Available online at http://scik.org

J. Math. Comput. Sci. 6 (2016), No. 5, 934-942

ISSN: 1927-5307

USING F-TRANSFORM TO REMOVE IMAGE NOISE

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Abstract: This paper is devoted to investigate fuzzy transform from the approximation point of view and to incorporate it as a technique to remove image noise. It will be applied to remove random variation Impulsive Noise. The Numerical Algorithm based on fuzzy transform will be implemented as a user-subroutine in the mathematical code MATLAB. The algorithm will be simulated to remove different ratios of Random Variation Impulsive Noise. An example will be given to show the efficiency of applying F-transform to data processing and results will be compared with Adaptive Wiener Filter.

Keywords: F-transform; Numerical Algorithm; Image processing; Random Variation Impulsive Noise.

2010 AMS Subject Classification: 65N22, 26A33, 41A55, 65B05, 65L05, 65L06, 65D30.

1. Introduction

The main concept of fuzzy set theory was set by L.A. Zadeh [13] provided mathematicians with solid infrastructure for modelling applied physical and engineering problems. The fuzzy transform has been developed as one of the fields depending on the fuzzy approximation models. Fuzzy transform has been proposed and used as an efficient and a pilot approximation technique for approximating numerical solutions for fractional and non-fractional differential equations [1-4].

The fuzzy transform has been introduced for the first time by I. Perfilieva in 2001, since this time it already proved itself in solving many physical, mathematical, financial problems, data compression and reconstruction, image processing and fusion, control systems, … etc. [1-4, 9, 10, 11, 12]. The success of using F-transform in image fusion encouraged us to use it in removing

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Received April 1, 2016

noise from images. In this paper, we developed I. Perfilieva algorithm [12] with f-transform applied to data processing in removing the Random Variation Impulsive Noise.

Image noise is one of the most well-known problems in image data processing field, there are many reasons to get an image with noise, it can be produced by the sensor of a scanner or digital camera. Noise can damage data in the original image, so removing noise is a very important task in image processing. There are many types of noises, each type has a different method to deal with. In this study we will apply the F-transform to remove Random Variation Impulsive Noise.

2. F-Transform

In this section we will only include main definitions of F-transform that will be used in the next sections of numerical implementations of image noise. Main definitions of fuzzy sets and transform, triangular shaped basic functions and Sinusoidal shaped basic functions can be found with details in [1, 3, 10].

Let $[a, b]$ as a common domain $\boldsymbol{\mathcal{D}}$ for all functions in our study, this domain is partitioned into subintervals "**Fuzzy Partition"** to define membership functions to introduce fuzzy sets.

Definition 1[10] Let $x_0 = x_1 < x_2 < \cdots < x_n = x_{n+1}$ be fixed nodes within $\mathcal{D} = [a, b]$ such that $x_1 = a, x_n = b$ and $n \geq 2$. We say that fuzzy sets $A_1, \dots, A_n$ are *basic functions* and form a fuzzy partition of $\boldsymbol{\mathcal{D}}$ if the following conditions hold true for each $i = 1, \dots, n$:

- $A_i: [a, b] \to [0, 1], A_i(x_i) = 1$,
- $A_i(x) = 0$ if $x \notin (x_{i-1}, x_{i+1})$,
- A_i is continuous on $\mathcal{D}$,
- A_i strictly increases on $[x_{i-1}, x_i]$ and strictly decreases on $[x_i, x_{i+1}]$,
- $\sum_{i=1}^{n} \mathbf{A}_\mathrm{i}(\mathrm{x}) = 1,\ \forall \mathrm{x} \in \mathcal{D}$.
    - If the nodes $x_1, \dots, x_n$ are equidistant, then $h = \frac{b-a}{n-1}$,and $x_i = a + h(i - 1),\ \forall\, i = 1, \dots, n$
    - so the following properties are hold:
- $A_i(x_i - c) = A_i(x_i + c), \forall c \in [0, h], i = 2, \dots, n-1, n > 2$,
- $A_{i+1}(x) = A_i(x - h), \forall x \in [a + h, b], i = 2, \dots, n-1, n > 2$,

- $A_i(x_i + c) = A_{i+1}(x_{i+1} - c), \forall c \in [0, h], i = 1, \dots, n-1, n \geq 2.$

The most popular basic functions are **triangular shaped** and **Sinusoidal shaped** [10]

Since in image processing we deal with images as two dimensions matrix, we will introduce the two dimensional forms of direct F-transform and inverse F-transform.

Definition 2 [10] Let a fuzzy partition of $\mathcal{D}^2$ be given by $\{A_1, \dots, A_n\} \times \{B_1, \dots, B_m\}$ and let $f \in \mathcal{C}(\mathcal{D}^2)$. We say that a real matrix $F^2[f] = [F_{ij}]_{n \times m}$ given by

$$F_{ij} = \frac{\int_c^d \int_a^b f(x,y) A_i(x)\, B_j(y) dxdy}{\int_c^d \int_a^b A_i(x)\, B_j(y) dxdy} \tag{1}$$

For $i = 1, \dots, n, j = 1, \dots, m$

is the F-transform of f with respect to the given fuzzy partition. The real number F_{ij} are called the components of the F-transform of f.

