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DEVELOPMENT OF A DISCRETE SIMULATION MODEL FOR TSUNAMI TIDAL WAVE

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Abstract. In this paper, we develop new discrete models for the numerical simulation of non-linear ordinary differential equation arising from the dynamics of a tidal wave. A new set of non-standard schemes are proposed using the technique of non-local approximation. Both one step and two step formats have been considered. The schemes were found to be suitable for the numerical simulation of the Tsunami equation as proposed.

Keywords: discrete simulation model; Tsunami tidal wave.

2010 AMS Subject Classification: 81T80.

Introduction

One of the most important questions in prognostic tsunami modeling is estimation of tsunami run-up heights at different points along the coastline. Methods for numerical simulation of tsunami wave propagation are well developed and are widely used by a great number of scientists. To date, only a few existing numerical models have met current standards, and these models remain the only choice for use for real-world forecasts

To some earlier modelers, the Tsunamis are assumed to be linear, long gravity waves at single frequencies. Several models that have proven very useful has employed non-linear views. Some of these works include: [1],[2],[3],[4],[5] and[6]. Since numerical applications has proven to produce alternative discrete model that can be used to approximate non-linear equations. We can

apply numerical experiments to simulate various scenario possible for any given dynamical system

A lot of work has been done in the area of numerical modeling of ordinary differential equations. Many of these techniques have been found to be useful for developing discrete solution for different type of equations.

Various researchers have come up with useful models in which the numerical simulations have been applied in the management and study of tidal waves among them are: [7],[8],[9],[10] and[11].

In this work we will use a combination of non-local approximation of the derivatives and the grid point estimates to develop one step and two step Non-standard finite difference schemes for the Tsunami equation.

Non-local approximations and renormalization of the denominator functions have been found to be appropriate for the solution of differential equations. The works of [12] and [13], [14],[15] and [16] have used these techniques extensively to develop discrete models that correctly follow the dynamics of the original differential equation. In many cases the schemes have been found to possess certain desirable qualitative properties like monotonicity of solutions, linear stability and preservation of the properties of the fixed points. The model of non-linear ordinary differential equation proposed in the book of [17] will be used for the numerical experiment. The significant of this numerical model is in the combination of several techniques in one scheme to simulate the original equation. Some denominator functions developed by these authors have been used for the purpose of comparison. These denominator functions have been developed based on the rule 2 of non-standard modeling technique proposed by [12].

2. The Tsunami model (Gill and Cullen 2005)

The Tsunami Model is given by

$$\frac{dy}{dx} = y\sqrt{4 - 2y} \quad (1)$$

$y(x) > 0$ is the height of the wave expressed as a function of its position relative to a point offshore. This is one of the simplest model for a tidal wave considering the complex dynamics of the phenomena being model. The function $y(x)$ has a lot of underlining assumption and as such in reality may possess more complex properties. The equilibrium points of this equation are $y= 0$ and $y=2$; and

if $y_0 = 2$ the analytic solution is $y(x) = 2\text{sech}^2x$ Otherwise $y(x) = 2\text{sech}^2(x - c)$ (2)

We will develop new schemes that possess the same qualitative properties as that of this differential equation.

3. Derivation of the Method

We will apply rule2&3 (see [12]) and their extensions in [14] to each of the components of the equation as shown below

Rule 2 (Mickens 1994)

Denominator function for the discrete derivatives must be expressed in terms of more complicated function of the step-sizes than those conventionally used. This rule allows the introduction of complex analytic function of h that satisfy certain conditions in the denominator . It must be stated here that the selection of an appropriate denominator is an ‘art’ (Mickens 1999). Close examinations of differential equation, for which the exact schemes are known, shows that the denominator function generally are functions that are related to particular solutions or properties of general solution to the differential equation. This therefore places great importance on the necessity of the modeler to obtain as much analytic knowledge as possible about the differential equation. A lot of such denominator functions have been developed by this author and many others, such will be used directly here.

Rule 3 (Mickens 1994)

The non-linear terms must in general be modeled (approximated) non-locally on the computational grid or lattice in many different ways.

Application of the combination of these two rules will give us the following transformations

$$\frac{dy}{dx} \equiv \frac{(y_{k+1} - y_k)}{\psi} \quad \text{where } \psi(h) \rightarrow h + 0(h^2) \text{ as } h \rightarrow 0 \quad (3)$$

$$\frac{dy}{dx} \equiv \frac{(y_{k+1} - \beta y_k)}{\psi} \quad \text{where } \psi(h) \rightarrow h + 0(h^2), \beta(h) \rightarrow 1 \text{ as } h \rightarrow 0 \quad (4)$$

$$\frac{dy}{dx} \equiv \frac{(y_{k+1} - \beta y_{k-1})}{2\psi} \quad \text{where } \psi(h) \rightarrow h + 0(h^2), \beta(h) \rightarrow 1 \text{ as } h \rightarrow 0 \quad (5)$$

