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MAGNETOHYDRODYNAMIC NATURAL CONVECTIVE FLOW PAST A VERTICAL CONE WITH VARIABLE SURFACE TEMPERATURE IN PRESENCE OF HEAT GENERATION

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Abstract: Natural convective flow past a vertical cone maintained at non-uniform surface temperature in presence of heat generation and magnetic field is considered. The governing boundary layer equations are first transformed into a non-dimensional form and the resulting nonlinear system of partial differential equations are then solved numerically using the finite difference method. The results of the surface shear stress in terms of local skin friction and rate of heat transfer in terms of local Nusselt number, velocity distribution as well as temperature distribution are shown graphically for a selection of parameter sets consisting of the surface temperature gradient n, Prandtl numbers Pr, the heat generation parameter Q and the magnetic parameter M.

Keywords: heat generation; magnetohydrodynamic; natural convection; vertical cone.
2010 AMS Subject Classification: 76W05.

1. INTRODUCTION

Natural convection flow and heat transfer problems are of important consideration in the thermal design of a variety of industrial equipment and also in nuclear reactors, geo-physical fluid dynamics. When a heated surface is in contact with the fluid, the result of temperature difference causes buoyancy force, which induces natural convection heat transfer. Merk and Prins [1-2] developed the general relations for similar solutions on isothermal axisymmetric forms and showed that the vertical cone has such a solution. Approximate boundary layer techniques were utilized to arrive at an expression for dimensionless heat transfer. Similarity solution for natural convection from the vertical cone has been exhausted by Hering and Grosh

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[3]. They showed that the similarity solutions to the boundary layer equations for a cone exist when the wall temperature distribution is a power function of distance along a cone ray. Later Hering [4] extended the analysis to investigate for low Prandtl number fluids. On the other hand Roy [5] has studied the same problem for high values of Prandtl number. Further, Pop and Takhar [6] have studied the compressibility effects in laminar free convection from a vertical cone, while Hossain and Paul [7], [8] have considered the effect of suction and injection when the cone surface is permeable.

The effect of slenderness on the natural convection flow over a slender frustum of a cone has studied. The problem of natural convection flow over a frustum of a cone without traverse curvature effect has been treated in the literature, even though the problem for a full cone has been considered quite extensively. On the other hand, the overall heat transfer in laminar natural convection flow from a vertical cone by using the integral method has investigated.

A study of the flow in presence of a magnetic field is important from the technical point of view, and such types of problems have received much attention by many researchers. Kuiken [9] studied the problem of MHD natural convection in a strong cross-field. Chowdhury and Islam [10] investigated MHD natural convection flow of visco-elastic fluid past an infinite porous plate. Hydromagnetic convection from a cone and a wedge with variable surface temperature and internal heat generation or absorption were studied. Hossain [11] introduced the viscous and joule heating effects on MHD-natural convection flow with variable plate temperature. Moreover, Hossain *etal.* [12-13] discussed both forced and natural convection boundary layer flow of an electrically conducting fluid in presence of magnetic field.

A large number of physical phenomena involve natural convection driven by heat generation. The study of heat generation in moving fluids is important in view of several physical problems such as those dealing with chemical reactions and those concerned with dissociating fluids. The heat transfer characteristics in the laminar boundary layer of a viscous fluid over a linearly stretching continuous surface have been studied with viscous dissipation or frictional heating and internal heat generation. In this study they considered that the volumetric rate of heat generation q'''[W/m^3], should be

$$q''' = \begin{cases} Q_0(T - T_\infty) & \text{for} \quad T \geq T_\infty \\ 0 & \text{for} \quad T < T_\infty \end{cases}$$

where $Q_0$is the heat generation constant. Hossain et al. [13] also discussed the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation. Following [13], Molla et al. [14], [15] investigated the natural convection with heat generation along a uniformly heated horizontal circular cylinder, and sphere, respectively.

The present work considers the natural convection boundary layer flow past a vertical cone with uniform surface temperature in presence of magnetic field and heat generation. The governing partial differential equations are reduced to partial differential equation, under the usual Boissinesq approximation. The transformed boundary layer equations are solved numerically by using finite difference method. Solutions are presented in terms of Prandtl numbers, different values of surface temperature gradient for the values of magnetic parameters with the heat generation parameters.

2. MATHEMATICAL FORMALISM

A steady two-dimensional laminar natural convection flow past a non-isothermal vertical permeable cone with variable surface temperature in presence of heat generation and magnetic field is considered. The effect of viscous dissipation on thermal boundary layer is neglected. The physical coordinates (x, y) are chosen such that x is measured from the leading edge, O, in the stream wide direction and y is measured normal to the surface of the cone. The co-ordinate system and flow configuration are shown in Fig: 1.

