Available online at http://scik.org
J. Math. Comput. Sci. 6 (2016), No. 5, 943-952
ISSN: 1927-5307

A FINITE ELEMENT MESH GENERATION FOR COMPLEX GEOMETRY

G. MANJULA, K.T. SHIVARAM* AND N.V. VIGHNESAM
Department of Mathematics, Dayananda Sagar College of Engineering, Bangalore, India

Abstract: A new approach to generate quadrilateral and pentagonal finite element meshes in convex, non convex and curved domain is presented. The method is based partially on triangle and pentagon, centroid of each triangle is considered joining the sides of its midpoints and discretised into quadrilaterals and pentagons. This procedure is applied to each triangle in the domain and a quadrilateral and pentagonal mesh is generated. For this a program has been developed in computer algebra and symbolic computational software MAPLE.

Keywords: Mesh generation; Finite element method; quadrilateral element; pentagonal element; Maple.
2010 AMS Subject Classification: 65L60.

1. Introduction

Finite element mesh generation is one of the most time consuming aspects for solving two dimensional partial differential equations in complex domain. The advent of modern computer technologies provided a powerful computational tool in numerical solutions of the partial differential equations over complex domains. A triangle quadrilateral and pentagonal mesh is required for finite element method as it uses in finite elements of a domain discretization. Finite Element Method (FEM) is widely used for many fields of engineering and applied science. Mesh generation technique is used in many industrial sectors such as automobile engineering, aerospace engineering, civil engineering, medical electronics, manufacturing and others. The Finite Element Analysis (FEA) and its applications comprises three phases i) Domain discretization or Mesh generation ii) Equation solving iii) Error analysis.

Mesh generation is the important role in the achievement of accurate solutions of the partial differential equation problems. FEM is a numerical solution technique that finds an approximate

*Corresponding author
Received June 13, 2016

solution by dividing the analysis region into smaller sub regions these smaller sub regions are the elements which are linked with adjacent elements at their nodel points. The procedure of the mesh generation technique depends on the geometric data of the elements and their nodes. First all the coordinate nodes are created and connecting nodes from arbitrary triangles. FEM is the computational basis of many computer aided design its usefulness is often hampered by the need to generate a quadrilateral or pentagonal mesh in complex domain. To creating a mesh in complex domain is the first step in a wide range of applications, including computer graphics, biomedicine, materials science and scientific and engineering computing. This can be very time consuming and error prone task when done manually. A large number of methods have been devised to automate the mesh generation task. Proposed paper is an attempt to create automatic mesh generation over convex, non convex and curved domain. From the literature survey a quadrilateral mesh generation in plane and curved surfaces in isoperimetric techniques presented by [1]. High quality of quadrilateral mesh gives the better accurate solution than a triangle mesh is given in [2],[3],[4]. A number of researchers [5],[6],[7] have made effort to develop adaptive Finite element method which integrates with error estimation and modification of automatic mesh generator. The research literature of various meshing methods and the applications of these meshes along with the development of mesh generation method are provided in [8]-[12]. The scope of the structured meshes and unstructured meshes by optimization method is discussed in [13]. There have been a number of approaches to develop automatic mesh generation based on partitioning the domain of interest into sets of sub domains and subsequently meshing those sub domains in medical electronics [14], finite element mesh can be usually categorized into structured and unstructured triangle or quadrilateral mesh. A structured mesh has a uniform defined structure that unstructured meshes lack the uniformity. The automatic generation of both structured and unstructured meshes is non trivial task and each comes with challenges of their own even though for most of the appropriate numerical solution [15], [16]. In this paper. we propose a new technique for generating quadrilateral and pentagonal meshes for geometry specified in the coordinate system. First we decompose the convex and non convex polygonal domain into sub triangles and every arbitrary triangle is again discretized into three quadrilaterals by adding three vertices in the middle of the edges and a node at the centre of the linear triangular element. The subsequent quadrilateral and pentagonal meshes are developed by Maple 13, we demonstrated. In section 2, we present arbitrary triangle divided into 2^2, 3^2, 4^2,

$5^2,\ldots.,n^2$ sub triangles of equal size to generate triangular mesh. In section 3, we explain the procedure to split these arbitrary triangle into quadrilaterals. In section 4, we present several examples to illustrate the quadrilateral and pentagonal meshes in complex domain.

