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BEST ∞−SIMULTANEOUS APPROXIMATION IN BANACH LATTICE FUNCTION SPACES

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Abstract. Let X be a conditional complete Banach lattice space. We are concerned with the proximinality problem for best simultaneous approximations to two functions in the Köthe Bochner function spaces in the l_∞sum sense. This characterization can be considered as a generalization of some analogous theorems concerning the Orlicz Bochner spaces and Lp Bochner spaces.

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1. Introduction

The problem of best simultaneous approximation for functions and operator spaces has been studied by many authors, e.g., [1]-[5] ,[12]-[15] and references herein. These works are mainly on the characterization and on the uniqueness of best simultaneous approximations. Results on best simultaneous approximation in Köthe Bochner function space concerning the l^1

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sum has been studied in [7]. Recent interests are focused on the study of the best simultaneous approximation in conditional complete lattice Banach spaces with strong unit 1, e.g., [12],[15].

In the following , we recall some notions concerning vector lattices.

Definition 1. *A lattice* $(L,\leq)$ *is said to be conditionally complete if it satisfies one of the following equivalent conditions:*

(i)*Every non-empty lower bound set admits an infimum.*

(ii)*Every non-empty upper bound set admits a supremum.*

(iii)*There exists a complete lattice* $\overline{L} = L \cup \{\top,\bot\}$*, which we shall call minimal completion of* L *with bottom element* $\top$*and top element* $\bot$ *such that* L *is a sublattice of* $\overline{L}$*,* $\inf L = \bot$ *and* $\sup L = \top$.

Definition 2. *Areal vector lattice* $(X,\leq,+,\cdot)$ *is a set* X *endowed with a partial order* $\leq$ *such that* $(X,\leq)$ *is lattice with a binary operation* "$+$"*and a scalar product* "$\cdot$" *such that* $(X,\leq,+,\cdot)$ *is a vector space.*

Definition 3. *A vector lattice* $(X,\leq,+,\cdot)$ *such that* $(X,\leq)$ *is a conditionally complete lattice is called a conditionally complete vector lattice.*

Definition 4. *A conditionally complete Banach lattice space* X *is a real Banach space which is also a conditionally complete vector lattice such that*

$$|x| \leq |y| \Longrightarrow \|x\| \leq \|y\|,$$

for all $x,y \in X$ *, where* $|x| = x^{+} + x^{-}$*,* $x^{+} = \max\{x,0\}, x^{-} = \max\{-x,0\}$.

Throughout this paper, we always assume that X is a conditionally complete real Banach lattice space with strong unit 1. Then by Lemma 1, p.18 in [9], one has $\|\,|x|\,\| = \|x\|$, for each $x \in X$, and the norm in X is monontonic, that is

$$-y \leq x \leq y \Longrightarrow \|x\| \leq \|y\|,$$

for all $x,y \in X$.

In this paper we give a new characterization of the best simultaneous approximation to two functions in the Köthe Bochner Banach lattice function spaces.

Let $(T,\sum,\mu)$ be a finite complete measure space and let $L^0 = L^0(T)$ denotes the space of all (equivalence classes) of Σ-measurable real-valued functions. For $f,\ g\in L^0,\ f\leq g$ means that $f(t)\leq g(t)\ \mu-$almost every where, $t\in T$.

A Banach space $(E,\|\cdot\|_E)$ is said to be a Köthe space if :

(1) For $f,\ g\in L^0,\ |f|\leq|g|$ and $g\in E$ implies that $f\in E$ and $\|f\|_E\leq\|g\|_E$.

(2) For each $A\in\Sigma$, if $\mu(A)$ is finite, then $\chi_A\in E$. See [11].

The most prominent examples on the Köthe Bochner function spaces are the Lebesgue-Bochner fsunction paces $L^p(X)$, $(1\leq p<\infty)$, and their generalization the Orlicz-Bochner function spaces $L^{\Phi}(X)$.

