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SOME NEW OPERATIONS OF FUZZY SOFT SETS

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Abstract: In this paper, we have defined disjunctive sum and difference of two fuzzy soft sets and study their basic properties. The notions of α - cut soft set and α - cut strong soft set of a fuzzy soft set have been put forward in our work. Some related properties have been established with proof, examples and counter examples.

Keywords: Fuzzy Set, Soft Set, Fuzzy Soft Set, Disjunctive Sum, Difference, α - cut soft set, α - cut strong soft set.

2000 AMS Subject Classification: 03E72

1. Introduction

Most of the real life problems have various uncertainties. The Theory of Probability, Evidence Theory, Fuzzy Set Theory, Intuitionistic Fuzzy Set Theory, Rough Set Theory etc. are mathematical tools to deal with such problems. In 1999, Molodtsov [4] introduced the Theory of Soft Set and established the fundamental results related to this theory. In comparison, this theory can be seen free from the inadequacy of parameterization tool. It is a general mathematical tool for dealing with problems in the fields of social science, economics, medical sciences etc. In 2003, Maji, Biswas

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and Roy **[2]** studied the theory of soft sets initiated by Molodtsov. They defined equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set, and absolute soft set with examples. Soft binary operations like AND, OR and also the operations of union, intersection were also defined. In recent times, researchers have contributed a lot towards fuzzification of Soft Set Theory. Combining fuzzy sets with soft sets, Maji et al. [3] introduced the notion of fuzzy soft sets. They studied some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, De Morgan Law etc. These results were further revised and improved by Ahmad and Kharal [1]. They defined arbitrary fuzzy soft union and intersection and proved De Morgan Inclusions and De Morgan Laws in Fuzzy Soft Set Theory. In 2011, Neog and Sut [6] put forward some propositions regarding fuzzy soft set theory. They studied the notions of fuzzy soft union, fuzzy soft intersection, complement of a fuzzy soft set and several other properties of fuzzy soft sets along with examples and proofs of certain results. In this paper, we have defined disjunctive sum and difference of two fuzzy soft sets. The notions of α - cut soft set and α - cut strong soft set of a fuzzy soft set have been put forward in our work. Some related properties have been established in our work with supporting proof, examples and counter examples.

2. Preliminaries

In this section, we first recall the basic definitions related to fuzzy soft sets which would be used in the sequel.

Definition 2.1 [4]

A pair (*F*, *E*) is called a soft set (over *U*) if and only if *F* is a mapping of *E* into the set of all subsets of the set *U*.

In other words, the soft set is a parameterized family of subsets of the set *U*. Every set $F(\varepsilon), \varepsilon \in E$, from this family may be considered as the set of ε - elements of the soft set (*F*, *E*), or as the set of ε - approximate elements of the soft set.

Definition 2.2 [3]

A pair (*F*, *A*) is called a fuzzy soft set over *U* where $F : A \to \widetilde{P}(U)$ is a mapping from

A into $\widetilde{P}(U)$.

Definition 2.3 [1]

Let U be a universe and E a set of attributes. Then the pair (U, E) denotes the collection of all fuzzy soft sets on U with attributes from E and is called a fuzzy soft class.

Definition 2.4 [3]

A soft set (F, A) over U is said to be null fuzzy soft set denoted by φ if $\forall \varepsilon \in A, F(\varepsilon)$ is the null fuzzy set $\overline{0}$ of U where $\overline{0}(x) = 0 \forall x \in U$.

We would use the notation (φ, A) to represent the fuzzy soft null set with respect to the set of parameters A.

Definition 2.5 [3]

A soft set (F, A) over U is said to be absolute fuzzy soft set denoted by $\tilde{A}$ if $\forall \varepsilon \in A, F(\varepsilon)$ is the absolute fuzzy set $\overline{1}$ of U where $\overline{1}(x) = 1 \forall x \in U$.

We would use the notation (U, A) to represent the fuzzy soft absolute set with respect to the set of parameters A.

Definition 2.6 [3]

For two fuzzy soft sets (F, A) and (G, B) in a fuzzy soft class (U, E), we say that (F, A) is a fuzzy soft subset of (G, B), if

(i) $A \subseteq B$

(ii) For all $\varepsilon \in A, F(\varepsilon) \subseteq G(\varepsilon)$ and is written as (F , A) $\tilde{\subseteq}$ (G, B).