2. Discretization of Triangles

2.1. Subdivision of an Linear Arbitrary Triangle

a) 2^2 sub triangle b) 4^2 sub triangles

Fig. 1 Division of an arbitrary triangle into sub triangles in Cartesian space

Figure 1a and 1b show the division of arbitrary triangle into 2^2, 3^2, 4^2, $5^2,\ldots.n^2$ sub triangles of equal size in the Cartesian coordinates. The sides of the triangle is divided into equal parts with same length, the area of the linear arbitrary triangle Δ with vertices $(\,(x_i, y_i),\ i = 1,2,3)$ is equal to sum of the area of the sub triangles Δ/n^2 where n is the equal division of all sides in the Cartesian space.

2.2. Division of Arbitrary triangle into quadrilaterals

a) 3 – Quadrilaterals b) 12 – Quadrilaterals

Fig. 2. Division of an arbitrary linear triangle into quadrilaterals

We first discretize arbitrary triangle into three quadrilaterals by joining the centroid of the midpoints of sides of the linear triangle. The area of the arbitrary linear triangle with vertices $(\,(x_i, y_i),\ i = 1,2,3)$ is same as the sum of the three quadrilaterals. to divide the arbitrary linear triangle into six node triangles of equal size, then by joining the centroid of

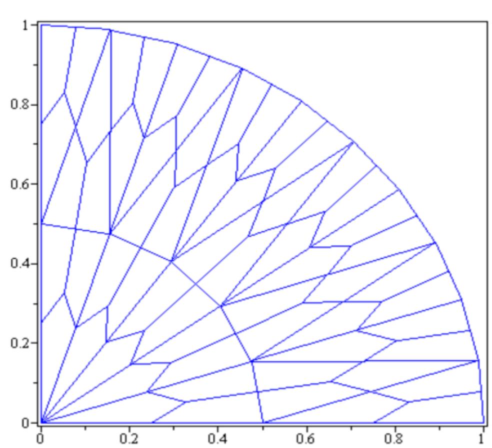
$C(\frac{x_i+x_j+x_k}{3}, \frac{y_i+y_j+y_k}{3})$ these six nodes, linear triangle to the midpoints of their sides. To illustrated the same process for the two or more divisions of sides of the arbitrary linear triangles to converted into quadrilaterals.

3. Subdivision of an Linear Arbitrary pentagon

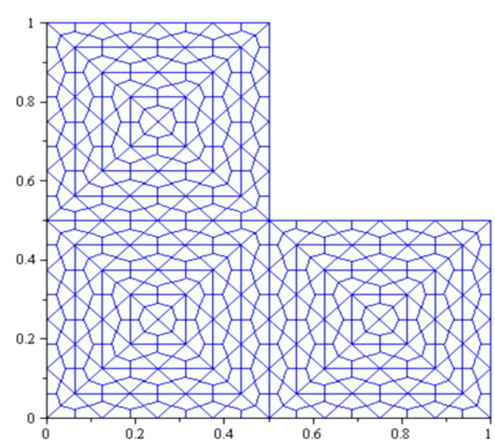
a) Regular pentagon b) 6 - Pentagons

Fig.3 Division of an arbitrary pentagon into sub pentagons in Cartesian space

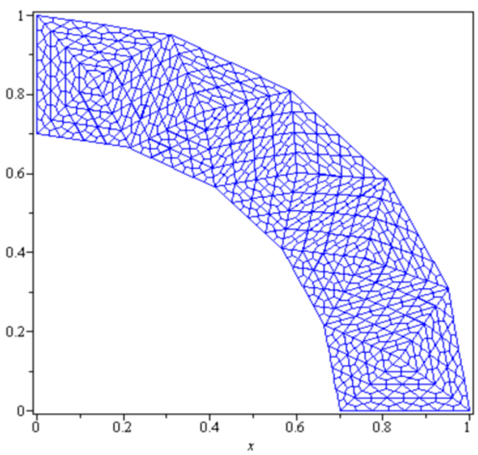
We first divided arbitrary pentagon into 6 sub pentagon of equal size in the Cartesian coordinates, the sides of the pentagon is divided into equal parts with same length, the area of the linear arbitrary pentagon with vertices ((x_i, y_i), $i = 1, 2, 3,4,5$) is equal to sum of the area of the sub pentagons. We can generate pentagonal mesh in polygonal domain. We first discretised the polygonal domain into pieces of arbitrary pentagons and each pentagonal domain is again discretised into pentagons. Finally, we can generate pentagonal mesh in polygonal domain. We have included some meshes generated by using maple program. We further illustrated the applications of the algorithm by generating quadrilateral and pentagonal meshes in convex, non-convex polygonal and curved domain.