Remark 5. *A Köthe space $(E,\|\cdot\|_E)$ is a Banach lattice under $\leq$,*

$(f\geqslant 0$ if $f(t)\geqslant 0$ for μ- alomost every where $t\in T)$.

Let E be a Köthe space on the measure space $(T,\sum,\mu)$, then $E(X)$ is the space of all equivalence classes of strongly measurable functions $f:T\to X$, such that $\|f(\cdot)\|_X\in E$ equipped with the norm:

$$|||f|||\ =\|\|f(\cdot)\|_X\|_E\,.$$

The space $(E(X),|||\cdot|||_E)$ is a Banach space called the Köthe Bochner function space induced by E and X [10].

A Köthe space E has absolutely continuous norm if, for each $f\in E$ and each sequence $(A_n)\searrow 0$, we have $\|\chi_{A_n}f\|_E\longrightarrow 0$.A Köthe space is said to be stictly monotone, if for $x\leq y$ and $\|x\|_E=\|y\|_E$ implies that $x=y$.

Remark 6. [4]. *If X is a Banach lattice, then the Köthe Bochner function space $E(X)$ is also a Banach lattice space.*

Let Y be a closed subspace of X. We define a norm on $X\times X$ by

$$\|(x_1,x_2)\|_\infty=\max\{\|x_1\|,\|x_2\|\}\,,for\ x_1,x_2\in X.$$

Where it is denoted by $X \underset{\infty}{\oplus} X$. Let $G = \{(y,y) : y \in Y\}$ with $\|(y,y)\|_\infty = \|y\|$, then $X \underset{\infty}{\oplus} X$ with $\|\cdot\|_\infty$ is a Banach lattice space with G a closed subspace of $X \underset{\infty}{\oplus} X$. We say that Y a simltaneous proximinal in X, if for any pair $x_1, x_2 \in X$, there exists an element $y_o \in Y$, satisfying

$$d(x_1, x_2, Y) = \inf_{y \in Y} \max\{\|x_1 - y\|, \|x_2 - y\|\} \tag{1}$$

$$= \max\{\|x_1 - y_0\|, \|x_2 - y_0\|\},$$

then y_o is called a best $\infty-$simultaneous proximinant of x_1, x_2 in X. We say that Y is $\infty-$simultaneously Chebyshev in X, if for any pair $x_1, x_2 \in X$, there exists a unique element $y_o \in Y$, satisfying inequality (1).

We note that Y is $\infty-$simultaneously proximinal in X if and only if Y is proximinal in $X \underset{\infty}{\oplus} X$.

Remark 7. *If $x_1 = x_2 \in X$, then $d(x_1, x_2, Y) = \inf_{y \in Y} \|x_1 - y\|$, where it seems that the best $\infty-$simultaneously proximanal is stronger than ordinary proximinality. Nevetheless Mendoza et. al. in* [12]*, shows that this is not the case in general for some characterization of simultaneous approximations.*

Theorem 8. [1]. *If X is a uniformly convex Banach space and Y is a closed subspace of X. Then Y is simultaneously Chebyshev in X.*

For a function $F = (f_1, f_2) \in (E(X))^2$, we define the norm of F by

$$|||F|||_\infty = \|\max\{\|f_1(\cdot)\|_X, \|f_2(\cdot)\|_X\}\|_E .$$

In this paper, for a given closed subspace Y of X and $F = (f_1, f_2) \in (E(X))^2$, we show that the existence of ordered pair $Y_0 = (g_0, g_0) \in (E(Y))^2$ such that the following infimum attained

$$|||F - Y_0|||_\infty = \inf_{g \in E(Y)} |||F - (g,g)|||_\infty$$

$$= \|\max\{\|f_1(\cdot) - g_0(\cdot)\|_X, \|f_2(\cdot) - g_0(\cdot)\|_X\}\|_E ,$$

means that the function g_0 is called a best $\infty-$simultaneous approximation of $F = (f_1, f_2)$.

