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PREDICTING THE VALUE OF AN OPTION BASE ON AN OPTION PRICE

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Abstract: The movement of the price of the stock (up or down) has a direct –although not equal- effect on the price of the option. As the price of a stock rises, the more likely the price of a call option rise and the price of a put will fall. The value of an option (and hence value the portfolio of an investor) decreases (or increases depending whether option is call or put) as its expiration date approaches and becomes worthless (or full of gain). The value of an option consists of intrinsic values. A careful projection of the value of options starting a few from a few years in the past up to the present will be a good way of predicting in future the portfolio of investor. In this paper, we predict the value of an investor's portfolio by solving a second order linear partial differential equation. We first describe the evolution of price of an option as stochastic volatility based and further derived a model equation for predicting the values of option based on the price.

Keywords: predicting value, investor's portfolio, option price, partial differential equation.

2000 AMS Subject Classification: 91B30, 98B28.

1. Introduction

Stock prices sky rocket with little reason, then plummet just as quickly and investors are frightened. Viewed by some authors as a sequence of temporary equilibra, (Follmer [5]) stock prices fluctuate widely in marked contrast to the stability of bank deposits or bonds; affecting not only the individual investor or household but also the economy on a large scale. Following Black and Sholes ([2], [3]) many authors have reached a significant plateau in the

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modelling of stock price dynamics. Some authors have priced options on stocks with stochastic volatility (Stein and Stein [12] and Lediot [8])others have determined the equilibrium price and the market growth rate assets (Ugbebor [12]).Others have applied the operation splitting method for pricing American option (Ikonen and Toivanen [7]).

In this paper instead, we aim at predicting the value of an option given the present price. It is nearly impossible to predict this value exactly, but we present herein a model which can predict an option value in future by solving the generalized Black-Sholes' pde for stocks already priced in the market. Furthermore, given different prices for different options we present a model for predicting in future the capacity output (or portfolio) of an investor with $i = 1,2, \ldots,$ investments by the method in Adagba [1] in solution of formulated second order linear partial differential equation.

2. Formulation of Problem

The work ofWentzel and Friedlin[13] has interesting implications for discussion of dynamics of gradient system. They considered such problems when the system is perturbed by a Wiener process: consider the model

$$dS = -\nabla F(S)dt + \sigma dW, \qquad (1)$$

where $S \in R^1$ is the price process and dW is white noise. This equation can be analysed in R^1 like the Markov chain (Hoppensteadt [6]).

The probability distribution function $\emptyset(S, t)$, describes the probability of the solution being near the point S at time t. This function is known to satisfy the Kolmogorov equation

$$\frac{\partial\emptyset}{\partial S} = \frac{\partial}{\partial S}\left(\frac{\sigma^2\partial\emptyset}{2\partial S} + \emptyset\nabla F\right). \qquad (2)$$

The equilibrium distribution is obtained by solving

$$\frac{\sigma^2\partial\emptyset}{2\partial S} = -\emptyset\nabla F,$$

and solution given as

$$\emptyset(S) = exp\left(-\frac{2F(S)}{\sigma^2}\right).$$

If σ is small, then the probability of being near S will be largest at values of S where $F(S)$ is smallest. The minima of F appears as the most likely places for the solution to be residing.

In order to find $F(S)$, assume $S_t$follows instead the Orntsein-Uhlenbeck process,

$$dS_t = -aS_t dt + \sigma dW_{t,} \tag{3}$$

with explicit solution

$$S_t = e^{-at}S_o + \sigma e^{-at}\int_o^t e^{-aX}\, dW_X. \tag{4}$$

Applying the Duhammel principle, equation (4) has a Gaussian distribution with mean $e^{-at}S_0$ and variance given by

$$\sigma^2\,(t) = \sigma^2 e^{-2at}\int_0^t e^{2aX}\, dX$$

$$= \frac{\sigma^2 e^{-2at}}{2a}\left[e^{2aX} + 1\right]_0^t$$

$$= \frac{\sigma^2}{2a}\left[1 + e^{-2at}\right] \tag{5}$$

Hence (5) has a markov process with stationary transition probability densities

$$F(t,S,y) = \frac{1}{\sigma(t)\sqrt{2\pi}}exp\frac{\left[-(y-e^{-at}S\right]}{2\sigma^2(t)}. \tag{6}$$

This is particularly interesting for $a > 0$(say a= 1), which is the stable case.

$$\alpha = \lim_{t\to\infty}\sigma^2\,(t) = \frac{\sigma^2}{2}, \tag{7}$$

and

$$\lim_{t\to\infty} F\,(t,s,y) = \frac{1}{\sqrt{2\alpha\pi}}\exp\left(\frac{-y^2}{2\alpha}\right). \tag{8}$$

Thus as $t\to\infty, S_t \xrightarrow{d} N(0,\frac{\sigma^2}{2})$.

