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VAGUE SOFT SET THEORY AND DECISION MAKING

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Abstract. Fuzzy max-minima and fuzzy soft max-minima approaches have been discussed in the literature. We introduce and discuss vague soft maxima-minima approach, and by using these techniques we solve the problems in decision making theory. When vague soft relations are made, among these relations which ever object have a higher membership grade is selected, that object plays a vital role in decision making. It contains miscellaneous conclusions for advanced research and scientific applications in Engineering.
We generalized a fuzzy soft max-minima approach towards vague soft maxima-minima approach and apply it towards decision making theory. Finally, we apply these techniques in decision making theory. We introduce vague soft Max-Min method and Minimization, We adopt soft set notions and operations towards applications in decision making theory.

Keywords: Soft sets; vague soft maxima-minima; vague soft relation.

2010 AMS Subject Classification: 08A72, 03E72.

1. Introduction

Theory of probability, theory of fuzzy sets [9], theory of intuitionistic fuzzy sets [10], theory of vague sets [3], theory of interval mathematics [4], and theory of rough sets [8] have been

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introduced in the literature and considered the best mathematical tools for dealing with uncertainties. But all most all these theories have their own difficulties, and the reasons for these difficulties are may be some imperfections in these theories. To fill the gapes, Molodtsov [7] introduced the concept of soft theory to deal with uncertainties which is free from the above difficulties. In [7], Molodtsov showed that to fix uncertainties, soft set theory works more efficient than any other tool.

In [2] authors discussed soft groups, soft subgroups and few of their results. In [1] soft rings, soft ideals of soft rings have been introduced, furthermore the authors also introduced idealistic soft rings.

Throughout R will represent a commutative ring or an integral domain, we will clarify it regularly in our discussion whenever it is required.

Many theories have been introduced to deal with uncertainties in the literature, among them theory of fuzzy sets [9], theory of vague sets [3], theory of interval mathematics [4], theory of rough sets [8] are very important and famous. But in [7], Molodtsov showed that to fix uncertainties, soft set theory works more efficient than any other tool.

Soft set theory is not only applicable to remove uncertainities but has its applications in decision making theory for instance see [5]. In [5] and [5] authors presented the applications of soft sets in a decision making problem.

Fuzzy soft matrix theory and max-minima approches have been introduced and applied in decision making theory [13].Vague soft set theory has been estabilished in [14]. Applications of fuzzy soft set have been discussd in [12]. Furthermore, soft matrix theory and its application in decision making theory has been introduced and discussed in [11]. In [11] authors also introduced some useful operations on soft matrices such as products of soft matrices and so on, finally a soft max-min method has been efficiently applied to the problems that contain uncertainties.

Presentely, the applications of soft set theory includes the solution of the problems on the basis of the rough sets or fuzzy soft sets. In this note, we utilize the soft matrices which represents the soft sets. We incorporate the vague set theory in the soft matries and introduce vague soft matrices, we also construct a vague soft decision making which is more can be successfully applied to many problems to remove uncertainties by using the vague soft sets. We introduce and discuss vague soft maxima-minima approach and by using these techniques we also solve the problems in decision making theory. When vague soft relations are made, among these relations we select the object having higher membership grade, because that object plays a vital role in decision making. This study contains various conclusions for advanced research and scientific applications in many branches of Engineering. Basically we generalized a fuzzy soft max-minima approach in terms of vague soft maxima-minima and further we apply it towards solving decision making problems.

For basic terminologies one may consult [7], [5] and [11]. However we recall few useful definitions and terminologies.

3. Main results

Definition 1. [7]Let U be an initial universe, $P(U)$be the power set of U, E be the set of all parameters and $A \subseteq E$. A soft set (f_A, E) on the universe U is defined by the set of ordered pairs $(f_A, E) = \{(e, f_A(e)) : e \in E, f_A(e) \in U\}$ where $f_A : E \to P(U)$ such that $f_A(e) = \emptyset$ if $e \notin A$. Here, f_A is called an approximate function of the soft set (f_A, E). The set f_A is called e-approximate value set or e-approximate set which consists of related objects of the parameter $e \in E$.