The following non-local approximations

$$y_{k+1} \equiv \frac{(y_{k+1} + \beta y_k)}{2} \quad \text{where } \beta(h) \rightarrow 1 \text{ as } h \rightarrow 0 \quad (6)$$

$$y_{k+1} \equiv \frac{(2y_k + \beta y_{k-1})}{3} \quad \text{where } \beta(h) \rightarrow 1 \text{ as } h \rightarrow 0 \quad (7)$$

$$y_{k+1} \equiv \frac{(y_k y_k)}{y_{k-1}} \quad (8)$$

$$y_{k+1} \equiv ay_{k+1} + by_k \quad a + b = 1 \quad (9)$$

Sample renormalisation functions to be employed are

$$\psi = \sin(\alpha h), \quad \alpha \in \mathbb{R} \quad \rightarrow h + O(h^2) \quad \text{as } h \rightarrow 0 \quad (10)$$

$$\psi = \frac{(e^{\lambda h} - 1)}{\lambda}, \quad \lambda \in \mathbb{R}, \quad \rightarrow h + O(h^2) \quad \text{as } h \rightarrow 0 \quad (11)$$

$$\beta = \cos(\alpha h), \quad \alpha \in \mathbb{R} \quad \rightarrow 1 \quad \text{as } h \rightarrow 0 \quad (12)$$

4. Derivation of the schemes

$$\frac{dy}{dx} = y\sqrt{4 - 2y} \quad (\text{Zill \&Cullen 2005}) \quad (13)$$

One step Schemes

Scheme A

Applying non-local approximation to grid points using the transformation equations (3) and (9) in (1)

we have the following

$$\frac{y_{k+1} - y_k}{\psi} = (ay_{k+1} + by_k)\sqrt{(4 - 2y_k)} \quad (14)$$

$$y_{k+1} = y_k + (a\psi y_{k+1} + b\psi y_k)\sqrt{(4 - 2y_k)} \quad (15)$$

$$y_{k+1} = \frac{y_k(1+b\psi)\sqrt{(4-2y_k)}}{(1-a\psi)\sqrt{(4-2y_k)}} \quad (16)$$

We can choose any ψ to form schemes of the form

$$y_{k+1} = \frac{y_k(1+b\psi)\sqrt{(4-2y_k)}}{(1-a\psi)\sqrt{(4-2y_k)}}, \quad \psi = h, \quad \beta = \cos(\alpha h), \quad \lambda, \alpha \in \mathbb{R} \quad \text{Scheme A1} \quad (17)$$

$$y_{k+1} = \frac{y_k(1+b\psi)\sqrt{(4-2y_k)}}{(1-a\psi)\sqrt{(4-2y_k)}}, \quad \psi = \frac{(e^{\lambda h} - 1)}{\lambda}, \quad \beta = \cos(\alpha h), \quad \lambda, \alpha \in \mathbb{R} \quad \text{Scheme A2} \quad (18)$$

$$y_{k+1} = \frac{y_k(1+b\psi)\sqrt{(4-2y_k)}}{(1-a\psi)\sqrt{(4-2y_k)}}, \quad \psi = \sin(h), \quad \beta = \cos(\alpha h), \quad \lambda, \alpha \in \mathbb{R} \quad \text{Scheme A3} \quad (19)$$

Scheme B

Applying non-local approximation to derivative using the transformation equations (3) in (1)

we have the following

$$\frac{y_{k+1}-\beta y_k}{\psi} = \sqrt{y_k^2(4-2y_k)} \quad (20)$$

$$y_{k+1} = \beta y_k + \psi y_k \sqrt{(4-2y_k)} \quad (21)$$

We can choose any ψ to form schemes of the form

$$y_{k+1} = \beta y_k + \psi y_k \sqrt{(4-2y_k)}, \quad \psi = h, \beta = \cos(\alpha h), \lambda, \infty \in \mathbb{R} \quad \textbf{Scheme B1} \quad (22)$$

$$y_{k+1} = \beta y_k + \psi y_k \sqrt{(4-2y_k)}, \quad \psi = \frac{(e^{\lambda h}-1)}{\lambda}, \beta = \cos(\alpha h), \lambda, \infty \in \mathbb{R} \quad \textbf{Scheme B2} \quad (23)$$

$$y_{k+1} = \beta y_k + \psi y_k \sqrt{(4-2y_k)}, \quad \psi = \sin(h), \beta = \cos(\alpha h), \lambda, \infty \in \mathbb{R} \quad \textbf{Scheme B3} \quad (24)$$

The direct substitution of a normalized denominator to replace h in the standard

Finite Difference Scheme $y_{k+1} = y_k + hf(x_k, y_k)$, will result in the simple scheme given below in (25).

This scheme will not involve the application of any other Nonstandard modeling rule except the replacement of the denominator.

$$y_{k+1} = y_k + \psi y_k \sqrt{(4-2y_k)}, \quad \psi = \frac{(e^{\lambda h}-1)}{\lambda} \quad \textbf{Scheme (DIRECT)} \quad (25)$$

Two Step Schemes

Method I

Scheme C

Applying non-local approximation to the original differential equation using the transformation equations (5) in (1) we obtain the following

$$\frac{y_{k+1}-\beta y_{k-1}}{2\psi} = \sqrt{y_k^2(4-2y_k)} \quad (26)$$

$$y_{k+1} = \beta y_{k-1} + (2\psi y_k) \sqrt{(4-2y_k)} \quad (27)$$

We can choose any ψ to form schemes of the form

$$y_{k+1} = \beta y_{k-1} + (2\psi y_k) \sqrt{(4-2y_k)}, \quad \psi = h, \beta = \cos(\alpha h), \lambda, \infty \in \mathbb{R} \quad \textbf{Scheme C1} \quad (28)$$

$$y_{k+1} = \beta y_{k-1} + (2\psi y_k) \sqrt{(4-2y_k)}, \quad \psi = \frac{(e^{\lambda h}-1)}{\lambda}, \beta = \cos(\alpha h), \lambda, \infty \in \mathbb{R} \quad \textbf{Scheme C2} \quad (29)$$

$$y_{k+1} = \beta y_{k-1} + (2\psi y_k) \sqrt{(4-2y_k)}, \quad \psi = \sin(h), \beta = \cos(\alpha h), \lambda, \infty \in \mathbb{R} \quad \textbf{Scheme C3} \quad (30)$$