Denote by $V(S_t,k_t)$ the value of the capacity output (portfolio) $k = (k_1,\dots,k_n)$ for given values of stock prices $S = (S_1,\dots,S_n)$. An investor with some initial endowment $V_0 \geq 0$ and invests it in $d+1$ assets. Let $N_i(k_t)$ denote the number of shares of assets

$i = 1,2,$ ...owned by the investor at time$t$. Then $V_0 = \sum_{i=0}^{n} N_i(0)\, S_i$ and the investor's portfolio at time t is

$$V_i = \sum_{i=0}^{n} N_i(k_i)\, S_i. \tag{9}$$

If the portfolio is adjusted (due to trading of shares) at discrete time points ... $t-h, t, t+h$ ..., say, and there is no infusion or withdrawal of fund[(Osu [9]), then

$$V_{t+h} = V_t + \sum_{i=0}^{n} N_i(k_i)\, [S_i(t+h) - S_i(t)]. \tag{10}$$

From time t to $t+\Delta t$, the dynamics growth of the portfolio is characterized by fluctuation due to the risks of stock price variation such that $N = {}^{V_t}\!/_{S_i}$, so that (10) becomes;

$$V_{t+h} = V_t + \sum_{i=0}^{n} {}^{V_t}\!/_{S_i}\, [S_i(t+h) - S_i(t)]. \tag{11}$$

3. ***The Value of Stock Return Under the Dynamics*** $\boldsymbol{dS_t = \alpha(t)S_t dt + \sigma(t)S_t dW(t)}$

The price evolution of risky assets are usually modelled as the trajectory of a diffusion process defined on some underlying probability space $(\Omega, \mathfrak{F}, P)$, with the geometric Brownian motion process the best candidate used as the canonical reference model. It had been shown inBrick [4] that the geometric Brownian motion can indeed be justified as the rational expectation equilibrium in a market with homogenous agents. But the evolution of the stock price process is well known to be described by the dynamics

$$dS_t = \alpha(t)S_t dt + \sigma(t)S_t dW(t), \tag{12}$$

with unique solution known to be (α and σ are the drift and volatility respectively, assumed continuous functions of time)

$$S_t = S_o exp\left\{\int_0^t \sigma(u)dW(u) + \int_0^t (\alpha(u) - \tfrac{1}{2}\sigma^2\,(u))du\right\}. \tag{13a}$$

Given equation (7), it is not difficult to see that (13a) becomes

$$S_t = S_o exp\left\{\int_0^t \sigma(u)dW(u)\right\}. \tag{13b}$$

By (7), we mean that the drift parameter α and future price of an option depend on volatility σ.

Ito's formula on (12) gives;

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2}{\partial S^2} V(S,t) + rS\frac{\partial}{\partial S}V(S,t) - rV(S,t) = \frac{\partial}{\partial t}V(S,t)\forall(S,t) < (0,\infty)\times(0,T), \quad (14)$$

which is the famous Black-Scholes parabolic partial differential equation.$V = V(S,t)$ is the value of option(s) or the portfolio value given different option values with different prices. We shall now solve the PDE (14) for stock which are already priced in the market to predict value of stock. In what follows we state;

Theorem 1

The value of asset returns of stock derivative which pays $S_j, j = 1,2,\ldots$ at future date t under the risk-adjusted probability measure

$$dS_t = \alpha(t)S_t dt + \sigma(t)S_t dW(t),$$

given equation (14) is

$$V(S,t) = V_0 exp\{rt\}\sum_{j=0}^{\infty}\frac{H_j\left(\frac{S}{\sqrt{2t}}\right)\left(\frac{t}{2}\right)^{j/2}}{j!}W^j, \qquad (15)$$

where$H_j(x) = ex^n \ \frac{d^n}{dx}(e^{-x})$ are the Hermite polynomial with

$$H_0(x) = 1, \ldots . H_4 = 16x^4 - 48x + 12.$$

Proof

We remove the discount rate r with the following transformations (Osu and Okoroafor[11]);

$$\bar{V} = Ve^{-rt}\text{and } \bar{S} = Se^{-rt}\ , \qquad (16)$$

and (14) becomes

$$\frac{1}{2}\sigma^2\bar{S}^2\frac{\partial^2\bar{V}(S,t)}{\partial S^2} = -\frac{\partial\bar{V}(S,t)}{\partial t}. \qquad (17)$$