Then $dist(f_1, f_2, E(Y))$ is defined by

$$dist(f_1, f_2, E(Y)) = \inf_{g\in E(Y)} \|\max\{\|f_1(\cdot) - g(\cdot)\|_X \,, \|f_2(\cdot) - g(\cdot)\|_X\}\|_E \,.$$

In this paper, we study the best $\infty-$simulataneous approximation on the Banach lattice space $E(X)$.

2. **Distance Formula**

For $f_1, f_2 \in E(X)$, the set $B_{E(Y)}(f_1, f_2, E(Y))$ is defined by

$$\{g \in E(Y) : \max\{\|f_1(\cdot) - g(\cdot)\|_X \,, \|f_2(\cdot) - g(\cdot)\|_X\} = dist(f_1, f_2, E(Y))\}.$$

It is clear that if $B_{E(Y)}(f_1, f_2, E(Y)) \neq \varphi$, then $E(Y)$ is $\infty-$simultaneous proximinal in $E(X)$.

Lemma 9. *Let $f_1, f_2 \in E(X)$, and $g : T \to Y$ be a strongly measurable function with $g(t) \in B_Y(f_1(t), f_2(t), Y)$, for almost all $t \in T$.Then $g \in E(Y) \cap B_{E(Y)}(f_1, f_2, E(Y))$.*

Proof. Since $g(t) \in B_Y(f_1(t), f_2(t), Y)$, for almost all $t \in T$, we have

$$\begin{aligned} \|g(t)\|_X &\leq \|f_1(t) - g(t)\|_X + \|f_1(t)\|_X \\ &\leq \max\{\|f_1(t) - g(t)\|_X \,, \|f_2(t) - g(t)\|_X\} + \|f_1(t)\|_X \\ &\leq \max\{\|f_1(t)\|_X \,, \|f_2(t)\|_X\} + \|f_1(t)\|_X \\ &\leq 2\|f_1(t)\|_X + \|f_2(t)\|_X \,, \end{aligned}$$

for almost all $t \in T$.Therefore,

$$|||g||| \leq 2|||f_1||| + |||f_2|||,$$

which shows that $g \in E(Y)$.

$$\max\{\|f_1(t) - g(t)\|_X \,, \|f_2(t) - g(t)\|_X\} \leq \max\{\|f_1(t) - h(t)\|_X \,, \|f_2(t) - h(t)\|_X\},$$

for all $h \in E(Y)$.

$$\|\max\{\|f_1(\cdot) - g(\cdot)\|_X \,, \|f_2(\cdot) - g(\cdot)\|_X\}\|_E \leq \|\max\{\|f_1(\cdot) - h(\cdot)\|_X \,, \|f_2(\cdot) - h(\cdot)\|_X\}\|_E \,.$$

Therefore, $g \in B_{E(Y)}(f_1, f_2, E(Y))$. □

We can now state and proof the main Theorem

Theorem 10. *Let $E(X)$ be a Köthe Bochner function space with absolutely continuous norm. If $f_1, f_2 \in E(X)$, then the distance function $dist_E(f_1, f_2, E(Y))$ belongs to E and*

$$\|d(f_1(\cdot), f_2(\cdot), Y)\|_E = dist(f_1, f_2, E(Y)).$$

Proof. Let $f_1, f_2 \in E(X)$, then there exist two sequence of simple functions in $E(X)$, $(f_{n,i})$, $i = 1, 2$, such that:

$$\|f_{n,i}(t) - f_i(t)\| \to 0,\ i = 1,2 \text{ , as } n \to \infty\text{, for almost all } t \text{ in } T.$$

The continuity of the distance function $d(x_1, x_2, Y)$, implies that:

$$\left|d(f_{n,1}(t),\ f_{n,2}(t), Y) - d(f_1(t),\ f_2(t), Y)\right| \to 0,\ \text{, as } n \to \infty,$$