Note that $\psi = h$ is the standard denominator function in Finite difference method

We will also compute values for the scheme

Definition 1: An initial value problem of a first order ODE can be represented as follows:

$$y' = f(t, y), \quad y(t_0) = y_0 \quad (31)$$

where y_0 is the value of y at time t_0

It is common fact to write the functional dependence y_{n+1} on the quantities x_n, y_n and h in the form;

$$y_{n+1} = y_n + h \varphi(x_n, y_n; h) \quad (32)$$

Where $\varphi(x_n, y_n; h)$ is called the increment function.

Let us denote y_k the approximate solution of (31) at grid point $t_k : y_k = y(t_k)$ then the sequence y_k is obtained as a solution to a finite difference equation of the form (32). Lets denote (32) by the sequence $y_k = F(h ; y_k)$ (33)

Definition 2: (Anguelov and Lubuma(2003).) Assume that the solution of equation (31) satisfy some property \mathcal{P} , The numerical scheme(32) is called qualitatively stable with respect to property \mathcal{P} or \mathcal{P} -stable, if for every value $h>0$ the set of solutions of (32) satisfies \mathcal{P} .

Definition 3: (Anguelov and Lubuma(2003)) A set $G(\Omega)$ of real-valued functions defined on a subset Ω of $[t_0, \infty)$ monotonically depend on the initial value(at t_0) if for every two functions $y, z \in G(\Omega)$ we have $y(t_0) \leq z(t_0) \Rightarrow y(t) \leq z(t), t \in \Omega$ (34)

Definition 4: (Anguelov and Lubuma(2003): The finite difference scheme(33) is stable with respect to the property of monotonicity of solutions if for every $y_0 \in \mathbb{R}$ the solution y_k of (33) is an increasing or a decreasing sequence according as the $y(t)$ of equation (31) is increasing or decreasing.

Definition 5: (Anguelov and Lubuma(2003): The finite difference method (32) is called elementary stable if for any value of the step size h , its only fixed points \bar{y} are those of the differential equation (31), the linear stability property of each \bar{y} being the same for both the differential equation and the discrete method.

The following theorems establish the conditions (sufficient) for the stability properties of the discrete equation (33). Proves of the theorems can be found in (Anguelov and Lubuma(2003)) . The authors have been able to prove the condition for stability of the fixed points and link the properties of linear stability to elementary stability of the fixed points.

Theorem 1: (Anguelov and Lubuma(2003): The difference Scheme (32) is stable with respect to monotone dependence on initial value if

$$\frac{\partial F}{\partial y}(h; y) \geq 0, y \in \mathbb{R}, h > 0 \quad (35)$$

Theorem 2: (Anguelov and Lubuma(2003): Assume that the difference scheme (32) is stable with respect to monotone dependence on initial value. Assume also that for every $h > 0$ the equations

$$y = F(h; y) \text{ and } f(y) = 0 \quad (36)$$

in y have the same roots considered their multiplicity. Then the difference scheme (33) is stable with respect to monotonicity of solutions.

Theorem 3: (Anguelov and Lubuma(2003)): Under the assumptions of theorem (2) the difference scheme (32) is elementary stable.

In the next subsection section we shall use the above theorems (1 –3) to establish the stability or otherwise of the non-exact schemes developed for the Logistic and Combustion equations. Please note that a major advantage of having an exact scheme for a differential equation is that questions related to the usual considerations of consistency, stability and convergence do not arise (see Mickens 1994).

In this Section we show that our schemes satisfy the sufficient condition for the stability properties described above.

Stability of Scheme A with respect to monotonicity of solutions.

$$y_{k+1} = \frac{y_k(1+b\psi)\sqrt{(4-2y_k)}}{(1-a\psi)\sqrt{(4-2y_k)}} \quad (37)$$

$$y = F(h, y) = \frac{y_k(1+b\psi)\sqrt{(4-2y_k)}}{(1-a\psi)\sqrt{(4-2y_k)}} \quad (38)$$

$$y = F(h, y) = \frac{y(1+b\psi)\sqrt{(4-2y)}}{(1-a\psi)\sqrt{(4-2y)}} \quad (39)$$

$$y(1-a\psi)\sqrt{(4-2y)} - y(1+b\psi)\sqrt{(4-2y)} = 0 \quad (40)$$

Have roots 0 and 2

Let $y \neq 0$ and $y \neq 2$

let $a + b = 1$

$$(1-a\psi)\sqrt{(4-2y)} = (1+b\psi)\sqrt{(4-2y)} \quad (41)$$

$$b\psi = -a\psi$$

$$b = -a \quad (42)$$

This contradicts the assumption of selecting parameters a, b s. t. $a + b = 1$

Hence the only roots of $y = F(h, y)$ is 0 and 2

$$f(y) = y\sqrt{(4-2y)} \text{ have roots 0 and 2}$$

The conditions of theorem.... are satisfied for all scheme A

We can choose our parameters on

Stability of Scheme B&C with respect to monotonicity of solutions.