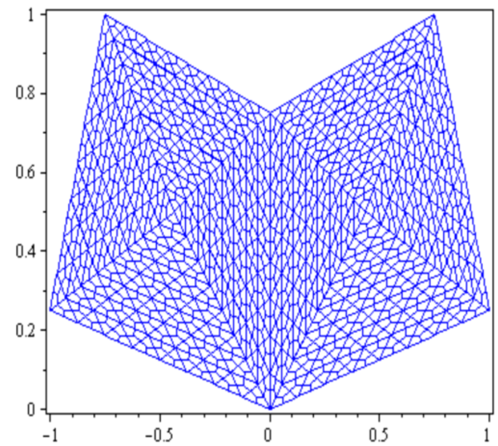
Mesh 1



Mesh 2



Mesh 3



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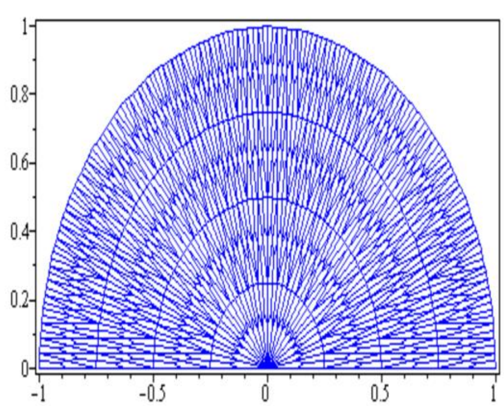
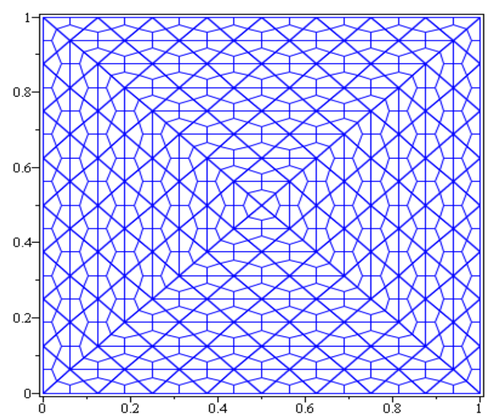
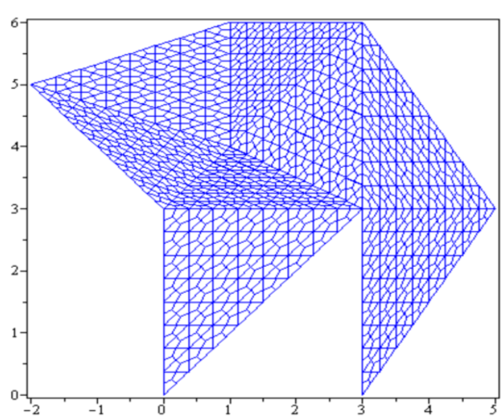