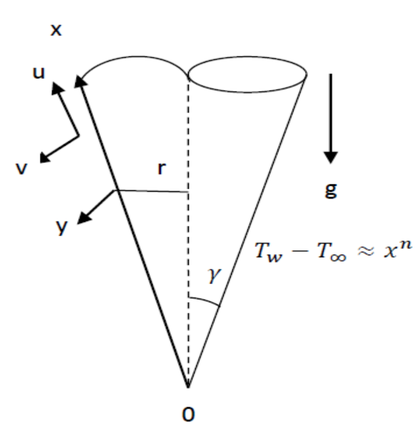

Fig. 1. Physical model and coordinate system

Under the Boussinesq and boundary layer approximations, the governing equations for mass continuity, momentum and energy take the following forms:

$$\frac{\partial(ur)}{\partial x}+\frac{\partial(vr)}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta\cos\cos\gamma\left(T-T_\infty\right)-\frac{\alpha_0\beta_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho C_p}\left(T-T_\infty\right) \tag{3}$$

Boundary conditions for the equations (1) to (3) are

$$u=0, v=-V, T=T_w \text{ at } y=0$$
$$u\to 0, T\to T_\infty \text{ at } y\to\infty \tag{4}$$

Volumetric rate of heat generation

$$q'''=\begin{cases} Q_0(T-T_\infty) & \text{for} \quad T\ge T_\infty \\ 0 & \text{for} \quad T<T_\infty \end{cases} \tag{5}$$

where u, v are the fluid velocity components in the x- and y-directions, respectively, $\nu=\mu/\rho$, is the kinematic coefficient of viscosity, g is the acceleration due to gravity, β is the coefficient of thermal expansion, α is the thermal diffusivity, γ is the cone apex half-angle, ρ is the density, C_p is the specific heat at constant pressure, α_0 is the electrical conduction, β_0 is the strength of the magnetic field and *T* is the temperature of the fluid. Here, *V* represents the transpiration velocity of the fluid through the surface of the cone. T_∞ is the ambient fluid temperature, T_w is the surface temperature with $T_w > T_\infty$. When *V* is positive, it stands for suction or withdrawal and *V* is negative for injection or blowing of fluid through the surface of the cone. In this investigation we have considered only suction case and therefore, *V* is taken as positive throughout. The amount of heat generated or absorbed per unit volume is, $Q_0\left(T-T_\infty\right)$, Q_0 being a constant, which may take either positive or negative. The source term represents the heat generation when $Q_0>0$ and the heat absorption when $Q_0<0$. To make the above equations dimensionless, we introduce the new variable follows as

$$\psi=\left[\nu r Gr_x^{1/4} f\left(\xi,\eta\right)+\frac{\xi}{2}\right], T-T_\infty=\left(T_w-T_\infty\right)\theta\left(\xi,\eta\right)$$

$$\xi = \frac{Vx}{\nu} Gr_x^{1/4}, \eta = \frac{y}{x} Gr_x^{-1/4}, r = x\sin\gamma \qquad (6)$$

$$Gr_x = \frac{g\beta\cos\gamma\left(T_w - T_\infty\right)x^3}{\nu^2}, T_w - T_\infty \approx x^n$$

where Gr_x is the local Grashof number, θ is the non-dimensional temperature, ψ is the stream function defined by

$$u = \frac{1}{r}\frac{\partial\psi}{\partial y}, v = -\frac{1}{r}\frac{\partial\psi}{\partial x} \qquad (7)$$

Finally, the functions $f(\xi,\eta)$ and $\theta(\xi,\eta)$ are, respectively, the dimensionless stream function and the temperature function of the fluid in the boundary-layer region.

Substituting the transformations given in (6) into (1) to (4), we obtained the following ordinary differential equations,

$$f''' + \frac{n+7}{4} ff'' - \frac{n+1}{2} f'^2 + \theta - Mf' + \xi f''$$

$$= \frac{1-n}{4}\xi\left(f'\frac{\partial f'}{\partial\xi} - f''\frac{\partial f}{\partial\xi}\right) \qquad (8)$$

and, $\frac{1}{\Pr}\theta'' + \frac{n+7}{4} f\theta' - nf'\theta + Q\theta + \xi\theta' = \frac{1-n}{4}\xi\left(f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi}\right)$ (9)

where,

$$M = \frac{\alpha_0\beta_0{}^2 x^2}{\rho\nu Gr_x^{1/2}} \quad \text{and} \quad Q = \frac{Q_0 x^2}{\rho C_p \nu Gr_x^{1/2}} \qquad (10)$$

The corresponding boundary conditions to be satisfied by are as given below

$$f' = 0, f = 0, \theta = 1 \text{ at } \eta = 0$$

$$f' = 0, \theta = 0 \text{ as } \eta \to \infty \qquad (11)$$

where, $\Pr = \nu/\alpha$, Prandtl number.

In the above equations primes denote differentiation with respect to η

In practical applications, the physical quantities of principal interest are the shearing stress and the rate of heat transfer in terms of skin-friction co-efficient C_{fx} and Nu_x the Nusselt number, respectively, which can be written as

$$C_{fx} = \frac{\tau_w}{\rho U_\infty^2} \text{ and } Nu_x = \frac{q_w x}{\kappa\left(T_w - T_\infty\right)} \qquad (12)$$

where $\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ and $q_w = -\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0}$ (13)

are respectively, the shear-stress and rate of heat flux at the surface and $U_\infty = \left(\frac{\nu}{x} Gr_x^{1/2}\right)$ being the reference velocity.