Definition 2.7 [3]

Union of two fuzzy soft sets (F, A) and (G, B) in a soft class (U, E) is a fuzzy soft set (H, C) where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

and is written as $(F, A) \tilde{\cup} (G, B) = (H, C)$.

Definition 2.8 [3]

Intersection of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*, *C*) where $C = A \cap B$ and $\forall \varepsilon \in C, H(\varepsilon) = F(\varepsilon)$ or $G(\varepsilon)$ (as both are same fuzzy set) and is written as $(F, A)\tilde{\cap}(G, B) = (H, C)$.

Ahmad and Kharal [1] pointed out that generally $F(\varepsilon)$ or $G(\varepsilon)$ may not be identical. Moreover in order to avoid the degenerate case, he proposed that $A \cap B$ must be non-empty and thus revised the above definition as follows -

Definition 2.9 [1]

Let (*F*, *A*) and (*G*, *B*) be two fuzzy soft sets in a soft class (*U*, *E*) with $A \cap B \neq \phi$. Then Intersection of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*,*C*) where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$. We write $(F, A)\tilde{\cap}(G, B) = (H, C)$.

Definition 2.10 [5]

The complement of a fuzzy soft set (*F*, *A*) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \to \tilde{P}(U)$ is a mapping given by $F^c(\alpha) = [F(\alpha)]^c$, $\forall \alpha \in A$.

Definition 2.11[3]

If (*F*, *A*) and (*G*, *B*) be two fuzzy soft sets, then "(*F*, *A*) AND (*G*, *B*)" is a fuzzy soft set denoted by $(F, A) \wedge (G, B)$ and is defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$, $\forall \alpha \in A$ and $\forall \beta \in B$, where $\cap$ is the operation intersection of two fuzzy sets.

Definition 2.12[3]

If (*F*, *A*) and (*G*, *B*) be two fuzzy soft sets, then "(*F*, *A*) OR (*G*, *B*)" is a fuzzy soft set denoted by $(F, A) \vee (G, B)$ and is defined by $(F, A) \vee (G, B) = (K, A \times B)$, where $K(\alpha, \beta) = F(\alpha) \cup G(\beta)$, $\forall \alpha \in A$ and $\forall \beta \in B$, where $\cup$ is the operation union of two fuzzy sets.

3. Some New Operations on Fuzzy Soft Sets

Definition 3.1 (Disjunctive Sum of Fuzzy Soft Sets)

Let (F,A) and (G,B) be two fuzzy soft sets over (U,E). We define the disjunctive sum of (F,A) and (G,B) as the fuzzy soft set (H,C) over (U,E), written as $(F,A)\tilde{\oplus}(G,B)=(H,C)$, where $C=A\cap B\neq\varphi$ and $\forall\varepsilon\in C$, $x\in U$,

$$\mu_{H(\varepsilon)}(x)=\max\left(\min\left(\mu_{F(\varepsilon)}(x),1-\mu_{G(\varepsilon)}(x)\right),\min\left(1-\mu_{F(\varepsilon)}(x),\mu_{G(\varepsilon)}(x)\right)\right).$$

Example 3.1

Let $U=\{a,b,c\}$ and $E=\{e_1,e_2,e_3,e_4\}$, $A=\{e_1,e_2,e_4\}\subseteq E$, $B=\{e_1,e_2,e_3\}\subseteq E$

$$(F,A)=\{F(e_1)=\{(a,0.1),(b,0.2),(c,0.4)\},\ F(e_2)=\{(a,0.7),(b,0),(c,0.3)\},$$
$$F(e_4)=\{(a,0.6),(b,0.1),(c,0.9)\}\}$$

$$(G,B)=\{G(e_1)=\{(a,0.6),(b,0.7),(c,0.8)\},\ G(e_2)=\{(a,0.1),(b,0.5),(c,0.4)\},$$
$$G(e_3)=\{(a,0),(b,0),(c,0.1)\}\}$$