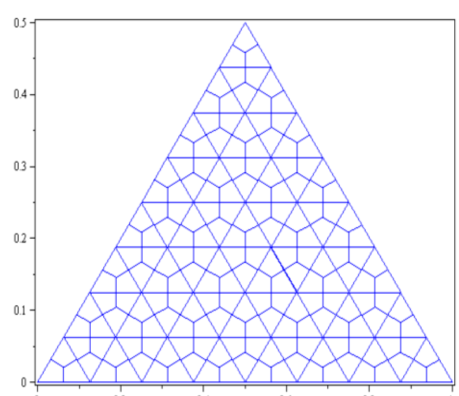
Mesh 5



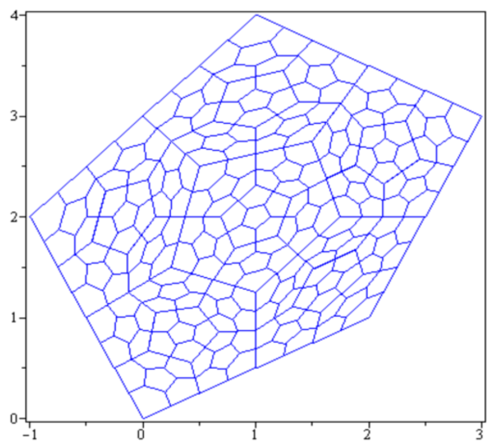
Mesh 6



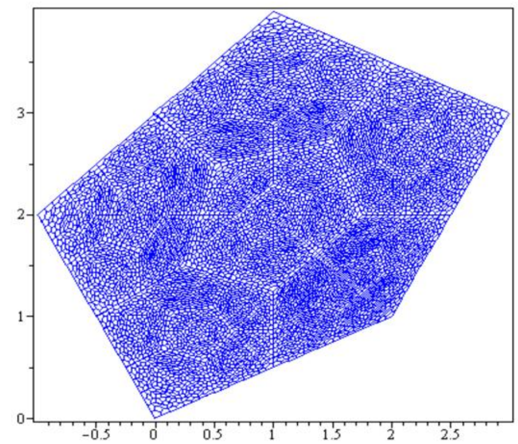
Mesh 7



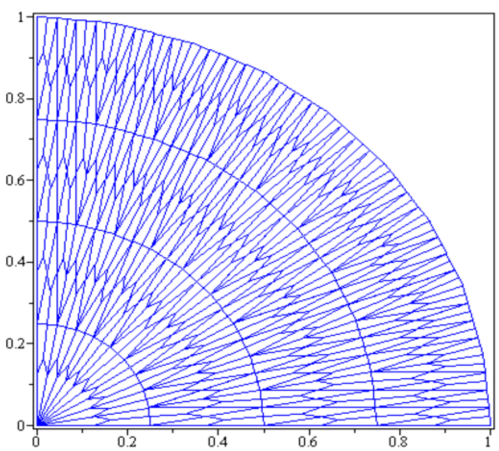
Mesh 8



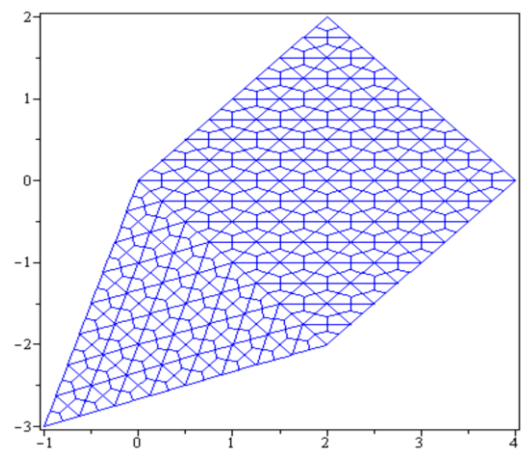
Mesh 9



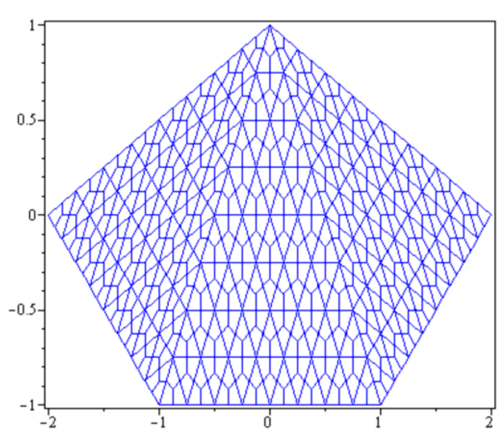
Mesh 10



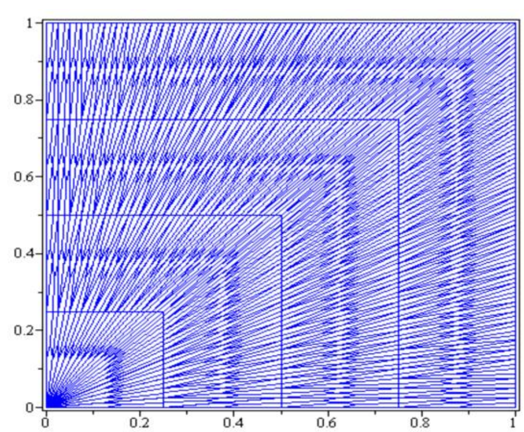
Mesh 11



Mesh 12



Mesh 13



Mesh 14

4. Application Examples

The mesh generation technique is applied to solve the Saint Venant Torsion problem in triangle and circle domain, the boundary value problem is described by

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=-2\,,\quad \text{within T or C} \qquad (1)$$

$$\phi=0\,,\ \text{on}\ \Delta_1\,,\quad \frac{\partial\phi}{\partial n}=0,\ \text{on}\ \Delta_2$$

Using Galerkin weighted residual method. The numerical solution of eq.(1) for the domain T or C expressed as

$$[K]_{M\times M}\ \{U\}_{M\times 1}=\{F\}_{M\times 1} \qquad (2)$$

Where $K_{i,j}=\iint_{T\ or\ C}\left(\frac{\partial N_i}{\partial u}\frac{\partial N_j}{\partial u}+\frac{\partial N_i}{\partial v}\frac{\partial N_j}{\partial v}\right)dx\,dy$

$$=K_{u,u}+K_{v,v} \qquad (3)$$

$$K_{u,u}=\int_{-1}^{1}\int_{-1}^{1}\left(\frac{\partial N_i}{\partial \xi}\frac{\partial v}{\partial \eta}+\frac{\partial N_i}{\partial \eta}\frac{\partial v}{\partial \xi}\right)*\left(\frac{\partial N_j}{\partial \xi}\frac{\partial v}{\partial \eta}+\frac{\partial N_j}{\partial \eta}\frac{\partial v}{\partial \xi}\right)\frac{1}{J}\ d\xi\,d\eta \qquad (3a)$$