Definition 2. [11]Let (f_A, E) be a soft set over U. Then a subset of $U \times E$ is uniquely defined by $R_A = \{u, e) : e \in A, u \in f_A(e)\}$, which is called a relation form of (f_A, E). The characteristic form of R_A is written by

$\chi_{R_A} : U \times E \to \{0, 1\}$, $\chi_{R_A}(u, e) = \{\frac{1, (u, e) \in R_A}{0, (u, e) \notin R_A}$. If $U = (u_1, u_2, ...u_n)$, $E = (e_1, e_2, ...e_n)$ and $A \subseteq E$, then R_A can be presented by the table as below

If $a_{ij} = \chi_{R_A}(u_i, e_j)$ we can define a matrix

$$\begin{matrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & & & & & \\ . & & & & & \\ . & & & & & \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{matrix}$$

which is called an $m \times n$ soft matrix of the soft set (f_A, E) over U.

According to this defination, a soft set (f_A, E) is uniquely characterized by the matrix$[a_{ij}]_{m\times n}$. It means that a soft set (f_A, E) is formally equal to its soft matrix$[a_{ij}]_{m\times n}$. Therefore, we shall identify any soft set with its soft matrix and use these two concepts as interchangeable.

[11]The set of all $m\times\ n$ soft matrices over U will be denoted by $SM_{m\times n}$.From now on we shall delete the subscripts $m\times\ n$ of $[a_{ij}]_{m\times n}$,we use $[a_{ij}]$ instead of $[a_{ij}]_{m\times n}$,since $[a_{ij}] \in SM_{m\times n}$ mean that $[a_{ij}]$ is an $m\times\ n$ soft matrix for $i = 1, 2, ..., n$ and $j = 1, 2, ..., m$.

Definition 3. Following [11] let $[a_{ij}] \in SM_{m\times n.}$Then $[a_{ij}]$ is called

(a) A zero soft matrix, denoted by$[0]$, if $a_{ij} = 0$ for all i and j.

(b) An A-universal soft matrix, denoted by$[\tilde{a_{ij}}]$, if $a_{ij} = 1$ for all $j \in\ I_A\ = \{j : e_j\ \in\ A\}$ and i.

(c) A universal soft matrix, denoted by$[1]$, if $a_{ij}\ = 1$ for all i and j.

Vague sets

Following [3], Let U be an initial universe set, $U = \{u_1, u_2, ...u_n\}$. A vague set over U is characterized by a truth-membership function t_v and a false-membership function f_v, $t_v :\ U \to [0, 1]$, where $t_v(u_i)$ is a lower bound on the grade of membership of u_i derived from the evidence for u_i, $f_v(u_i)$ is a lower bound on the negation of u_i derived from the evidence against u_i, and $t_v(u_i) + f_v(u_i) \leq 1$. The grade of membership of u_i in the vague set is bounded to a subinterval $[t_v(u_i), 1 - f_v(u_i)]$ of $[0, 1]$. The vague value $[t_v(u_i),\ 1 - f_v(u_i)]$ indicates the exact grade grade of membership $\mu_v(u_i)$ of u_i may be unknown, but is bounded by $t_v(u_i) \leq \mu_v(u_i) \leq 1 - f_v(u_i)$, where $t_v(u_i) + f_v(u_i) \leq 1$.

when the universe U is continuous, a vague set A can be written as

$A = \int_U [t_A(u_i),\, 1 - f_A(u_i)]/u_i, \qquad u_i \in U.$

when the universe U is discrete, a vague set A can be written as

$A = \sum_{i=1}^{n} [t_A(u_i),\, 1 - f_A(u_i)]/u_i, \qquad u_i \in U.$

For a vague set, Gau and Buehrer have introduced the following definitions, which will be useful to understand the subsequent discussion.

Definition 4. [3]Let x be a vague value, $x = [t_x, 1 - f_x]$, where $t_x \in [0,1]$, $f_x \in [0,1]$, and $0 \le t_x \le 1 - f_x \le 1$. if $t_x = 1$ and $f_x = 0$ ($i.e.$, $x = [1,1]$), then x is called unit vague value. If $t_x = 0$ and $f_x = 1$($i.e.$, $x = [0,0]$), then x is called a zero vague value.