Using transformation (6) in the above expressions for expressions for C_{fx} and Nu_X take the following form:

$$C_{fx} Gr_x^{1/4} = f''(\xi, 0) \tag{14}$$

$$\frac{Nu_x}{Gr_x^{1/4}} = -\theta'(\xi, 0) \tag{15}$$

2.1. Finite Difference Method:

In the present analysis, we shall employ an efficient solution technique, known as implicit finite difference method together with Keller-box elimination technique, which is well-documented and widely used by Keller [16] and Cebeci [17] and also by Hossain et al. [13], Hossain and Paul[7], [8] etc.

To apply the finite difference method, we first convert the equations (8) to (11) into the following system of first order equations with dependent variables u (ξ, η), v (ξ, η) and p (ξ, η)

$$\frac{\partial f}{\partial \eta} = u \tag{16}$$

$$\frac{\partial u}{\partial \eta} = v \tag{17}$$

$$\frac{\partial \theta}{\partial \eta} = p \tag{18}$$

$$v' + p_1 f v + p_2 u^2 + p_3 \theta - p_4 u + p_5 v = p_0 \xi\left(u\frac{\partial u}{\partial \xi} - v\frac{\partial f}{\partial \xi}\right) \tag{19}$$

$$\frac{1}{\Pr} p' + p_1 f p + p_5 p + p_6 \theta - p_7 u \theta = p_0 \xi\left(u\frac{\partial \theta}{\partial \xi} - p\frac{\partial f}{\partial \xi}\right) \tag{20}$$

where,

$$p_0 = \frac{1-n}{4},\ p_1 = \frac{n+7}{4},\ p_2 = \frac{n+1}{2},$$
$$p_3 = 1,\ p_4 = M,\ p_5 = \xi,\ p_6 = Q,\ p_7 = n \tag{21}$$

With the boundary conditions

$$f = f' = 0, \theta = 1 \quad \text{at} \quad \eta = 0$$
$$f' = 0, \theta = 0 \text{ at } \quad \eta \to \infty \tag{22}$$

We now consider the net rectangle on the (ξ, η) plane and denote the net points by

$$\xi^0 = 0, \xi^n = \xi^{n-1} + k_n; n = 1, 2, \ldots\ldots\ldots, N \tag{23}$$

$$\eta_0 = 0, \eta_j = \eta_{j-1} + h_j; j = 1, 2, \ldots\ldots\ldots, J, \eta_J = \eta_\infty \tag{24}$$

Here n and j are just sequence of numbers on the (ξ, η) plane, k_n and h_j be the variable mesh widths.

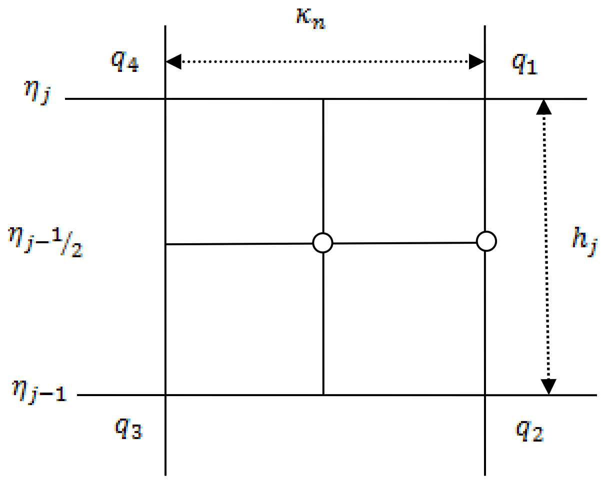

Fig: 2.Net rectangle of the difference approximation

We approximate the quantities (f, u, v, θ , p) at points (ξ^n, η_j) of the net by $(f_j^n, u_j^n, v_j^n, \theta_j^n, p_j^n)$, which we call net function. It is also employed the notation g_j^n for the quantities midway between net points shown in fig. 5 and for any net function as

$$\xi^{n-1/2} = \frac{1}{2}\left(\xi^n + \xi^{n-1}\right) \tag{25}$$

$$\eta_{j-1/2} = \frac{1}{2}\left(\eta_j + \eta_{j-1}\right) \tag{26}$$

$$g_j^{n-1/2} = \frac{1}{2}\left(g_j^n + g_j^{n-1}\right) \tag{27}$$

$$g_{j-1/2}^n = \frac{1}{2}\left(g_j^n + g_{j-1}^n\right) \tag{28}$$

Now we write the difference equations that are to approximate equations (16) to (21) by considering one mesh rectangle, we start by writing the finite difference approximation of the equations (16) to (21) using central difference quotients and average about the mid-point to obtained