$$y_{k+1} = \beta y_k + \psi y_k \sqrt{(4 - 2y_k)} \quad (43)$$

$$y = F(h, y) = \beta y + \psi y \sqrt{(4 - 2y)} \quad (44)$$

$$y - \beta y - \psi y \sqrt{(4 - 2y)} = 0 \quad \text{have root } 0$$

It will have root 2 iff $\beta = 1$

Hence if $\beta = \cos(h)$ and $h \neq 0$

$$\text{and } f(y) = y \sqrt{(4 - 2y)} \quad \text{have roots } 0 \text{ and } 2$$

The condition of theorem 2 is not satisfied we cannot conclude on the property of monotonicity of solutions.

However the Scheme given by

$$y_{k+1} = y_k + \psi y_k \sqrt{(4 - 2y_k)} \quad (45)$$

Clearly satisfy the conditions of theorem 2 and we conclude that the scheme Direct has the property of monotonicity of solutions

The above also confirm Elementary Stability as stated in Theorem 3.

Stability of Scheme A with respect to monotone dependence on initial values of solutions.

$$\frac{\partial F}{\partial y}(h; y) \geq 0$$

$$F(h, y) = \frac{y(1+b\psi)\sqrt{(4-2y)}}{(1-a\psi)\sqrt{(4-2y)}} \quad (46)$$

Let $\alpha = (1 + b\psi)$ and $\beta = (1 - a\psi)$

$$\Rightarrow F(h, y) = \frac{y\alpha\sqrt{(4-2y)}}{\beta\sqrt{(4-2y)}}$$

$$\frac{\partial F}{\partial y}(h; y) = -\alpha \beta y - \beta[-\alpha y + \alpha(4 - 2y)] \quad (47)$$

$$= -\alpha \beta + \beta \alpha y - \alpha \beta(4 - 2y)$$

$$\Rightarrow -\alpha \beta(4 - 2y) \geq 0 \quad (48)$$

$$\Rightarrow -\alpha \beta \geq 0$$

$$\Rightarrow -(1 + b\psi)(1 - a\psi) \geq 0 \quad (49)$$

$$\text{Suppose } (1 + b\psi) \leq 0 \text{ AND } (1 - a\psi) \geq 0 \quad (50)$$

$$\begin{aligned}
&\Rightarrow 1 + (1 - a)\psi \leq 0 \quad a + b = 1 \\
&\Rightarrow (1 - a)\psi \leq -1 \\
&\Rightarrow 1 - a \leq -\frac{1}{\psi} \\
&\Rightarrow a \geq 1 + \frac{1}{\psi} \quad \text{and} \quad b \leq -\frac{1}{\psi}
\end{aligned} \tag{51}$$

Suppose $(1 + b\psi) \geq 0$ AND $(1 - a\psi) \leq 0$

$$\Rightarrow a \geq \frac{1}{\psi} \quad \text{and} \quad b \leq 1 - \frac{1}{\psi} \tag{52}$$

The condition for theorem is satisfied with equation (50) and the second condition is contained in the first for $\psi > 0$

Stability of Scheme B&C with respect to monotone dependence on initial values of solutions.

$$F(h, y) = \beta y + \psi y \sqrt{(4 - 2y)} \tag{53}$$

$$\frac{\partial F}{\partial y}(h; y) \geq 0$$

$$\frac{\partial F}{\partial y}(h; y) = \beta - \frac{\psi y}{\sqrt{(4 - 2y)}} \geq 0 \tag{54}$$

Let $y \neq 0$ and $y \neq 2$ and $y > 0$

$$\Rightarrow \beta \geq \frac{\psi y}{\sqrt{(4 - 2y)}}$$

$$\Rightarrow \beta \sqrt{(4 - 2y)} \geq \psi y \tag{55}$$

Taking the positive root, the above is true for $0 < y < 2$, $\beta = \cos(h)$, ψ as defined

$$\frac{\partial F}{\partial y}(h; y) \geq 0, \text{ for } 0 < y < 2$$

This is also true when $\beta = 1$

$$\Rightarrow \text{The Direct Scheme } y_{k+1} = y_k + \psi y_k \sqrt{(4 - 2y_k)} \tag{56}$$

Satisfies

$$\frac{\partial F}{\partial y}(h; y) \geq 0, \text{ for } 0 < y < 2$$

NUMERICAL EXPERIMENT

The schemes have been tested using step size $h=0.01$ and for about 100 iterations. The result of the numerical simulation is here presented in 3D graphs