We guess a solution of the form $\bar{V} = f(t)g(S)$ to get

$$g(S) = V_0 e^{2WS} \text{ and } f(t) = e^{-\sqrt{\frac{W}{Z}}t}$$

(using the negative characteristic root). Hence

$$\bar{V} = V_0 exp\left\{2WS - \sqrt{\frac{w}{Z}\, t}\right\}. \quad (18a)$$

Assume $Z = \frac{t}{|W|^{\gamma+1}}$ (see Osu[10]), then (18) becomes with $\gamma = 2$.

$$\bar{V} = V_0 exp\{2WS - W^2 t^{1/2}\}. \quad (18b)$$

Using the Rodriguez formula $e^{2\theta y^2 - \theta^2} = \sum_{j=0}^{\infty} \frac{H_j(y)\theta^j}{j!}$, we obtain (15)

4. Predicting the Value of an investor’s portfolio given at least two Prices

In this section, we attempt to predict the value of an investor’s portfolio (s) by presenting a model based on the solution of a linear second order partial differential equation.We begin by writing down the equation of the stock price movement. Thus, the equations characterizing the stock price movement are:

$$\frac{\partial u}{\partial s_1} + \frac{\partial v}{\partial s_2} = 0 \quad (19)$$

$$\sigma\left\{\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial s_1} + V\frac{\partial u}{\partial s_2} = \frac{\partial S_1}{\partial s_1}\right\} \quad (20)$$

$$\sigma\left\{\frac{\partial v}{\partial t} + U\frac{\partial u}{\partial s_1} + V\frac{\partial v}{\partial s_2} = -\frac{\partial S_2}{\partial s_2}\right\} \quad (21)$$

σ is the volatility, S_i is the stock price, where $i = 0,1,2$. $U$and V are values respectively of stock 1 and stock 2 with the intrinsic values of stock at prices s_1 and s_2.

- We have used upper case S for the stock price and lower case s for the intrinsic value.
- The intrinsic value is the perceived value of the security which has no one value, but varies by investor.

Since we are interested in the case of small volatility, we neglect the product terms in equation (20) and (21) to obtain

$$\sigma\frac{\partial U}{\partial t} = -\frac{\partial S_1}{\partial s_2} \quad (22)$$

$$\sigma\frac{\partial V}{\partial t}=-\frac{\partial S_2}{\partial s_2} \tag{23}$$

We represent here the portfolio of the investor with investment $i=1{,}2$ by ϕ with $\nabla\phi=(\mathrm{U},\mathrm{V})$ (the gradient function), where U and V are the S_1 and S_2 value stock, and r_i the interest rate of stock i.

Now let

$$\frac{\partial U}{\partial t}=\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial s_i}\right)=\frac{\partial}{\partial s_i}\left(\frac{\partial\phi}{\partial t}\right) \tag{24}$$

$$\frac{\partial V}{\partial t}=\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial s_2}\right)=\frac{\partial}{\partial s_2}\left(\frac{\partial\phi}{\partial t}\right) \tag{25}$$

Equation (19) is satisfied identically, by introducing this function $\phi(s_1,s_2)$ such that

$$U=\frac{\partial\phi}{\partial s_1},\text{ and } V=\frac{\partial\phi}{\partial s_2} \tag{26}$$

Hence substituting equation (25) into (19) gives

$$\frac{\partial \mathrm{U}}{\partial s_1}+\frac{\partial \mathrm{V}}{\partial s_2}=\frac{\partial^2\phi}{\partial {s_1}^2}+\frac{\partial^2\phi}{\partial {s_2}^2}={\nabla_{(1)}}^2\phi=0\ . \tag{27}$$

It then follows that in the equilibrium fluctuation of two different portfolios that

$$\nabla^2_{(1)}\varphi_1=\nabla^2_{(1)}\phi_2=0\ , \tag{28}$$

where $\nabla^2_{(1)}=\frac{\partial^2}{\partial^2 \mathrm{s}_1}+\frac{\partial^2}{\partial^2 \mathrm{s}_2}$.