Then $(F,A)\tilde{\oplus}(G,B)=(H,C)$ where $C=A\cap B=\{e_1,e_2\}$ and

(H,C)

$$=\{H(e_1)=\{(a,\max(\min(\mu_{F(e_1)}(a),1-\mu_{G(e_1)}(a)),\min(1-\mu_{F(e_1)}(a),\mu_{G(e_1)}(a)))),$$
$$(b,\max(\min(\mu_{F(e_1)}(b),1-\mu_{G(e_1)}(b)),\min(1-\mu_{F(e_1)}(b),\mu_{G(e_1)}(b)))),$$
$$(c,\max(\min(\mu_{F(e_1)}(c),1-\mu_{G(e_1)}(c)),\min(1-\mu_{F(e_1)}(c),\mu_{G(e_1)}(c))))\},$$
$$H(e_2)=\{(a,\max(\min(\mu_{F(e_2)}(a),1-\mu_{G(e_2)}(a)),\min(1-\mu_{F(e_2)}(a),\mu_{G(e_2)}(a)))),$$
$$(b,\max(\min(\mu_{F(e_2)}(b),1-\mu_{G(e_2)}(b)),\min(1-\mu_{F(e_2)}(b),\mu_{G(e_2)}(b))))$$
$$(c,\max(\min(\mu_{F(e_2)}(c),1-\mu_{G(e_2)}(c)),\min(1-\mu_{F(e_2)}(c),\mu_{G(e_2)}(c))))\}$$

$$=\{H(e_1)=\{(a,\max(\min(0.1,1-0.6),\min(1-0.1,0.6))),$$
$$(b,\max(\min(0.2,1-0.7),\min(1-0.2,0.7))),$$

$$\left(c,\max\left(\min\left(0.4,1-0.8\right),\min\left(1-0.4,0.8\right)\right)\right)\},$$

$$H(e_2)=\{\left(a,\max\left(\min\left(0.7,1-0.1\right),\min\left(1-0.7,0.1\right)\right)\right),$$

$$\left(b,\max\left(\min\left(0,1-0.5\right),\min\left(1-0,0.5\right)\right)\right),$$

$$\left(c,\max\left(\min\left(0.3,1-0.4\right),\min\left(1-0.3,0.4\right)\right)\right)\}\}$$

$$=\{H(e_1)=\{\left(a,\max\left(\min\left(0.1,0.4\right),\min\left(0.9,0.6\right)\right)\right),$$

$$\left(b,\max\left(\min\left(0.2,0.3\right),\min\left(0.8,0.7\right)\right)\right),$$

$$\left(c,\max\left(\min\left(0.4,0.2\right),\min\left(0.6,0.8\right)\right)\right)\},$$

$$H(e_2)=\{\left(a,\max\left(\min\left(0.7,0.9\right),\min\left(0.3,0.1\right)\right)\right),$$

$$\left(b,\max\left(\min\left(0,0.5\right),\min\left(1,0.5\right)\right)\right),\ \left(c,\max\left(\min\left(0.3,0.6\right),\min\left(0.7,0.4\right)\right)\right)\}\}$$

$$=\{H(e_1)=\{(a,0.6),\ (b,0.7),\ (c,0.6)\},\ H(e_2)=\{(a,0.7),\ (b,0.5),\ (c,0.4)\}\}$$

Proposition 3.1

Let (F,A) and (G,B) be two fuzzy soft sets over (U,E). Then the following results hold.

(i) $(F,A)\tilde{\oplus}(G,B)=(G,B)\tilde{\oplus}(F,A)$

(ii) $(F,A)\tilde{\oplus}\left((G,B)\tilde{\oplus}(H,C)\right)=\left((F,A)\tilde{\oplus}(G,B)\right)\tilde{\oplus}(H,C)$

Proof. The proof is obvious and follows from definition.

Proposition 3.2

(i) $(F,A)\tilde{\oplus}(\varphi,A)=(F,A)$

(ii) $(F,A)\tilde{\oplus}(U,A)=(F,A)^c$

Proof.

(i) Let $(F,A)\tilde{\oplus}(\varphi,A)=(H,C)$, where $C=A\cap A=A$ and $\forall\varepsilon\in C=A, x\in U$, we have

$$\mu_{H(\varepsilon)}(x)=\max\left(\min\left(\mu_{F(\varepsilon)}(x),1-0\right),\min\left(1-\mu_{F(\varepsilon)}(x),0\right)\right)$$

$$=\max\left(\min\left(\mu_{F(\varepsilon)}(x),1\right),\min\left(1-\mu_{F(\varepsilon)}(x),0\right)\right)$$

$$= \max\left(\mu_{F(\varepsilon)}(x), 0\right)$$

$$= \mu_{F(\varepsilon)}(x)$$

It follows that $(F, A)\tilde{\oplus}(\varphi, A) = (F, A)$

(ii) Let $(F, A)\tilde{\oplus}(U, A) = (H, C)$, where $C = A \cap A = A$ and $\forall \varepsilon \in C = A, x \in U$, we have