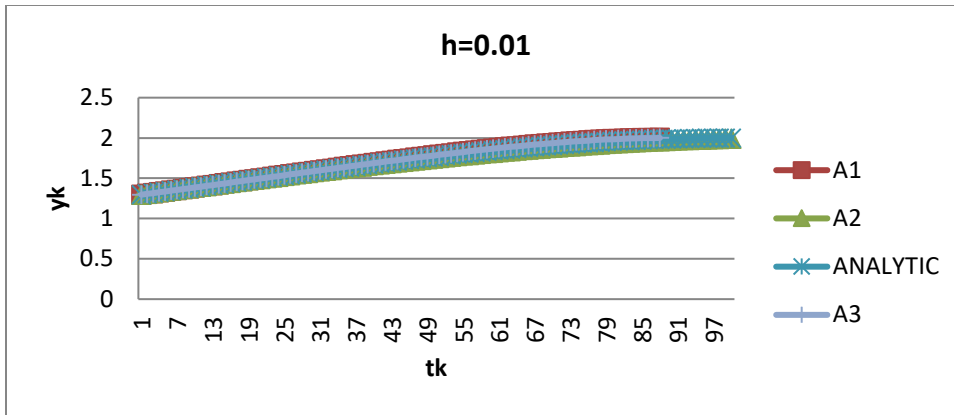


Fig 1: Graph of scheme A with the Analytic solution

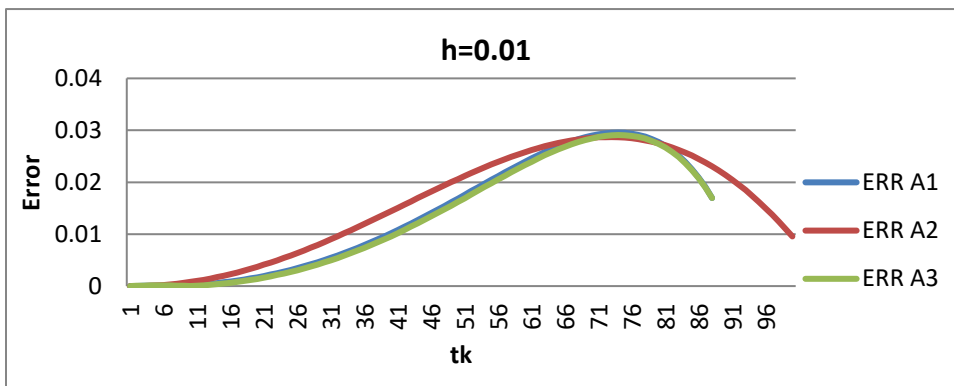


Fig 2: Graph of Error of deviation of scheme A from the Analytic solution

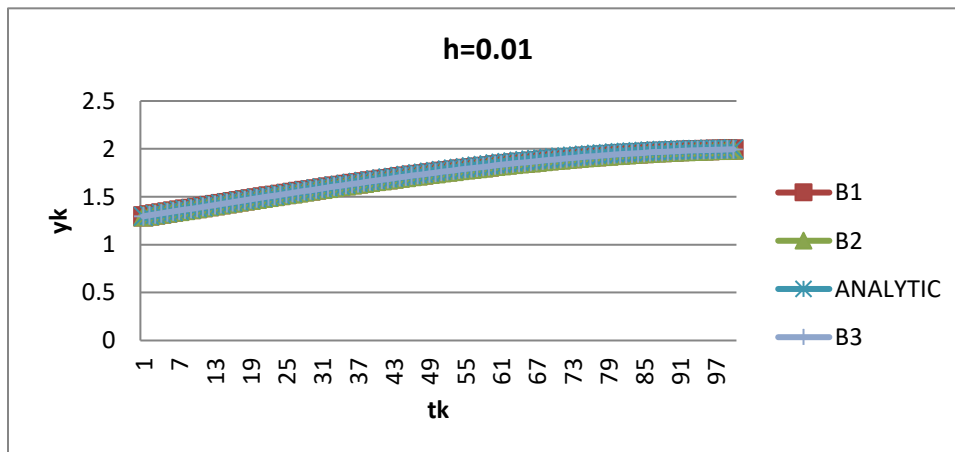


Fig 3: Graph of scheme B with the Analytic solution

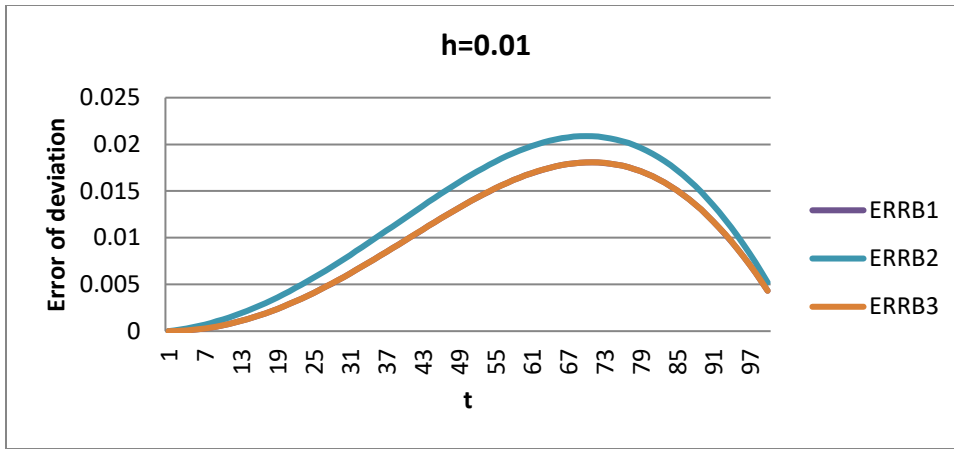