And to transform the integral f-transform back we use the inverse F-transform formula

Definition 3 [10] Let $F^2[f]$ be the F-transform of a function $f \in \mathcal{C}(\mathcal{D}^2)$ with respect to a given partition$\{A_1, \dots, A_n\} \times \{B_1, \dots, B_m\}$. Then the function

$$f_{n,m}^F(x,y) = \sum_{i=1}^{n} \sum_{j=1}^{m} F_{ij} A_i(x)\, B_j(y) \tag{2}$$

is called the inverse F-transform of f

3. Random Variation Impulsive Noise

Random Variation Impulsive Noise is also called the Gaussian noise, it is randomly occurs as white intensity values [7].

There are several ways through which noise can be introduced into an image, depending on how the image is created such as Satellite images.

It can be generated by

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Where x represents the gray level, μ the mean value and σ the standard deviation.

4. F-transform Algorithm

To remove the noise from image we will develop the algorithm used in [12] for images fusion, Taking into consideration the difference between the two processes.

Table 1 The developed algorithm for removing image noise

- Add the noise to image
- Compute the inverse F-transform
- Compute the first absolute difference between the image with noise and the inverse F-transform of it
- Compute the second absolute difference between the first one and its inverse F-transform and set them as weights of pixels
- **For** each pixel in the image with noise **do**

  Compute the value of *sow* – the sum of weights over all input images-

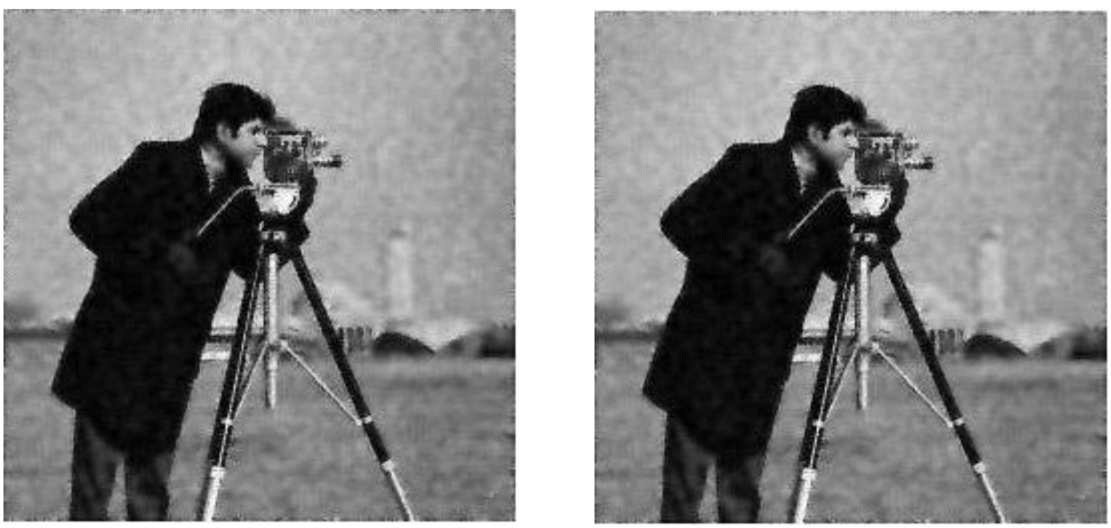
  Compute the value of *wr*- the ratio between the weight of the current pixel and *sow*

  End for
- Remove the noise from the resulting image

5. Example and Results

Fig. 1 The original cameraman image

Fig. 2 The image with 5% noise



(a) Using F-transform (b) Using Adaptive Wiener Filter

Fig. 3 Resulting images

Fig. 4 The image with 10% noise

(a) Using F-transform

(b) Using Adaptive Wiener Filter

Fig. 5 Resulting images

Fig. 6 The image with 20% noise

(a) Using F-transform

(b) Using Adaptive Wiener Filter

Fig. 7 Resulting images

To compare between results from using the F-transform and Adaptive Wiener Filter, we compute the quality of each resulting image by comparing it with the original image, we use the following relation [9].

$$Q(f) = \left(1 - \frac{\sum_{i=1}^{M}\sum_{j=1}^{N}|f(i,j) - I(i,j)|}{\sum_{i=1}^{M}\sum_{j=1}^{N} I(i,j)}\right) \times 100 \quad (3)$$

Where I the original image of size $M \times N$.

Table 2 Variation for different noise rates.

variation	5%	10%	20%
Adaptive Wiener Filter	92.66%	89.81%	86.45%
F-transform	92.72%	89.91%	86.56%

The algorithm in table (1) is implemented as a user subroutine in the mathematical code MATLAB [8]. Fig. (1) shows the original cameraman image, figures (2), (4), and (6) show cameraman image after adding Random Variation Impulsive Noise with rates 5%, 10%, and 20% respectively. Figures (3.a), (5.a), and (7.a) show images after applying F-transform Algorithm. Figures (3.b), (5.b), and (7.b) show images after applying Adaptive Wiener Filter.

As we see in Table 2, results obtained by applying the F-transform gives better results than ones obtained by Adaptive Wiener Filter for different noise rates.

6. Conclusions

In this paper; We developed Perfilieva I. F-transform algorithm for image fusion to be used for removing Random Variation Impulsive Noise. Results obtained were slightly better than using Adaptive Wiener Filter, our results encouraged us to consider the F. transform as a powerful tool for removing noise and to curry out future research with a modified F-transform Algorithm to get better results, and to develop the algorithm to remove other types of noises.

Conflict of Interests

The authors declare that there is no conflict of interests.

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