$$\frac{f_j^n - f_{j-1}^n}{h_j} = u_{j-1/2}^n \tag{29}$$

$$\frac{u_j^n - u_{j-1}^n}{h_j} = v_{j-1/2}^n \tag{30}$$

$$\frac{\theta_j^n - \theta_{j-1}^n}{h_j} = p_{j-1/2}^n \tag{31}$$

$$\frac{v_j^n - v_{j-1}^n}{h_j} + \left(p_1^n + \alpha_n\right)\left(fv\right)_{j-1/2}^n - \left(p_2^n + \alpha_n\right)(u^2)_{j-1/2}^n$$
$$+ p_3^n \theta_{j-1/2}^n - p_4^n u_{j-1/2}^n + (p_5^n - \alpha_n f_{j-1/2}^{n-1}) v_{j-1/2}^n + \alpha_n \left(v_{j-1/2}^{n-1} f_{j-1/2}^n \right) = R_{j-1/2}^{n-1} \tag{32}$$

$$\frac{1}{\Pr} \frac{p_j^n - p_{j-1}^n}{h_j} + \left(p_1^n + \alpha_n\right)\left(fp\right)_{j-1/2}^n$$
$$+ (p_5^n - \alpha_n f_{j-1/2}^{n-1}) p_{j-1/2}^n - (p_6^n - \alpha_n u_{j-1/2}^{n-1}) \theta_{j-1/2}^n$$
$$-\left(p_7^n + \alpha_n\right)(u\theta)_{j-1/2}^n + \alpha_n (f_{j-1/2}^n p_{j-1/2}^{n-1} + u_{j-1/2}^n \theta_{j-1/2}^{n-1}) = T_{j-1/2}^{n-1} \tag{33}$$

where,

$$\alpha_n = \frac{p_0 \xi^{n-1}}{k_n} \tag{34}$$

$$R_{j-1/2}^{n-1} = -L_{j-1/2}^{n-1} + \alpha_n [\left(fv\right)_{j-1/2}^{n-1} - (u^2)_{j-1/2}^{n-1}] \tag{35}$$

$$L_{j-1/2}^{n-1} = \frac{v_j^{n-1} - v_{j-1}^{n-1}}{h_j} + p_1 \left(fv\right)_{j-1/2}^{n-1} + p_2 (u^2)_{j-1/2}^{n-1} + p_3 \theta_{j-1/2}^{n-1} - p_4 u_{j-1/2}^{n-1} + p_5 v_{j-1/2}^{n-1} \tag{36}$$

$$T_{j-1/2}^{n-1} = -M_{j-1/2}^{n-1} + \alpha_n [\left(fp\right)_{j-1/2}^{n-1} - \left(u\theta\right)_{j-1/2}^{n-1}] \tag{37}$$

$$M_{j-1/2}^{n-1} = [\frac{1}{\Pr} \frac{p_j - p_{j-1}}{h_j} + p_1 \left(fp\right)_{j-1/2} + p_5 p_{j-1/2} + p_6 \theta_{j-1/2} - p_7 \left(u\theta\right)_{j-1/2}]^{n-1} \tag{38}$$

The boundary conditions becomes

$$f_0^n = 0, u_0^n = 0, p_0^n = 0$$
$$u_j^n = 1, \theta_j^n = 1 \tag{39}$$

If we assume that $f_j^{n-1}, u_j^{n-1}, v_j^{n-1}, \theta_j^{n-1}$ and p_j^{n-1} to be known for $? \le j \le J$ equations (29) to (35) are a system of *5J+5* equations for the solutions of *5J+5* unknowns $(f_j^n, u_j^n, v_j^n, \theta_j^n, p_j^n)$, *j*=0, 1, 2,…, *J*. These nonlinear systems of algebraic equations are to be linearized by Newton's Quassy linearization method.

We define the iterates $[f_j^{(i)}, u_j^{(i)}, v_j^{(i)}, \theta_j^{(i)}$ and $p_j^{(i)}]$, i=0, 1, 2, ….. with initial values equal to those at the previous ξ station (which is usually the best initial guess available). For higher iterates we set

$$f_j^{(i+1)} = f_j^{(i)} + \delta f_j^{(i)} \tag{40}$$

$$u_j^{(i+1)} = u_j^{(i)} + \delta u_j^{(i)} \tag{41}$$

$$v_j^{(i+1)} = v_j^{(i)} + \delta v_j^{(i)} \tag{42}$$

$$\theta_j^{(i+1)} = \theta_j^{(i)} + \delta \theta_j^{(i)} \tag{43}$$

$$p_j^{(i+1)} = p_j^{(i)} + \delta p_j^{(i)} \tag{44}$$

We then insert the right hand side of the expression (36) to (40) in place of f_j, u_j, v_j, θ_j and p_j in equations (24) to (29) and (35) and dropping the terms that are quadratic in $\delta f_j^{(i)}, \delta u_j^{(i)}, \delta v_j^{(i)}, \delta\theta_j^{(i)}$ and $\delta p_j^{(i)}$. This procedure yields the following linear system (The subscript i and δ quantities is dropped for simplicity):