$$\mu_{H(\varepsilon)}(x) = \max\left(\min\left(\mu_{F(\varepsilon)}(x), 1-1\right), \min\left(1-\mu_{F(\varepsilon)}(x), 1\right)\right)$$

$$= \max\left(\min\left(\mu_{F(\varepsilon)}(x), 0\right), \min\left(1-\mu_{F(\varepsilon)}(x), 1\right)\right)$$

$$= \max\left(0, 1-\mu_{F(\varepsilon)}(x)\right)$$

$$= 1-\mu_{F(\varepsilon)}(x)$$

It follows that $(F, A)\tilde{\oplus}(U, A) = (F, A)^c$

Definition 3.2 (Difference of Fuzzy Soft Sets)

Let (F, A) and (G, B) be two fuzzy soft sets over (U, E). We define the difference of (F, A) and (G, B) as the fuzzy soft set (H, C) over (U, E), written as $(F, A)\tilde{\Theta}(G, B) = (H, C)$, where $C = A \cap B \neq \varphi$ and $\forall \varepsilon \in C$, $x \in U$, $\mu_{H(\varepsilon)}(x) = \min\left(\mu_{F(\varepsilon)}(x), 1-\mu_{G(\varepsilon)}(x)\right)$.

Example 3.2

We take the fuzzy soft sets (F, A) and (G, B) given in **Example 3.1**.

Let $(F, A)\tilde{\Theta}(G, B) = (H, C)$, where $C = A \cap B = \{e_1, e_2\}$. Then

$$(H, C)$$

$$= \{H(e_1) = \{(a, \min(\mu_{F(e_1)}(a), 1-\mu_{G(e_1)}(a))), (b, \min(\mu_{F(e_1)}(b), 1-\mu_{G(e_1)}(b))),$$
$$(c, \min(\mu_{F(e_1)}(c), 1-\mu_{G(e_1)}(c)))\}, H(e_2) = \{(a, \min(\mu_{F(e_2)}(a), 1-\mu_{G(e_2)}(a))),$$
$$(b, \min(\mu_{F(e_2)}(b), 1-\mu_{G(e_2)}(b))), (c, \min(\mu_{F(e_2)}(c), 1-\mu_{G(e_2)}(c)))\}$$

$$= \{H(e_1) = \{(a, \min(0.1, 1-0.6)), (b, \min(0.2, 1-0.7)), (c, \min(0.4, 1-0.8))\},$$

$$H(e_2)=\{(a,\min(0.7,1-0.1)),\ (b,\min(0,1-0.5)),\ (c,\min(0.3,1-0.4))\}$$

$$=\{H(e_1)=\{(a,\min(0.1,0.4)),\ (b,\min(0.2,0.3)),\ (c,\min(0.4,0.2))\},$$

$$H(e_2)=\{(a,\min(0.7,0.9)),\ (b,\min(0,0.5)),\ (c,\min(0.3,0.6))\}$$

$$=\{H(e_1)=\{(a,0.1),\ (b,0.2),\ (c,0.2)\},\ H(e_2)=\{(a,0.7),\ (b,0),\ (c,0.3)\}\}$$

Also, let $(G,B)\tilde{\Theta}(F,A)=(I,C)$ where $C=A\cap B=\{e_1,e_2\}$ and

$$(I,C)\quad=\{I(e_1)=\{(a,\min(0.6,0.9)),\ (b,\min(0.7,0.8)),\ (c,\min(0.8,0.6))\},$$

$$I(e_2)=\{(a,\min(0.1,0.3)),\ (b,\min(0.5,1)),\ (c,\min(0.4,0.7))\}$$

$$=\{I(e_1)=\{(a,0.6),\ (b,0.7),\ (c,0.6)\},\ I(e_2)=\{(a,0.1),\ (b,0.5),\ (c,0.4)\}\}$$

It follows that $(F,A)\tilde{\Theta}(G,B)\neq(G,B)\tilde{\Theta}(F,A)$.

Proposition 3.3

(i) $(F,A)\tilde{\Theta}(\varphi,A)=(F,A)$

(ii) $(F,A)\tilde{\Theta}(U,A)=(\varphi,A)$

Proof.