Fig 4: Graph of Error of deviation of scheme B from the Analytic solution

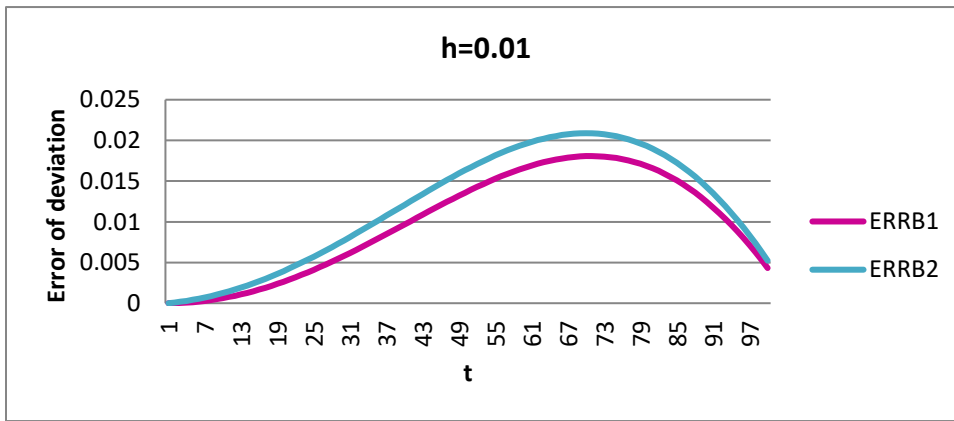


Fig 5: Graph of Error of deviation of scheme B1 and B2 from the Analytic solution

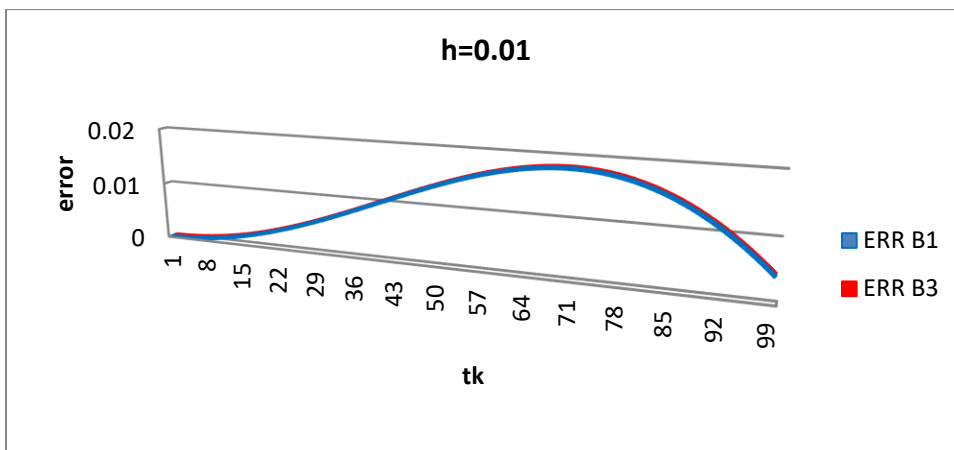


Fig 6: Graph of Error of deviation of scheme B1 and B2 from the Analytic solution