$$\delta f_j - \delta f_{j-1} - \frac{h_j}{2}\left(\delta u_j - \delta u_{j-1}\right) = \left(r_1\right)_j \tag{45}$$

$$\delta u_j - \delta u_{j-1} - \frac{h_j}{2}\left(\delta v_j - \delta v_{j-1}\right) = \left(r_4\right)_{j-1} \tag{46}$$

$$\delta \theta_j - \delta \theta_{j-1} - \frac{h_j}{2}\left(\delta \theta_j - \delta \theta_{j-1}\right) = \left(r_5\right)_{j-1} \tag{47}$$

$$\left(s_1\right)_j \delta v_j + \left(s_2\right)_j \delta v_{j-1} + \left(s_3\right)_j \delta f_j + \left(s_4\right)_j \delta f_{j-1} + \left(s_5\right)_j \delta u_j + \left(s_6\right)_j \delta u_{j-1} + \left(s_7\right)_j \delta\theta_j + \left(s_8\right)_j \delta\theta_{j-1} = \left(r_2\right)_j \tag{48}$$

$$\left(t_1\right)_j \delta p_j + \left(t_2\right)_j \delta p_{j-1} + \left(t_3\right)_j \delta f_j + \left(t_4\right)_j \delta f_{j-1} + \left(t_5\right)_j \delta\theta_j + \left(t_6\right)_j \delta\theta_{j-1} + \left(t_7\right)_j \delta u_j + \left(t_8\right)_j \delta u_{j-1} = \left(r_3\right)_j \tag{49}$$

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where,

$$\left(r_1\right)_j = f_{(j-1)}^{(i)} - f_j^{(i)} + h_j u_{j-1/2}^{(i)} \tag{50}$$

$$\left(r_4\right)_{j-1} = u_{(j-1)}^{(i)} - u_j^{(i)} + h_j v_{j-1/2}^{(i)} \tag{51}$$

$$\left(r_5\right)_{j-1} = \theta_{(j-1)}^{(i)} - \theta_j^{(i)} + h_j p_{j-1/2}^{(i)} \tag{52}$$

$$\begin{aligned}\left(r_2\right)_j = R_{j-1/2}^{n-1} - [h_j^{-1}\left(v_j^{(i)} - v_{j-1}^{(i)}\right) + \frac{p_1^n + \alpha_n}{2} + \\ \left[\left(fv\right)_j^{(i)} + \left(fv\right)_{j-1}^{(i)}\right] - \frac{p_2^n + \alpha_n}{2}\left[\left(u^2\right)_j^{(i)} + \left(u^2\right)_{j-1}^{(i)}\right] \\ + \frac{p_3^n}{2}\left(\theta_j^{(i)} + \theta_{j-1}^{(i)}\right) - \frac{p_4^n}{2}\left(u_j^{(i)} + u_{j-1}^{(i)}\right) + \frac{p_5^n - \alpha_n f_{j-1/2}^{n-1}}{2}\left(v_j^{(i)} + v_{j-1}^{(i)}\right) \\ + \frac{\alpha_n}{2}[v_{j-1/2}^{n-1}\left(f_j^{(i)} + f_{j-1}^{(i)}\right)]\end{aligned} \tag{53}$$

$$\begin{aligned}\left(r_3\right)_j = T_{j-1/2}^{n-1} - [\frac{h_j^{-1}}{\Pr}\left(p_j^{(i)} - p_{j-1}^{(i)}\right) + \frac{p_1^n + \alpha_n}{2}\left[\left(fp\right)_j^{(i)} + \left(fp\right)_{j-1}^{(i)}\right] \\ + \frac{p_5^n - \alpha_n f_{j-1/2}^{n-1}}{2}\left(p_j^{(i)} + p_{j-1}^{(i)}\right) + \frac{p_6^n - \alpha_n u_{j-1/2}^{n-1}}{2}\left(\theta_j^{(i)} + \theta_{j-1}^{(i)}\right) \\ - \frac{p_7^n + \alpha_n}{2}\left[\left(u\theta\right)_j^{(i)} + \left(u\theta\right)_{j-1}^{(i)}\right] + \frac{\alpha_n}{2}[p_{j-1/2}^{n-1}\left(f_j^{(i)} + f_{j-1}^{(i)}\right) \\ + \theta_{j-1/2}^{n-1}\left(u_j^{(i)} + u_{j-1}^{(i)}\right)]\end{aligned} \tag{54}$$

