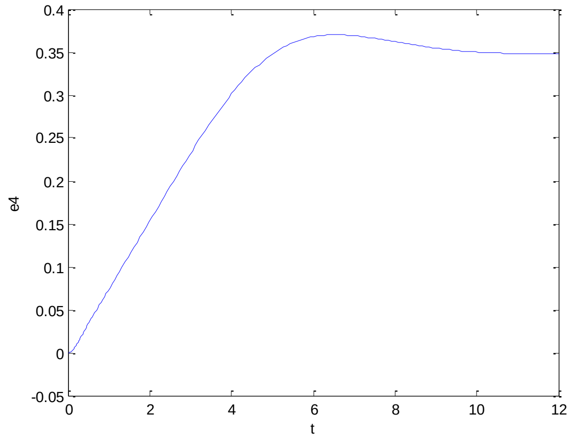

Fig. 2(b). Synchronization between e_4 and time t

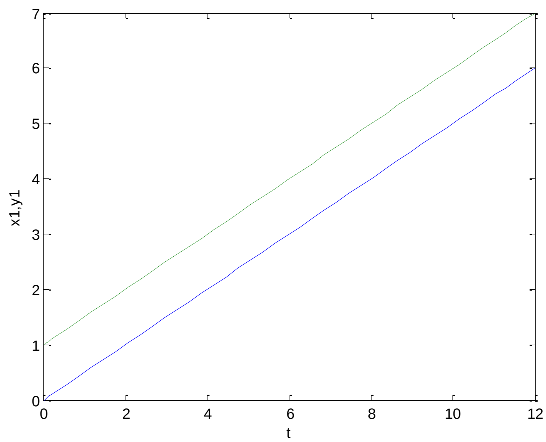

Fig. 3(a). Synchronization between x_1 and y_1 with time t

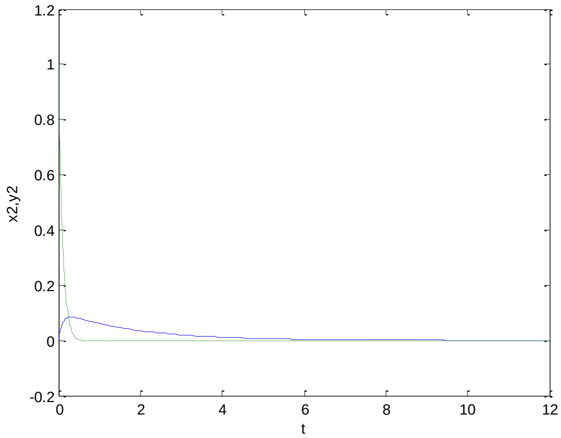

Fig. 3(b). Synchronization between x_2 and y_2 with time t

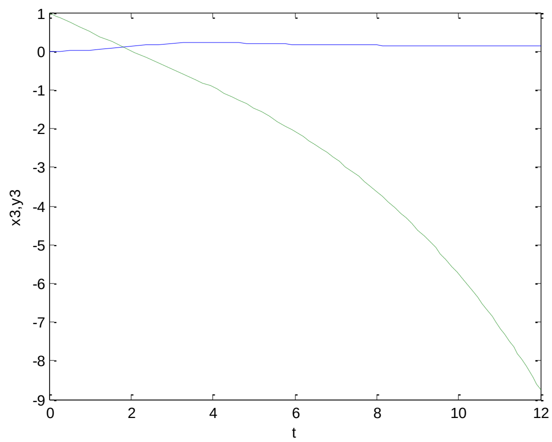

Fig. 3(c). Synchronization between x_3 and y_3 with time t

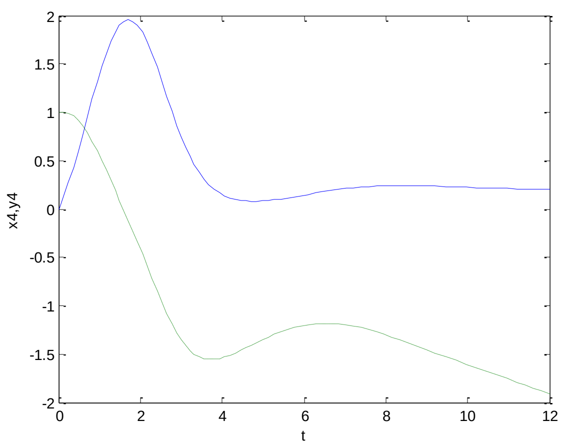

Fig. 3(d). Synchronization between x_4 and y_4 with time t

V. CONCLUSION

Chaos synchronization of two systems is presented in this paper by using Backstepping control method. However, due to the effectiveness of the cost and density with one controller backstepping is effective. The advantage of this procedure is that there is only one controller no matter how much dimension are there in the system to be synchronized.

Conflict of Interests

The authors declare that there is no conflict of interests.

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