(i) Let $(F,A)\tilde{\Theta}(\varphi,A)=(H,C)$, where $C=A\cap A=A$ and $\forall\varepsilon\in C=A, x\in U$, we have

$$\mu_{H(\varepsilon)}(x)=\min(\mu_{F(\varepsilon)}(x),1-0)\quad=\min(\mu_{F(\varepsilon)}(x),1)\quad=\mu_{F(\varepsilon)}(x)$$

It follows that $(F,A)\tilde{\Theta}(\varphi,A)=(F,A)$

(ii) Let $(F,A)\tilde{\Theta}(U,A)=(H,C)$, where $C=A\cap A=A$ and $\forall\varepsilon\in C=A, x\in U$, we have

$$\mu_{H(\varepsilon)}(x)=\min(\mu_{F(\varepsilon)}(x),1-1)\quad=\min(\mu_{F(\varepsilon)}(x),0)\quad=0$$

It follows that $(F,A)\tilde{\Theta}(U,A)=(\varphi,A)$

Definition 3.3 ($\alpha-$Cut Soft Set of a Fuzzy Soft Set)

Let (F,A) be a fuzzy soft set over (U,E). We define the $\alpha-$cut soft set of the fuzzy soft set (F, A), denoted by $(F, A)_\alpha$ as the soft set (F_α, A), where

$\forall \varepsilon \in A, F_\alpha(\varepsilon) = \{x : \mu_{F(\varepsilon)}(x) \geq \alpha; x \in U, \alpha \in [0,1]\}$

Example 3.3

Let $U = \{a,b,c\}$ and $E = \{e_1, e_2, e_3, e_4\}$, $A = \{e_1, e_2, e_4\} \subseteq E$. Let us consider a fuzzy soft set (F, A) as

$$(F, A) = \{F(e_1) = \{(a,0.5),(b,0.4),(c,0.9)\}, F(e_2) = \{(a,0.7),(b,0.2),(c,0.6)\},$$
$$F(e_4) = \{(a,0.6),(b,0.1),(c,0.8)\}\}$$

Let $\alpha = 0.4 \in [0,1]$. Then

$$(F, A)_{0.4} = (F_{0.4}, A) = \{F_{0.4}(e_1) = \{a,b,c\}, F_{0.4}(e_2) = \{a,c\}, F_{0.4}(e_4) = \{a,c\}\}$$

Definition 3.4 ($\alpha-$Cut Strong Soft Set of a Fuzzy Soft Set)

Let (F, A) be a fuzzy soft set over (U, E). We define the $\alpha-$cut strong soft set of the fuzzy soft set (F, A), denoted by $(F, A)_{\alpha+}$ as the soft set $(F_{\alpha+}, A)$, where

$\forall \varepsilon \in A, F_{\alpha+}(\varepsilon) = \{x : \mu_{F(\varepsilon)}(x) > \alpha; x \in U, \alpha \in [0,1]\}$

Example 3.4

We take the fuzzy soft set (F, A) given in **Example 3.3** Then

$$(F, A)_{0.4+} = (F_{0.4+}, A) = \{F_{0.4+}(e_1) = \{a,c\}, F_{0.4+}(e_2) = \{a,c\}, F_{0.4+}(e_4) = \{a,c\}\}$$

Proposition 3.4

Let $(F, A), (G, B)$ be two fuzzy soft sets over (U, E). Then the following results hold for all $\alpha \in [0,1]$.

(i) $(F, A) \tilde{\subseteq} (G, B) \Rightarrow (F, A)_\alpha \tilde{\subseteq} (G, B)_\alpha$, $(F, A)_{\alpha+} \tilde{\subseteq} (G, B)_{\alpha+}$

(ii) $((F, A) \tilde{\cup} (G, B))_\alpha = (F, A)_\alpha \tilde{\cup} (G, B)_\alpha$,

$((F, A) \tilde{\cup} (G, B))_{\alpha+} = (F, A)_{\alpha+} \tilde{\cup} (G, B)_{\alpha+}$

(iii) $((F, A) \tilde{\cap} (G, B))_\alpha = (F, A)_\alpha \tilde{\cap} (G, B)_\alpha$,

$$((F,A)\tilde{\cap}(G,B))_{\alpha+} = (F,A)_{\alpha+}\tilde{\cap}(G,B)_{\alpha+}$$

(iv) $(F,A)^c{}_{\alpha} = (F,A)^c{}_{(1-\alpha)+}$

Proof.