Definition 5. [3]Let x and y be two vague values, where $x = [t_x, 1 - f_x]$ and $y = [t_y, 1 - f_y]$. If $t_x = t_y$ and $f_x = f_y$, then vague value x and y are called equal (i.e.,$[t_x, 1 - f_x] = [t_y, 1 - f_y]$).

Definition 6. [3] Let $A$and B be two vague sets of the universe $U = \{u_1, u_2, ..., u_n\}$,where $A = [t_A(u_1), 1 - f_A(u_1)]/u_1 + [t_A(u_2), 1 - f_A(u_2)]/u_2 + ... + [t_A(u_n), 1 - f_A(u_n)]/u_n$, $B = [t_B(u_1), 1 - f_B(u_1)]/u_1 + [t_B(u_2), 1 - f_B(u_2)]/u_2 + ... + [t_B(u_n), 1 - f_B(u_n)]/u_n$. Let A be a vague set of the universe U. If $\forall u_i \in U$, $t_A(u_i) = 1$ and $f_A(u_i) = 0$, then A is called a unit vague set, Where $1 \le i \le n$. If $\forall u_i \in U$, $t_A(u_i) = 0$ $\forall u_i \in U$, $t_A(u_i) = 1$, then A is called a zero vague set, where $1 \le i \le n$.

Definition 7. [3]The complement of a vague set A is denoted by A^c and is defined by $t_{A^c} = f_A$, $1 - f_{A^c} = 1 - t_A$.

Definition 8. [3]Let A and B two vague sets of the universe U. If $\forall u_i \in U, [t_A(u_i), 1 - f_A(u_i)] = [t_B(u_i), 1 - f_B(u_i)]$,then the vague set A and B are called equel, where$1 \le i \le n$.

Definition 9. [3]Let A and B be two vague set of universe U. If $\forall u_i \in U, t_A(u_i) \le t_B(u_i)$ and $1 - f_A(u_i) \le 1 - f_B(u_i)$, then the vague set A are included by B, denoted by $A \subseteq B$, where $1 \le i \le n$.

Definition 10. [3]The union of two vague set$A$ and B is a vague set C, written as$C = A \cap B$, whose truth-membership and false-membership function are related to those of A and B by $t_C = max(t_A, t_B), 1 - f_C = max(1 - f_A, 1 - f_B) = 1 - min(f_A, f_B)$.

Definition 11. [3]The intersection of two vague sets A and B is a vague set C, written as $C = A \cap B$, whose truth-membership and false-membership function are related to those of A and B by $t_C = min(t_A, t_B), 1 - f_C = min\ (1 - f_A, 1 - f_B) = 1 - max(f_A, f_B)$.

Here we persent few relevent definitions from the literarure about Vague soft sets. A soft set is a mapping from a set of parameters to the power set of a universe set. However, the notion of soft set, as given in its definition, cannot be used to represent the vagueness of the associated parameters. In this section, we introduced the concept of a vague soft set based on soft set theory and vague set theory. The basic properties of a vague soft set will be discissed. Let U be a universe,E a set of parameters, $V(U)$ the power set of vague sets on U, and $A \subseteq E$.The concept of a vague soft set is given by the following definition.

Definition 12. [14]A pair $(\widehat{F}, A)$ is called a vague soft set over U, where $\widehat{F}$ is a mapping given by $\widehat{F} : A \to V(U)$. In other words, a vague soft set over U is a parameterized family of vague set of the universe U. For $\varepsilon \in A$, $\mu_{\widehat{F}(\varepsilon)} : U \to [0,1]^2$ is regarded as the set of ε-approximate elements of the vague soft set$(\widehat{F}, A)$.