Set

$$H_n(t) = d(f_{n,1}(t),\ f_{n,2}(t), Y),$$

then each H_n is a measurable function. Therefore, $d(f_1(\cdot), f_2(\cdot), Y)$ is measurable and

$$d(f_1(t),\ f_2(t), Y) \leq \max\{\|f_1(t) - z\|_X\ , \|f_2(t) - z\|_X\},$$

for all $z \in Y$, for each $t \in T$.
a consequence, we can write

$$d(f_1(t),\ f_2(t), Y) \leq \max\{\|f_1(t) - g(t)\|_X\ , \|f_2(t) - g(t)\|_X\},$$

for all $g \in E(Y)$, then

$$\|d(f_1(\cdot), f_2(\cdot), Y)\|_E \leq \|\max\{\|f_1(\cdot) - g(\cdot)\|_X\ , \|f_2(\cdot) - g(\cdot)\|_X\}\ \|_E\ .$$

This implies that $d(f_1(\cdot), f_2(\cdot), Y) \in E$ and

$$\|d(f_1(\cdot), f_2(\cdot), Y)\|_E \leq dist(f_1, f_2, E(Y)). \tag{2}$$

Fix $\varepsilon > 0$. Since $E(X)$ is a Köthe Bochner function space with absolutely continuous norm, then the simple functions are dense in $E(X)$, [10], therefore, there exist simple functions f_i^* in $E(X)$ such that

$$|||f_i - f_i^*||| < \frac{\varepsilon}{2}, \; for\; i = 1,2.$$

Assume that

$$f_i^*(t) = \sum_{k=1}^{m} x_k^i \; \chi_{A_k}(t), i = 1,2.$$

where the A_k's are pairwise disjoint measurable sets of T with $\bigcup_{k=1}^{m} A_k = T$, χ_{A_k}'s are characteristic functions of A_k's and $x_k^i \in X$, $k = 1,2,...,m$, $i = 1,2$. Here $\mu(T)$ is finite, so let $\alpha = |||\chi_T|||$. For each $k = 1,2,...,m$, let $y_k \in Y$ satisfy

$$\max\left\{\left\|x_k^1 - y_k\right\|_X, \left\|x_k^2 - y_k\right\|_X\right\} \leq d\left(x_k^1, x_k^2, Y\right) + \frac{\varepsilon}{\alpha}.$$

Mean while, by setting

$$g(t) = \sum_{k=1}^{m} y_k \; \chi_{A_k}(t),$$

for each $t \in T$, we can obtain the following inequality:

$$\begin{aligned}
&\max\{\|f_1^*(t) - g(t)\|_X, \|f_2^*(t) - g(t)\|_X\} \\
&= \sum_{k=1}^{m} \chi_{A_k}(t) \max\left\{\left\|x_k^1 - y_k\right\|_X, \left\|x_k^2 - y_k\right\|_X\right\} \\
&\leq \sum_{k=1}^{m} \chi_{A_k}(t) \left[d\left(x_k^1, x_k^2, Y\right) + \frac{\varepsilon}{\alpha}\right] \\
&= d\left(f_1^*(t), f_2^*(t), Y\right) + \frac{\varepsilon}{\alpha} \sum_{k=1}^{m} \chi_{A_k}(t)
\end{aligned}$$

Therefore,

$$\begin{aligned}
\|\max\{\|f_1^*(\cdot) - g(\cdot)\|_X, \|f_2^*(\cdot) - g(\cdot)\|_X\}\|_E &\leq \|d(f_1^*(\cdot), f_2^*(\cdot), Y)\|_E + \frac{\varepsilon}{\alpha} \left|\left|\left|\sum_{k=1}^{m} \chi_{A_k}\right|\right|\right| \\
&\leq \|d(f_1^*(\cdot), f_2^*(\cdot), Y)\|_E + \frac{\varepsilon}{\alpha} |||\chi_T||| \\
&= \|d(f_1^*(\cdot), f_2^*(\cdot), Y)\|_E + \varepsilon.
\end{aligned}$$