The coefficients of the momentum equation are

$$\left(s_1\right)_j = h_j^{-1} + \frac{p_1^n + \alpha_n}{2} f_j^{(i)} - \frac{p_5^n - \alpha_n f_{j-1/2}^{n-1}}{2} \tag{55}$$

$$\left(s_2\right)_j = -h_j^{-1} + \frac{p_1^n + \alpha_n}{2} f_{j-1}^{(i)} - \frac{p_5^n - \alpha_n f_{j-1/2}^{n-1}}{2} \tag{56}$$

$$\left(s_3\right)_j = \frac{p_1^n + \alpha_n}{2} v_j^{(i)} + \frac{\alpha_n}{2} v_{j-1/2}^{n-1} \tag{57}$$

$$\left(s_4\right)_j = \frac{p_1^n + \alpha_n}{2} v_{j-1}^{(i)} + \frac{\alpha_n}{2} v_{j-1/2}^{n-1} \tag{58}$$

$$(s_5)_j = -\left(p_2^n + \alpha^n\right)u_j^{(i)} - \frac{p_4^n}{2} \tag{59}$$

$$(s_6)_j = -\left(p_2^n - \alpha^n\right)u_{j-1}^{(i)} - \frac{p_4^n}{2} \tag{60}$$

$$(s_7)_j = \frac{p_3^n}{2} \tag{61}$$

$$(s_8)_j = \frac{p_3^n}{2} \tag{62}$$

The coefficients of the energy equation are

$$(t_1)_j = \frac{1}{\Pr}h_j^{-1} + \frac{p_1^n + \alpha_n}{2} f_j^{(i)} + \frac{p_5^n - \alpha_n f_{j-1/2}^{n-1}}{2} \tag{63}$$

$$(t_2)_j = -\frac{1}{\Pr}h_j^{-1} + \frac{p_1^n + \alpha_n}{2} f_{j-1}^{(i)} + \frac{p_5^n - \alpha_n f_{j-1/2}^{n-1}}{2} \tag{64}$$

$$(t_3)_j = \frac{p_1^n + \alpha_n}{2} p_j^{(i)} + \frac{\alpha_n}{2} p_{j-1/2}^{n-1} \tag{65}$$

$$(t_4)_j = \frac{p_1^n + \alpha_n}{2} p_{j-1}^{(i)} + \frac{\alpha_n}{2} p_{j-1/2}^{n-1} \tag{66}$$

$$(t_5)_j = \frac{p_6^n - \alpha_n u_{j-1/2}^{n-1}}{2} - \frac{p_7^n + \alpha_n}{2} u_j^{(i)} \tag{67}$$

$$(t_6)_j = \frac{p_6^n - \alpha_n u_{j-1/2}^{n-1}}{2} - \frac{p_7^n + \alpha_n}{2} u_{j-1}^{(i)} \tag{68}$$

$$(t_7)_j = -\frac{p_7^n + \alpha_n}{2} \theta_j^{(i)} + \frac{\alpha_n}{2} \theta_{j-1/2}^{n-1} \tag{69}$$

$$(t_8)_j = -\frac{p_7^n + \alpha_n}{2} \theta_{j-1}^{(i)} + \frac{\alpha_n}{2} \theta_{j-1/2}^{n-1} \tag{70}$$

The boundary conditions becomes

$$\delta f_0 = 0, \delta u_0 = 0, \delta p_0 = 0,\ \delta u_j = 0, \delta\theta_j = 0 \tag{71}$$

which just express the requirement for the boundary conditions to remain during the iteration process.

Now the system of linear equation (45) to (54) together with the boundary conditions (71) can be written in a block-matrix form a coefficient matrix, which are solved by using modified ‘Keller Box’ methods specially introduced by Keller [16]. Later this method has been used most efficiently by Cebeci and Bradshaw [17], Hossain and Paul [7], [8], Hossain [11] and Hossain et al. [13], taking the initial iteration to be given by convergent solution at $\xi = \xi_{j-1}$. To initiate the process with ξ= 0, we first prescribe the initial profiles for the functions u, v and p from the exact solutions. Here η_j are chosen so that the outer boundary $\eta_e \cong 10$ and were sufficiently dense in the vicinity of the boundary layer. In the present integration scheme maximum values of ξ has been accounted till the asymptotic scheme maximum values of ξ has been accounted till the asymptotic values for the local skin friction as well as for the surface temperature number are reached.

3. RESULTS AND DISCUSSION

In this work, we have investigatedthe magnetohydrodynamic natural convection flow on a vertical circular cone with non-uniform surface temperature in presence of heat generation. The solutions obtained by solving the momentum and energy equations employing the finite difference method. The results are presented in terms of the local skin friction, local Nusselt number, velocity profile and temperature profile.

Solutions are obtained for different values of Prandtl numbers and different values of surface temperature gradient for a wide range of values of magnetic parameterswith the heat generation parameters.

Table 1: Comparison of the present numerical values of f'' and $-\theta'$ for Pr = 0.1 and n = 0.5 while Q = 0, M = 0 with the results obtained by Hossain and Paul [8]

$f''(\xi,0)$			$?\theta'(\xi\ 0)$		
ξ =0.0	ξ =4.0	ξ =10.0	ξ =0.0	ξ =4.0	ξ =10.0
1.01332* 1.01350	1.66712* 1.67292	0.97304* 0.98585	0.24584* 0.24584	0.47828* 0.47697	1.01145* 1.00903

*represents the values from Hossain and Paul[8].