Substituting equations (24) and (25) into equations (22) and (23) respectively we obtain we obtain

$$\frac{\partial}{\partial s_1}\left(\frac{\partial\varphi}{\partial t}\right)+\frac{1}{\sigma}\frac{\partial S_1}{\partial s_1}=\frac{\partial}{\partial s_1}\left(\frac{\partial\varphi}{\partial t}+\frac{S_1}{\sigma}\right)=0\ , \tag{29}$$

$$\frac{\partial}{\partial s_2}\left(\frac{\partial\phi}{\partial t}\right)+\frac{1}{\sigma}\frac{\partial S_2}{\partial s_2}=\frac{\partial}{\partial s_2}\left(\frac{\partial\phi_2}{\partial t}+\frac{S_2}{\sigma}\right)=0\ . \tag{30}$$

It is not difficult to see that equations (29) and (30) at the up and down movement of price at investment time $0\le t\le T$ reduce to

$$\sigma \frac{\partial \varphi_1}{\partial t} + S_1 = 0 \tag{31}$$

$$\sigma \frac{\partial \varphi_2}{\partial t} + S_2 = 0\,. \tag{32}$$

The boundary condition are: For $s_1 = L$, we take

$$\frac{\partial \varphi_1}{\partial s_1} = \frac{\partial \varphi_2}{\partial s_1} = 0\,. \tag{33}$$

For $s_2 = l, \frac{\partial \varphi_1}{\partial s_2} = 0,$ and for $s_2 = -l, \frac{\partial \phi_2}{\partial s_2} = 0.$

We seek now a solution such that the field variables will be proportional to $e^{iwt} = e^{rt}$, with r the cumulative interest rate at the time of expiration.

We write the transformation equations;

$$\bar{\varphi} = \varphi e^{rt}, \tag{34a}$$

$$\bar{S_i} = S_i e^{rt}, \tag{34b}$$

and (28) becomes

$$\nabla^2_{(1)} \overline{\varphi}_1 = 0,\ \nabla^2_{(1)} \overline{\varphi}_2 = 0. \tag{35}$$

Equations (31) and (32) now become

$$S_1 + \sigma r \bar{\varphi}_1 = 0\ ;\ S_2 + \sigma r \bar{\varphi}_2 = 0\,. \tag{36}$$

Theorem 2.

The future portfolio of an investor with different investment $i = 1,2, ...,$ given the prices S_i under the underlying second order linear partial differential equation

$$\frac{\partial^2 \overline{\phi}}{\partial {s_1}^2} + \frac{\partial^2 \overline{\phi}}{\partial {s_2}^2} = 0,$$

is

$$\varphi = \begin{cases} \frac{\beta \cos \lambda(L-S_1) \cosh \lambda(l-S_2)}{\cos \lambda L \cosh \lambda \rho} e^{-rt}, if\ s_2 = l \\ \frac{\gamma\{\cos \lambda(L-S_1) \cos \lambda(L-S_2)\}}{\cos \lambda L \cosh \lambda L} e^{-rt}\ , if\ s_2 \neq l \end{cases}\ , \tag{37}$$

for some arbitrary constant $\beta, \gamma \neq 0$, and $s_1 = L$.

Proof

Using the method of solution in Adagba [1], we have

$$\bar{\varphi} = \begin{cases} \frac{\beta \cos \lambda(L-S_1) \cosh \lambda(l-S_2)}{\cos \lambda L \cosh \lambda \rho} \\ \frac{\gamma\{\cos \lambda(L-S_1) \cos \lambda(L-S_2)\}}{\cos \lambda L \cosh \lambda L} \end{cases}. \qquad (38)$$

Combining (34a) and (38) we have (37) as required.

Conclusion

The results proposed in this work enable us to predict the future value of an investor's portfolio given the investor's option position with future price. Discover that the future price will be (using (13) and (34b));

$$S_t = S_o exp\left\{\sigma(t)W(t) + \left[(\alpha - r) - \frac{\sigma^2}{2}\right] t\right\}, \qquad (39)$$

where $\alpha - r$ is the expected market risk premium. Risk is an important factor in determining how to efficiently manage a portfolio of investments because it determines the variation in returns on the asset and (or) portfolio and gives investors a mathematical bases for investment decisions. Using (7) in (39) we have

$$S_t = S_o exp\{\sigma(t)W(t) + (r)t\}. \qquad (40)$$

Notice that (39) becomes (16b) as $W(t) \to 0$. r can be calculate by formula (Osu[11]);

$$eps = \frac{Eps}{1-r}, \qquad (41)$$

where eps is the earning per share after dividends and Eps the earning per share before dividends. φ as in (37) is a circular function of s_1 and s_2, therefore repetition of fluctuation will occur every time s_1 increases by $L - 2\pi$ and s_2 increases by $l \pm 2\pi$.

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