(i) Let $(F,A)\tilde{\subseteq}(G,B)$. Then $A\subseteq B$ and $\forall\varepsilon\in A, x\in U, \mu_{F(\varepsilon)}(x)\le\mu_{G(\varepsilon)}(x)$

We assume that there is $\alpha_\circ\in[0,1]$ such that $(F,A)_{\alpha_\circ}\tilde{\not\subset}(G,B)_{\alpha_\circ}$.

Now $(F,A)_{\alpha_\circ} = (F_{\alpha_\circ},A) = \{F_{\alpha_\circ}(\varepsilon):\varepsilon\in A\}$

Then there exists $x_\circ\in F_{\alpha_\circ}(\varepsilon), x_\circ\in U$ such that $x_\circ\notin G_{\alpha_\circ}(\varepsilon)$ for at least one $\varepsilon\in A$.

i.e. $\mu_{F(\varepsilon)}(x_\circ)\ge\alpha_\circ$ and $\mu_{G(\varepsilon)}(x_\circ)<\alpha_\circ$. This is a contradiction, since $\forall\varepsilon\in A, x\in U,$

$\mu_{F(\varepsilon)}(x)\le\mu_{G(\varepsilon)}(x)$

Thus for all $\alpha\in[0,1]$ and $\forall\varepsilon\in A, F_\alpha(\varepsilon)\subseteq G_\alpha(\varepsilon)$.

It follows that $(F,A)_\alpha\tilde{\subseteq}(G,B)_\alpha$.

The reverse inclusion here is not valid as is obvious from the following example –

Example 3.4

Let $U=\{a,b,c\}$ and $E=\{e_1,e_2,e_3,e_4\}, A=\{e_1,e_2\}\subseteq E, B=\{e_1,e_2,e_4\}\subseteq E$

$$(F,A) = \{F(e_1)=\{(a,0.1),(b,0.2),(c,0.4)\},\ F(e_2)=\{(a,0.6),(b,0.1),(c,0.9)\}\}$$

$$(G,B) = \{G(e_1)=\{(a,0.6),(b,0.7),(c,0.8)\},\ G(e_2)=\{(a,0.3),(b,0.1),(c,0.4)\},$$
$$G(e_4)=\{(a,0),(b,0),(c,0.6)\}\}$$

Here

$$(F,A)_{0.2} = (F_{0.2},A)$$
$$= \{F_{0.2}(e_1)=\{b,c\},\ F_{0.2}(e_2)=\{a,c\}\}$$

$$(G,B)_{0.2} = (G_{0.2},B)$$
$$= \{G_{0.2}(e_1)=\{a,b,c\},\ G_{0.2}(e_2)=\{a,c\},\ G_{0.2}(e_4)=\{c\}\}$$

It is clear that $(F,A)_{0.2}\tilde{\subseteq}(G,B)_{0.2}$ but $(F,A)\tilde{\not\subset}(G,B)$ as

$0.6 = \mu_{F(e_2)}(a) > \mu_{G(e_2)}(a) = 0.3$

and $0.9 = \mu_{F(e_2)}(c) > \mu_{G(e_2)}(c) = 0.4$

The second result follows similar lines.

(*ii*) Let $(F,A)\tilde{\cup}(G,B) = (H,C)$. Then $C = A \cup B$ and $\forall \varepsilon \in C,$

$$H(\varepsilon) = \begin{cases} F(\varepsilon) \text{ if } \varepsilon \in A-B \\ G(\varepsilon) \text{ if } \varepsilon \in B-A \\ F(\varepsilon) \cup G(\varepsilon) \text{ if } \varepsilon \in A \cap B \end{cases}$$

i.e.

$$\mu_{H(\varepsilon)}(x) = \begin{cases} \mu_{F(\varepsilon)}(x) \text{ if } \varepsilon \in A-B \\ \mu_{G(\varepsilon)}(x) \text{ if } \varepsilon \in B-A \\ \max\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right) \text{if } \varepsilon \in A \cap B \end{cases}$$

Now $\left((F,A)\tilde{\cup}(G,B)\right)_\alpha = (H,C)_\alpha = (H_\alpha, C)$, where $C = A \cup B$ and $\forall \varepsilon \in C,$

$$H_\alpha(\varepsilon) = \begin{cases} \left\{x : x \in U, \mu_{F(\varepsilon)}(x) \geq \alpha\right\} \text{ if } \varepsilon \in A-B \\ \left\{x : x \in U, \mu_{G(\varepsilon)}(x) \geq \alpha\right\} \text{ if } \varepsilon \in A-B \\ \left\{x : x \in U, \max\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right) \geq \alpha\right\} \text{ if } \varepsilon \in A \cap B \end{cases}$$