This gives the following inequality

$$\begin{aligned} dist\left(f_1, f_2, E\left(Y\right)\right) &\leq dist\left(f_1^*, f_2^*, E\left(Y\right)\right) + |||f_1 - f_1^*||| + |||f_2 - f_2^*||| \\ &< \left\|\max\left\{\left\|f_1^*\left(\cdot\right) - g\left(\cdot\right)\right\|_X, \left\|f_2^*\left(\cdot\right) - g\left(\cdot\right)\right\|_X\right\}\right\|_E + \varepsilon \\ &\leq \left\|dist\left(f_1^*\left(\cdot\right), f_2^*\left(\cdot\right), Y\right)\right\|_E + 2\varepsilon \\ &\leq \left\|dist\left(f_1\left(\cdot\right), f_2\left(\cdot\right), Y\right)\right\|_E + |||f_1 - f_1^*||| + |||f_2 - f_2^*||| + 2\varepsilon \\ &< \left\|dist\left(f_1\left(\cdot\right), f_2\left(\cdot\right), Y\right)\right\|_E + 3\varepsilon. \end{aligned}$$

So, we have

$$dist_E\left(f_1, f_2, E\left(Y\right)\right) < \left\|d\left(f_1\left(\cdot\right), f_2\left(\cdot\right), Y\right)\right\|_E + 2\varepsilon.$$

It holds that

$$dist_E\left(f_1, f_2, E\left(Y\right)\right) \leq \left\|dist\left(f_1\left(\cdot\right), f_2\left(\cdot\right), Y\right)\right\|_E. \tag{3}$$

Using inequalities (2) and (3) we get the required results. □

As a direct consequence of the previous is the following result:

Corollary 11. *Let $E\left(X\right)$ be the Köthe lattice Bochner function space with absolutely continuous and strictly monotone norm. For $f_1, f_2, \in E\left(X\right)$, $g \in B_{E(Y)}\left(f_1, f_2, E\left(Y\right)\right)$, it is necessary and sufficient that $g\left(t\right) \in B_Y\left(f_1\left(t\right), f_2\left(t\right), Y\right)$, for almost all $t \in T$.*

Next, we give the ∞−simultaneous proximinality of simple functions in $E(X)$:

Theorem 12. *If Y is ∞−simultaneously proximinal in X, then for every simple functions f_1, f_2 in $E(X)$, we have $B_{E(Y)}\left(f_1, f_2, E\left(Y\right)\right) \neq \emptyset$.*

Proof. Let f_1, f_2 be two simple functions in $E\left(X\right)$. Then f_1, f_2 can be written as

$$f_i(t) = \sum_{k=1}^{m} u_k^i\, \chi_{A_k}(t),\ i = 1, 2,$$

where A_k's are pairwise disjoint measurable sets of T with $\bigcup_{k=1}^{m} A_k = T$. Also, we assume that $\mu\left(A_k\right) > 0$ for each $k = 1, 2, ..., m$. By the assumption we know that for each $k = 1, 2, ..., m$, there exists a best $\infty-$simultaneous approximation w_k in Y of the pair of elements $\left(u_k^1\ ,u_k^2\ \right) \in X \underset{\infty}{\oplus} X$ such that

$$dist\left(x_k^1, x_k^2\ , Y\right) = \max\left\{\left\|u_k^1 - w_k\right\|_X, \left\|u_k^2 - w_k\right\|_X\right\}.$$

Set

$$g(t) = \sum_{k=1}^{m} w_k\ \chi_{A_k}(t),$$

then for any $\alpha > 0$ and $h \in E\left(Y\right)$, we obtain that

$$\begin{aligned}
&\left\|\max\left\{\left\|f_1\left(\cdot\right) - h\left(\cdot\right)\right\|_X\ , \left\|f_2\left(\cdot\right) - h\left(\cdot\right)\right\|_X\right\}\right\|_E \\
&\geq \left\|\sum_{k=1}^{m} \chi_{A_k}(\cdot)\left[\max\left\{\left\|u_k^1 - w_k\right\|_X, \left\|u_k^2 - w_k\right\|_X\right\}\right]\right\|_E \\
&= \left\|\max\left\{\left\|f_1\left(\cdot\right) - g\left(\cdot\right)\right\|_X\ , \left\|f_2\left(\cdot\right) - g\left(\cdot\right)\right\|_X\right\}\right\|_E.
\end{aligned}$$