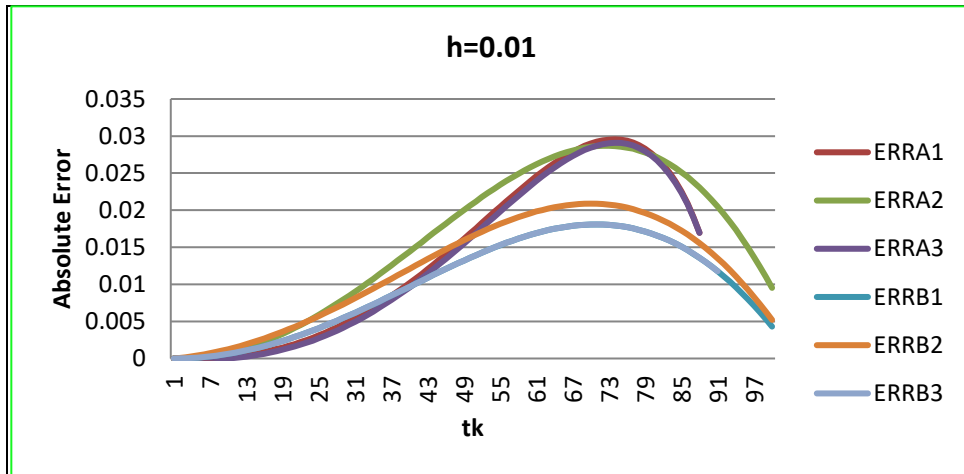


Fig 7: Graph of Error of deviation of scheme A and B from the Analytic solution

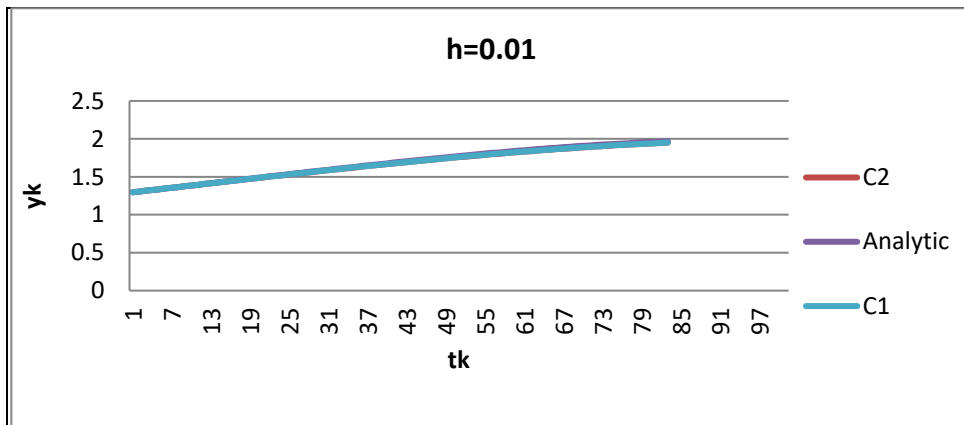


Fig 8: Graph of scheme C AND the Analytic solution

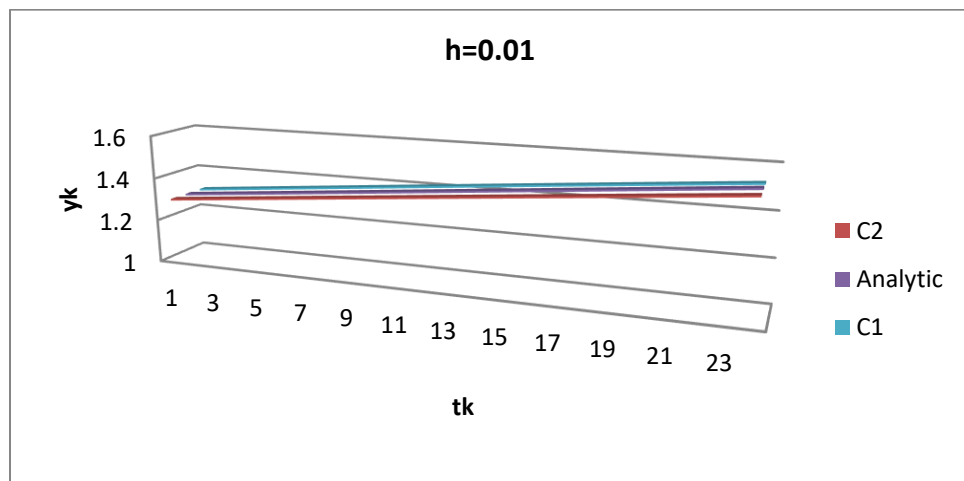


Fig 9: Graph of scheme C AND the Analytic solution (25 Iterations)