Definition 13. [14]For two vague soft sets $(\widehat{F}, A)$ and $(\widehat{G}, B)$ over a universe U, we say that $(\widehat{F}, A)$ is a vague soft subset of $(\widehat{G}, B)$, if A$\subseteq B$ and $\forall \varepsilon \in A$, $\widehat{F}(\varepsilon)$ and $\widehat{G}(\varepsilon)$ are identical approximations. This relationship is denoted by $(\widehat{F}, A) \tilde{\subseteq} (\widehat{G}, B)$. similarly,$(\widehat{F}, A)$ is said to be a vague soft super set $(\widehat{G}, B)$, if $(\widehat{G}, B)$ is a vague soft subset of $(\widehat{F}, A)$. we denoted by $(\widehat{F}, A) \tilde{\supseteq} (\widehat{G}, B)$.

Definition 14. [14]Two vague soft sets $(\widehat{F}, A)$ and $(\widehat{G}, B)$ over a universe $U$are said to be vague soft equal if $(\widehat{F}, A)$ is a vague soft subset of $(\widehat{G}, B)$ and $(\widehat{G}, B)$ is a vague soft subset of $(\widehat{F}, A)$.

Definition 15. [14]Let $E = \{e_1, e_2, ..., e_n\}$ be a parameter set. The not set of E denoted by $\neg E$ is defined by $\neg E = \{\neg e_1, \neg e_2, ..., \neg e_n\}$ where $\neg e_i$ =not e_i.

Definition 16. [14]The complement of vague soft set $(\widehat{F}, A)$ is denoted by $(\widehat{F}, A)^c$ and is define by $(\widehat{F}, A)^c = (\widehat{F}^c, \neg A)$ where $\widehat{F}^c ; \neg A \to V(U)$ is a mapping given by $t_{\widehat{F}(\alpha)^c}(x) = f\widehat{F}_{(\neg\alpha)}(x), 1 - t_{\widehat{F}(\alpha)^c}(x) = 1 - t_{\widehat{F}_{(\neg\alpha)}}(x), \forall \alpha \in \neg A, x \in U$.

For the rest of definitions and concepts I refer [14]. Finally here we represent an application of vague soft matrix in decision making. Assume that a real estate agent has a set of different types of houses$U = \{u_1, u_2, ...u_n\}$ hich may be characterized by a set of parameters $E = \{e_1, e_2, ...e_n\}$ where $j = 1, 2, ...n$ the parameters e_j stand for “in good location”, “cheap”, “modern”, “large”, respectively. Then we can give the following example.

Definition 17. Suppose that a married couple, $Mr.X$ and $Mrs.X$, come to the real estate agent to buy a house. If each partner has to consider their own set of parameters, then we select a house on the basis of the sets of partners’ parameters by using the SVMmDM (Vague soft matrix) as follows. Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $\{e_1, e_2, e_3, e_4, e_5\}$ is a set of all parameters.

Step 1: First, Mr.X and Mrs.X have to choose the sets of their parameters, $A = \{e_2, e_3, e_4\}$ *and* $B = \{e_1, e_3, e_4\}$ *respectively.*

Step 2: Then we can write the following vague soft matrices which are constructed according to their parameters

$$[a_{ij}] = \begin{matrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{matrix} \qquad [b_{ij}] = \begin{matrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{matrix}$$

Step 3: Now, we can find a product of the soft matrices $[a_{ij}]$ *and* $[b_{ij}]$ *by using And-Product as follows*

$$[a_{ij}] \wedge [b_{ij}] \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{matrix}$$

Example 1. Here, we use And-product since both Mr. X and Mrs. X’s choices have to be considered.

Step 4: We can find a $max - min$ decision soft matrix as; $Mm([a_{ij}] \wedge [b_{ij}]) = [1\ 0\ 0\ 0\ 0]^T$

Step 5: Finally, we can find an optimum set of U according to $Mm([a_{ij}] \wedge [b_{ij}])$ $\mathrm{opt}_{Mm([a_{ij}] \wedge [b_{ij}])}$ where u_1 is an optimum house to buy for $Mr.X$ and $Mrs.X$. Note that the optimal set of U may contain more than one element.

Similarly, we can also use the other products $[a_{ij}] \wedge [b_{ij}]$ and $[a_{ij}] \overline{\wedge} [b_{ij}]$ for the other convenient problem.

Conflict of Interests

The authors declare that there is no conflict of interests.

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