The results of this above mentioned method for the numerical values of local skin friction and local Nusselt number agaist $\xi$are depictedin tabular form in table: 1 forPr = 0.1 and n = 0.5 while Q = 0, M = 0.

Now we shall give our attention to the effect of pertinent parameters i.e. Pr, n, Q, M on the dimensionless velocity profile $f'(\xi,\eta)=\frac{ux}{\nu}Gr_x^{-1/2}$, and temperature profile, $\theta(\xi,\eta)=\frac{T-T_\infty}{T_w-T_\infty}$, in the flow field, computed by finite difference method.

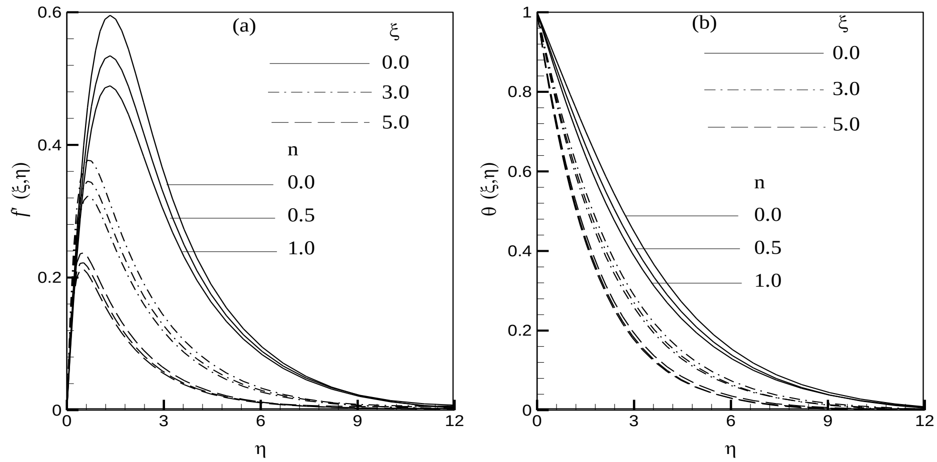

Fig.3. Velocity distribution (a) and temperature distribution (b) against η for Pr = 0.1, 0.2, 0.3 and ξ= 0.0, 3.0, 8.0 while n = 0.1, Q = 0.2, M = 0.2.

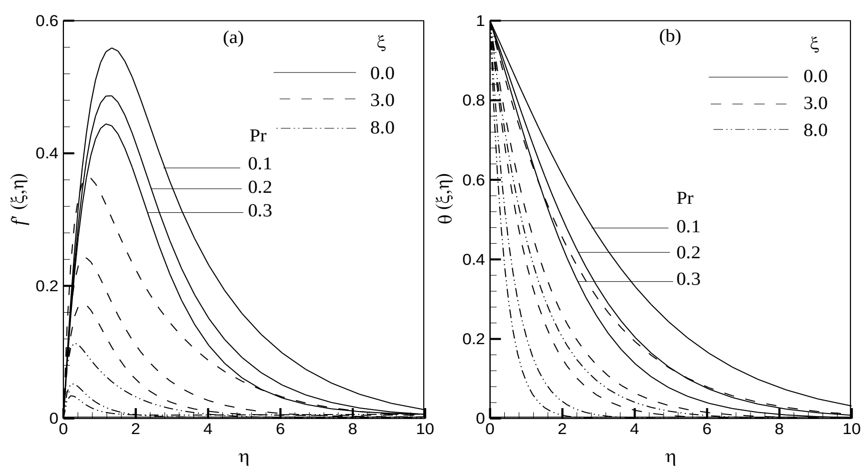

Fig.4. Velocity distribution (a) and temperature distribution (b) against η for n = 0.0, 0.5, 1.0 and ξ= 0.0, 3.0, 5.0 while Pr = 0.1, Q = 0.0, M = 0.2

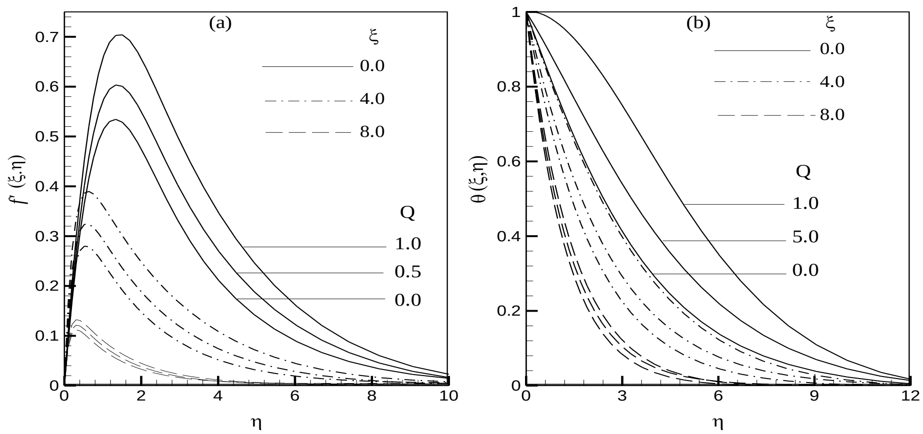

Fig.5. Velocity distribution (a) and temperature distribution (b) against η for Q = 0.0, 0.5, 1.0 and ξ= 0.0, 4.0, 8.0 while Pr = 0.1, n = 0.5, M = 0.2.