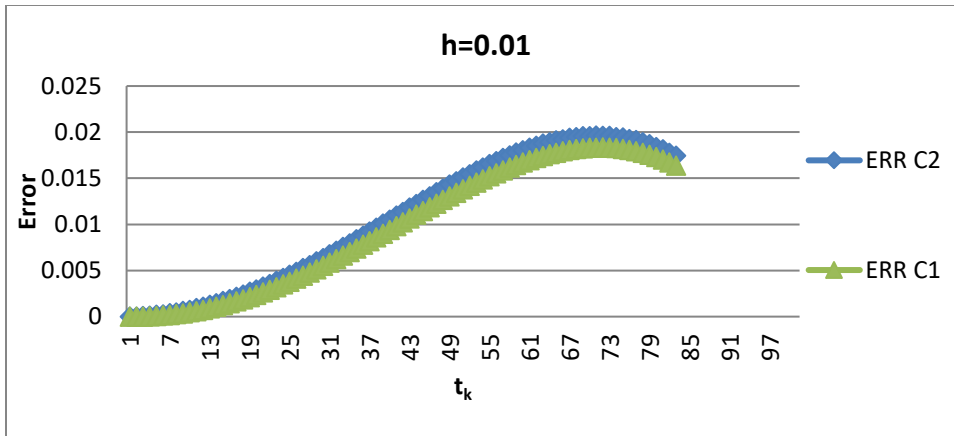


Fig 10: Graph of Error of deviation of scheme C from the Analytic solution

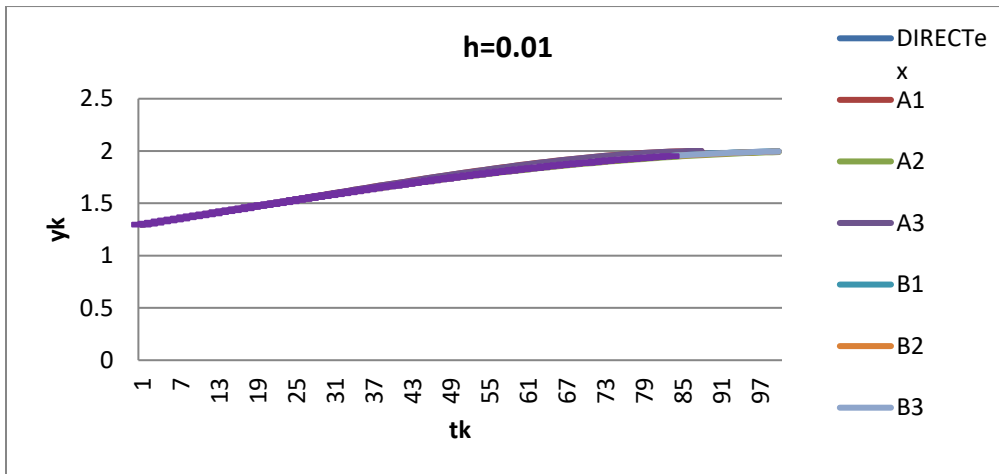


Fig 11: Graph of scheme A,B,C, Scheme Direct AND the Analytic solution

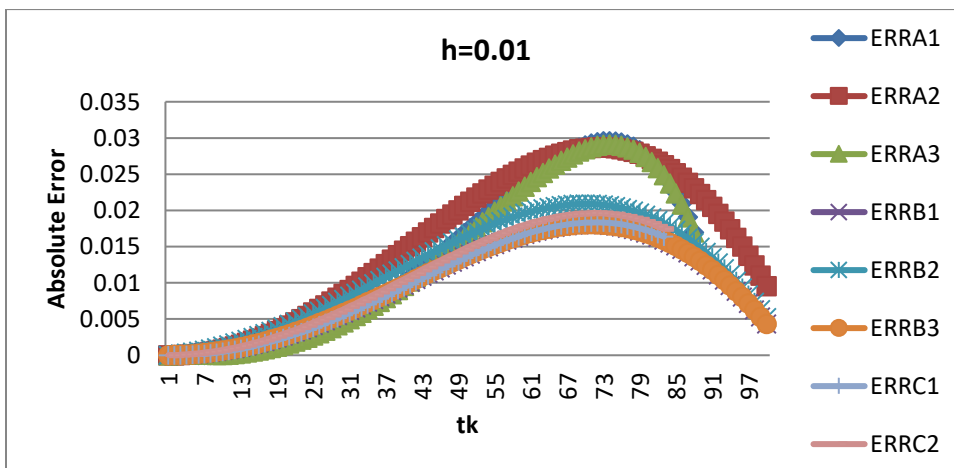


Fig 12: Graph of Error of deviation of scheme A, B and C from the Analytic solution

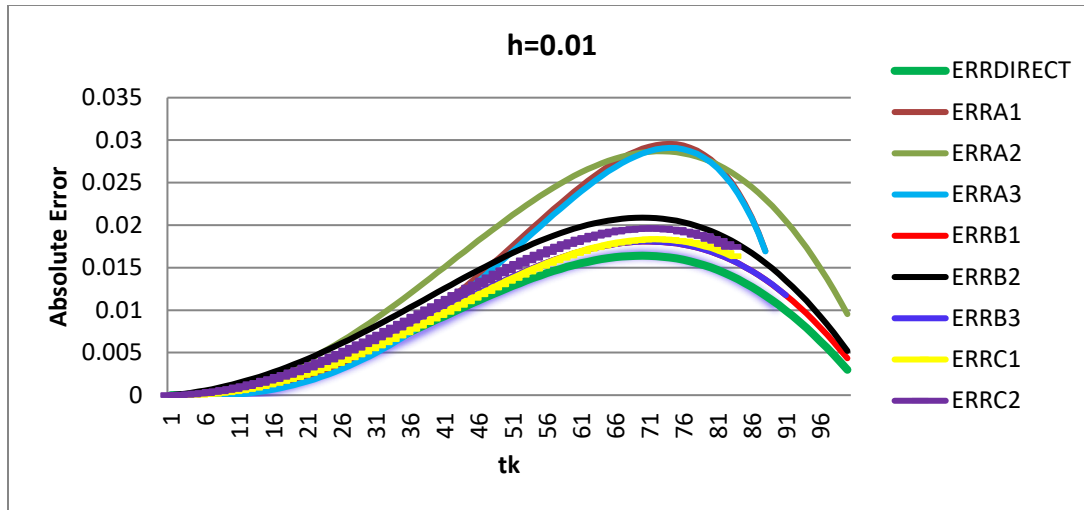


Fig 13: Graph of Error of deviation of scheme Direct, A, B & C from the Analytic solution

DISCUSSION AND CONCLUSION

We observe that the one step Finite Difference schemes (A and B) performed better as a nonstandard scheme because the curves followed correctly the monotonicity of the solution of the original equation (see fig 2,3,4 and 13). The schemes of B and Scheme (DIRECT), also produce the smallest absolute error of deviation (see fig 5,7,13). We also observed that the two step scheme (C) also perform well like the schemes (A and B) (see fig 11 and 12) but the scheme requires a strong predictor for the back steps here we have used the analytic solution as a predictor in which is not always available in practice. Runge kutta method of order 4 can however be used as such predictor. We also observed that the Schemes of C breaks down after few iterations. The schemes B perform better than A interms of absolute error of deviation (see fig 3-7), this confirms earlier results that the combination of both non-local approximation and renormalization of denominator function produces better schemes than use of any of these techniques alone. The scheme C3 was found to be unsuitable for this model because the use of denominator function $\psi = \sin(h)$ has produced solution that are very inconsistent with the original equation even when combined with non-local approximations. Surprisingly the direct substitution of h with normalized denominator function perform better than any of the other schemes in the long run (see the errors of scheme (DIRECT) marked green in fig. 13). This Scheme also possess the three stability properties examined. This is a

pointer to the fact that the application of a combination of the rules does not necessarily lead to a better scheme.

We can however conclude that the schemes are suitable for the simulation of the Tsunami model as proposed.

Conflict of Interests

The authors declare that there is no conflict of interests.

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