Taking infimum over all $h \in E\left(Y\right)$, we get

$$dist\left(f_1, f_2, E\left(Y\right)\right) = \left\|\max\left\{\left\|f_1\left(\cdot\right) - g\left(\cdot\right)\right\|_X\ , \left\|f_2\left(\cdot\right) - g\left(\cdot\right)\right\|_X\right\}\ \right\|_E.$$

This implies that the simple functions $f_1,\ f_2$ admits a best $\infty-$simultaneous approximation in $E(X)$. □

Theorem 13. *Let $E\left(X\right)$ be the Köthe lattice Bochner function space with absolutely continuous and strictly monotone norm. If $E\left(Y\right)$ is $\infty-$simultaneous proximinal in $E\left(X\right)$, then Y is $\infty-$simultaneous proximinal in X.*

Proof. Let $x_1,\ x_2 \in X$. Set $f_i\left(t\right) = x_i\ \ (i = 1,2)$ for almost all $t \in T$. Since

$$\begin{aligned}
\left|\left|\left|f_i\right|\right|\right| &= \left\|\left\|f_i\left(\cdot\right)\right\|_X\right\|_E = \left\|\left\|x_i\ \ \chi_T\left(\cdot\right)\right\|_X\right\|_E \\
&= \left\|x_i\right\|_X\ \left|\left|\left|\chi_T\right|\right|\right|,\ \ (i = 1,2),
\end{aligned}$$

which is finite, then $f_i \in E\left(X\right)$, $(i = 1,2)$.

assumption there exists $g \in E\left(Y\right)$ such that

$$\left\|\max\left\{\left\|f_1\left(\cdot\right) - g\left(\cdot\right)\right\|_X\ , \left\|f_2\left(\cdot\right) - g\left(\cdot\right)\right\|_X\right\}\ \right\|_E \leq \left\|\max\left\{\left\|f_1\left(\cdot\right) - h\left(\cdot\right)\right\|_X\ , \left\|f_2\left(\cdot\right) - h\left(\cdot\right)\right\|_X\right\}\right\|_E$$

for all $h \in E(Y)$.

$E(X)$ is a Köthe Bochner function space with a strictly monotone norm, then for almost $t \in T$, we have

$$\max\{\|f_1(t)-g(t)\|_X \ , \|f_2(t)-g(t)\|_X\}$$
$$\leq \max\{\|f_1(t)-h(t)\|_X \ , \|f_2(t)-h(t)\|_X\}.$$

Fix $t_0 \in T$ and $y = g(t_0)$, then $y \in Y$ and

$$\max\{\|x_1-y\|_X \ , \|x_2-y\|_X\} \leq \max\{\|x_1-h(t)\|_X \ , \|x_2-h(t)\|_X\},$$

for all $h \in E(Y)$.

Y is embedded isometrically into $E(Y)$, then

$$\max\{\|x_1-y\|_X \ , \|x_2-y\|_X\} \leq \max\{\|x_1-z\|_X \ , \|x_2-z\|_X\},$$

for all $z \in Y$. □

Theorem 14. *If X and E are uniformly convex lattice real Banach spaces and Y is a closed subspace of X. Then $E(Y)$ is simultaneously Chebyshev in $E(X)$.*

Proof. If X and E are uniformly convex lattice real Banach spaces , then [6], implies that $E(X)$ is uniformly convex. Therefore, Theorem 8 gives the required result.

Conclusion 15. *In this paper we discuss the best simultaneous approximations to two functions in the Köthe Bochner function spaces in the l_∞sum sense. We give some results concerning the relation between the best simultaneous proximinality of Y the closed subspace of X and the best simultaneous proximinality of $E(Y)$ in $E(X)$. This characterization can be considered as a generalization of some analogous theorems concerning the Orlicz Bochner spaces and Lp Bochner spaces.*

□

Conflict of Interests

The authors declare that there is no conflict of interests.

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