Let $x \in H_\alpha(\varepsilon)$ for some $\varepsilon \in C$. Then $\mu_{H(\varepsilon)}(x) \geq \alpha$

$$\Rightarrow \begin{cases} \mu_{F(\varepsilon)}(x) \geq \alpha \text{ if } \varepsilon \in A-B \\ \mu_{G(\varepsilon)}(x) \geq \alpha \text{ if } \varepsilon \in A-B \\ \max\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right) \geq \alpha \text{ if } \varepsilon \in A \cap B \end{cases}$$

$$\Rightarrow \begin{cases} \mu_{F(\varepsilon)}(x) \geq \alpha \text{ if } \varepsilon \in A-B \\ \mu_{G(\varepsilon)}(x) \geq \alpha \text{ if } \varepsilon \in A-B \\ \mu_{F(\varepsilon)}(x) \geq \alpha \text{ or } \mu_{G(\varepsilon)}(x) \geq \alpha \text{ if } \varepsilon \in A \cap B \end{cases}$$

$$\Rightarrow x \in \begin{cases} F_\alpha(\varepsilon) \text{ if } \varepsilon \in A-B \\ G_\alpha(\varepsilon) \text{ if } \varepsilon \in A-B \\ F_\alpha(\varepsilon) \cup G_\alpha(\varepsilon) \text{ if } \varepsilon \in A \cap B \end{cases}$$

$\Rightarrow x \in (F,A)_\alpha \tilde{\cup} (G,B)_\alpha$. Thus $(H,C)_\alpha \tilde{\subseteq} (F,A)_\alpha \tilde{\cup} (G,B)_\alpha$

For the converse part, let $(F,A)_\alpha \tilde{\cup} (G,B)_\alpha = (F_\alpha, A)\tilde{\cup}(G_\alpha, B) = (I,C)$, where

$C = A \cup B$ and $\forall \varepsilon \in C,$

$$I(\varepsilon) = \begin{cases} F_\alpha(\varepsilon) \text{ if } \varepsilon \in A - B \\ G_\alpha(\varepsilon) \text{ if } \varepsilon \in A - B \\ F_\alpha(\varepsilon) \cup G_\alpha(\varepsilon) \text{ if } \varepsilon \in A \cap B \end{cases}$$

Let $x \in I(\varepsilon)$ for some $\varepsilon \in C$.

Then $x \in \begin{cases} F_\alpha(\varepsilon) \text{ if } \varepsilon \in A - B \\ G_\alpha(\varepsilon) \text{ if } \varepsilon \in A - B \\ F_\alpha(\varepsilon) \cup G_\alpha(\varepsilon) \text{ if } \varepsilon \in A \cap B \end{cases}$

$$\Rightarrow \begin{cases} \mu_{F(\varepsilon)}(x) \geq \alpha \text{ if } \varepsilon \in A - B \\ \mu_{G(\varepsilon)}(x) \geq \alpha \text{ if } \varepsilon \in A - B \\ \mu_{F(\varepsilon)}(x) \geq \alpha \text{ or } \mu_{G(\varepsilon)}(x) \geq \alpha \text{ if } \varepsilon \in A \cap B \end{cases}$$

$$\Rightarrow \begin{cases} \mu_{F(\varepsilon)}(x) \geq \alpha \text{ if } \varepsilon \in A - B \\ \mu_{G(\varepsilon)}(x) \geq \alpha \text{ if } \varepsilon \in A - B \\ \max\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right) \geq \alpha \text{ if } \varepsilon \in A \cap B \end{cases}$$

$\Rightarrow x \in H_\alpha(\varepsilon)$.

Thus $I(\varepsilon) \subseteq H_\alpha(\varepsilon) \forall \varepsilon \in C \Rightarrow (F,A)_\alpha \tilde{\cup} (G,B)_\alpha \tilde{\subseteq} (H,C)_\alpha$ and the result follows immediately.

The proof of second result is similar.

(*iii*) Let $(F,A) \tilde{\cap} (G,B) = (H,C)$. Then $C = A \cap B$ and $\forall \varepsilon \in C,$

$\mu_{H(\varepsilon)}(x) = \min\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right)$

Now $\left((F,A) \tilde{\cap} (G,B)\right)_\alpha = (H,C)_\alpha = (H_\alpha, C)$, where $C = A \cap B$ and $\forall \varepsilon \in C,$

$H_\alpha(\varepsilon) = \{x : x \in U, \mu_{H(\varepsilon)}(x) \geq \alpha\}$

Let $x \in H_\alpha(\varepsilon)$ for some $\varepsilon \in C$.