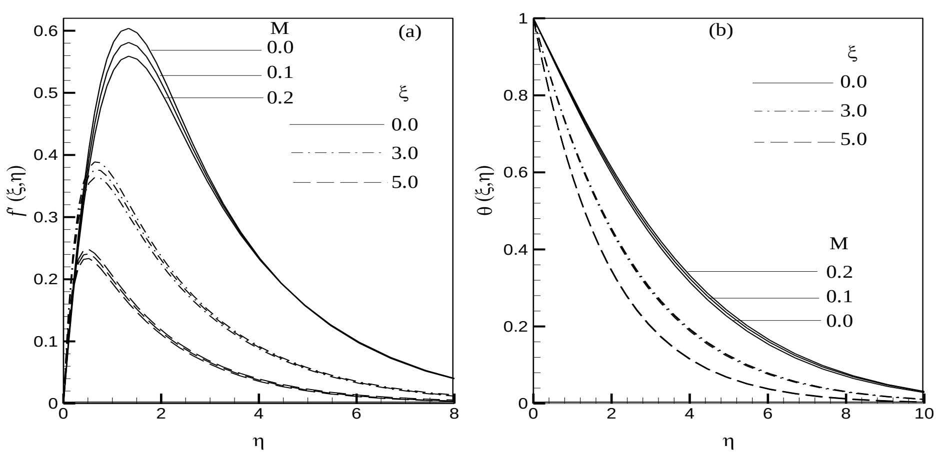

Fig.6. Velocity distribution (a) and temperature distribution (b) against η for M = 0.0, 0.1, 0.2 and ξ= 0.0, 3.0, 5.0 while Pr = 0.1, n = 0.5, Q = 0.2.

From figure 3(a), we observed that the fluid velocity decreases with the increase in suction parameterξ. It can also be observed that at each value of ξ there exist local maxima in velocity profile within boundary layer region. These maximum values are 0.55893, 0.36352, 0.11385 at η = 1.33565, 0.63665 and 0.30452, respectively, for Pr = 0.1. For Pr = 0.2, the maximum values occur at η= 1.33565, 0.63665 and 0.30452 and they are 0.48669, 0.24208 and 0.05214, respectively. The velocity attains maximum values at η = 1.17520, 0.52110 and

0.20134 for Pr = 0.3 and they are 0.44427, 0.17360 and 0.03437, respectively. From Fig: 3(b) it can be seen that the temperature profile decreases owing to the increase in the suction parameterξ. In addition, we see from Fig: 3 that both the velocity and temperature profiles decrease as the value of Pr increases. We further observe that at a given value of Pr, both the momentum and thermal boundary-layer thickness decreases with the increasing values ofξ.

From figure 4 we see that when the surface temperature gradient, n, increases, both the velocity and temperature profiles decrease. We further observed that both the value of velocity and temperature decrease as the suction parameter, ξ, increases. From fig: 4, we also see that for larger values ofξ , the velocity and temperature profile come closures for different values of n. From these we conclude that, both the momentum and thermal boundary-layer thickness decrease with the increasing values ofξ.

Figure 5 shows that if the heat generation parameter increases, both the velocity and temperature profiles increase. We further observed that both the value of velocity and temperature decrease as thesuction parameter,ξ, increases. As Q increases, the velocity gradient at the surface increases, which enhance the fluid velocity. On the other hand, when Q increases, fluid temperature also gradually increases.

The figure6 shows that when the magnetic parameter M increases, the velocity profile decreases but the temperature profile increases. But near the surface of the cone the velocity increases and then decreases slowly and finally approaches zero according to outer boundary condition. We further noticed that both the value of velocity and temperature decrease as the suction parameter, ξ, increases.

4. CONCLUSIONS

MHD natural convection flow from a vertical circular cone with heat generation has been investigated here numerically. The governing boundary layer equation are transformed into a non-dimensional form and the resulting non-linear systems of partial differential equations are reduced to local non-similarity boundary layer equations, which are solved numerically by using finite difference method. From the present investigation the following conclusions may be drawn:

- Both the velocity and temperature profiles decrease gradually when the values of Prandtl number increases within the boundary layer region.

- For increased values of surface temperature gradient, *n*, both the velocity and temperature profiles decrease slightly.
- For increasing values of heat generation parameter, *Q*, both the velocity and temperature profiles increase gradually.
- An increase in the values of magnetic parameter, *M*, the velocity profile decreases slightly for increasing values of magnetic parameter, *M*, whereas the temperature profile increases.

Conflict of Interests

The authors declare that there is no conflict of interests.

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