$$K_{v,v}=\int_{-1}^{1}\int_{-1}^{1}\left(\frac{\partial N_i}{\partial \xi}\frac{\partial u}{\partial \eta}+\frac{\partial N_i}{\partial \eta}\frac{\partial u}{\partial \xi}\right)*\left(\frac{\partial N_j}{\partial \xi}\frac{\partial u}{\partial \eta}+\frac{\partial N_j}{\partial \eta}\frac{\partial u}{\partial \xi}\right)\frac{1}{J}\ \mathrm{d}\xi\ \mathrm{d}\eta \qquad (3b)$$

$$F_i=\int_{-1}^{1}\int_{-1}^{1}f\big(u(\xi,\eta),v(\xi,\eta)\big)\,J\,N_i(\xi,\eta)\,d\xi\,d\eta \qquad (3c)$$

Integrals of Eqn.3a-c are calculated for each quadrilateral elements by Gauss Legendre quadrature rule and assembling is performed to add the effect of all quadrilateral elements into account with boundary condition, finding all unknown's of u of eqn. [2] and these are contour plotted in Fig. 4- 5.

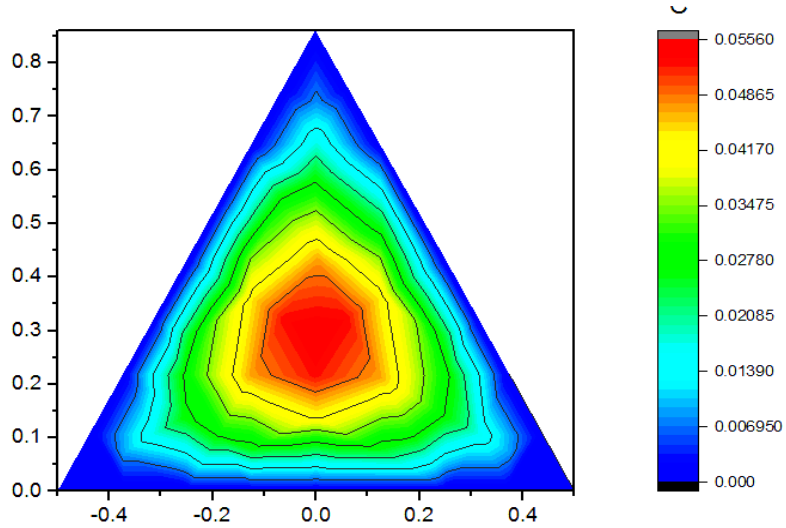


Fig. 4 Stress function for bar of triangle cross section of Mesh 8

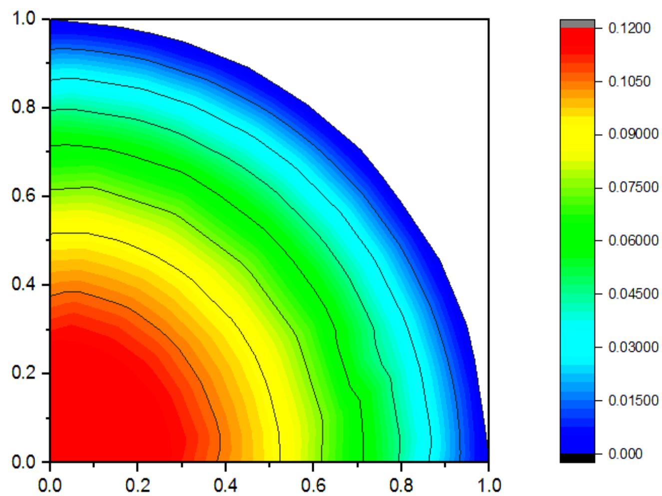


Fig. 5 Stress function for bar of circle cross section of Mesh 1

5 . Conclusions

In this paper, a new technique for automatic quadrilateral and pentagonal meshes are generated in convex, non convex and curved domain is proposed. The method is based on considering arbitrary triangle and each triangle is discretized into three quadrilaterals by adding three vertex in the middle of the edges and a vertex at the centroid of the linear triangular element. A simplified version of the method is demonstrated using MAPLE program. is found that more accuracy is obtained by when dense quadrilateral mesh is considered comparing with triangle mesh for solving in partial differential equation problems

Conflict of Interests

The authors declare that there is no conflict of interests.

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Appendix