Then $\mu_{H(\varepsilon)}(x) \geq \alpha \Rightarrow \min\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right) \geq \alpha \Rightarrow \mu_{F(\varepsilon)}(x) \geq \alpha$ and $\mu_{G(\varepsilon)}(x) \geq \alpha \Rightarrow x \in F_\alpha(\varepsilon)$ and $x \in G_\alpha(\varepsilon) \Rightarrow x \in (F,A)_\alpha \tilde{\cap} (G,B)_\alpha$.

Thus $(H,C)_\alpha \tilde{\subseteq} (F,A)_\alpha \tilde{\cap} (G,B)_\alpha$

For the converse part, let $(F,A)_\alpha \tilde{\cap} (G,B)_\alpha = (F_\alpha, A) \tilde{\cap} (G_\alpha, B) = (I, C)$

Where $C = A \cap B$ and $\forall \varepsilon \in C,\ I(\varepsilon) = F_\alpha(\varepsilon) \cap G_\alpha(\varepsilon)$.

Let $x \in I(\varepsilon)$ for some $\varepsilon \in C$.

$\Rightarrow x \in F_\alpha(\varepsilon)$ and $x \in G_\alpha(\varepsilon)$

$\Rightarrow \mu_{F(\varepsilon)}(x) \geq \alpha$ and $\mu_{G(\varepsilon)}(x) \geq \alpha$

$\Rightarrow \min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)) \geq \alpha$

$\Rightarrow x \in H_\alpha(\varepsilon)$.

Thus $I(\varepsilon) \subseteq H_\alpha(\varepsilon) \forall \varepsilon \in C \Rightarrow (F,A)_\alpha \tilde{\cap} (G,B)_\alpha \tilde{\subseteq} (H,C)_\alpha$ and the result follows.

The proof of second result similarly follows.

(*iv*) We have $(F,A)^c = (F^c, A)$, where $\forall \varepsilon \in A,\ F^c(\varepsilon) = (F(\varepsilon))^c$

i.e. $\forall \varepsilon \in A, x \in U \Rightarrow \mu_{F^c(\varepsilon)}(x) = 1 - \mu_{F(\varepsilon)}(x)$

Now $(F,A)^c{}_\alpha = (F^c, A)_\alpha = (F^c{}_\alpha, A)$ where $\forall \varepsilon \in A,$

$F^c{}_\alpha(\varepsilon) = \{x : x \in U, \mu_{F^c(\varepsilon)}(x) \geq \alpha\}$

Let $x \in F^c{}_\alpha(\varepsilon)$ for some $\varepsilon \in C$.

Then $\mu_{F^c(\varepsilon)}(x) \geq \alpha \Rightarrow 1 - \mu_{F(\varepsilon)}(x) \geq \alpha \Rightarrow \mu_{F(\varepsilon)}(x) \leq 1 - \alpha$

This means $x \notin F_{(1-\alpha)+}(\varepsilon)$ *i.e.* $x \in (F_{(1-\alpha)+}(\varepsilon))^c$.

It follows that $(F,A)^c{}_\alpha \tilde{\subseteq} (F,A)^c{}_{(1-\alpha)+}$. It can also be verified that $(F,A)^c{}_{(1-\alpha)+} \tilde{\subseteq} (F,A)^c{}_\alpha$ and the result follows immediately.

Proposition 3.5

Let (F,A) be a fuzzy soft set over (U, E) and $\alpha, \beta \in [0,1]$. Then the following results hold.

(*i*) $(F,A)_{\alpha+} \tilde{\subseteq} (F,A)_\alpha$

$(ii)\ \alpha \leq \beta \Rightarrow (F,A)_{\alpha} \tilde{\subseteq} (F,A)_{\beta}, (F,A)_{\alpha+} \tilde{\subseteq} (F,A)_{\beta+}$

Proof. The proof is straight forward and follows from definition.

4. Conclusion

In our work, we have put forward some new concepts such as disjunctive sum, difference, α - cut soft set and α - cut strong soft set of a fuzzy soft set. Some related properties have been established with examples and counter examples. It is hoped that our work would enhance this study in fuzzy soft sets in near future.

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