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# EDGE DOMINATION ON $S-$ VALUED GRAPHS

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**Abstract.** In this paper, we introduce the notion of edge domination on $S-$valued graphs and study some properties.

**Keywords:** semirings; graphs; $S-$valued graphs; weight dominating edge set.

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## 1. Introduction

In[5], the authors introduced the notion of $S-$ valued graphs, where $S$ is a semiring. In graph theory, domination of graphs is the most powerful area of research for, it has several applications in other areas of sciences. It was inicticted by Berge [1]. In [6], the authors have studied the vertex domination on $S-$ valued graphs. In this paper we discuss the notion of edge domination on $S-$ valued graphs.

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## 2. Preliminaries

In this section we recall some basic definitions that are needed for our work.

**Definition 2.1.** [3] *A semiring* $(S,+,\cdot)$ *is an algebraic system with a non-empty set S together with two binary operations* $+$ *and* $\cdot$ *such that*

1. $(S,+,0)$ *is a monoid.*
2. $(S,\cdot)$ *is a semigroup.*
3. *For all* $a,b,c \in S$, $a\cdot(b+c) = a\cdot b + a\cdot c$ *and* $(a+b)\cdot c = a\cdot c + b\cdot c$.
4. $0\cdot x = x\cdot 0 = 0\ \forall\, x \in S$.

**Definition 2.2.** [3] *Let* $(S,+,\cdot)$ *be a semiring.* $\preceq$ *is said to be a Canonical Pre-order if for* $a,b \in S$, $a \preceq b$ *if and only if there exists an element* $c \in S$ *such that* $a+c = b$.

**Definition 2.3.** [1] *A set F of edges in a graph* $G = (V,E)$ *is called an edge dominating set in G if for every edge* $e \in E - F$ *there exist an edge* $f \in F$ *such that e and f have a vertex in common.*

**Definition 2.4.** [1] *A dominating set S is a minimal edge dominating set if no proper subset of S is an edge dominating set in G.*

**Definition 2.5.** [1]

$$A\ set\ M \subseteq E\ is\ an\ Independent\ edge\ set\ of\ G\ if\ f,g \in M,\ N(f) \cap \{g\} = \phi.$$

**Definition 2.6.** [1] *A set* $M \subseteq E$ *is an Independent edge dominating set of G if M is both an independent edge set and a dominating edge set.*

**Definition 2.7.** [5] *Let* $G = (V, E \subset V \times V)$ *be a given graph with* $V,E \neq \phi$. *For any semiring* $(S,+,\cdot)$, *a semiring-valued graph (or a* $S-$*valued graph),* $G^S$, *is defined to be the graph* $G^S = (V,E,\sigma,\psi)$ *where* $\sigma : V \to S$ *and* $\psi : E \to S$ *are defined to be*

$$\psi(x,y) = \begin{cases} min\{\sigma(x),\sigma(y)\} & if\ \sigma(x) \preceq \sigma(y)\ or\ \sigma(y) \preceq \sigma(x) \\ 0 & otherwise \end{cases}$$

*for every unordered pair* $(x,y)$ *of* $E \subset V \times V$. *We call* $\sigma$, *a* $S-$*vertex set and* $\psi$, *a* $S-$*edge set of S-valued graph* $G^S$.

**Definition 2.8.** [4] *Let $G^S = (V, E, \sigma, \psi)$ be a $S-$ valued graph. Let $e \in E$.The open neighbourhood of $e$, denoted by$N_S(e)$, is defined to be the set*

$$N_S(e) = \{(e_i, \psi(e_i)) \ / \ e \text{ and } e_i \in E \text{ are adjacent}\}$$

*The closed neighbourhood of $e$, denoted by$N_S[e]$, is defined to be the set*

$$N_S[e] = N_S(e) \cup (e, \psi(e))$$

**Definition 2.9.** [6]*A vertex $v$ in $G^S$ is said to be a weight dominating vertex if $\sigma(u) \preceq \sigma(v)$, $\forall\ u \in N_S[v]$.*

**Definition 2.10.** [6]*A subset $D \subseteq V$ is said to be a weight dominating vertex set if for each $v \in D, \sigma(u) \preceq \sigma(v), \forall\ u \in N_S[v]$.*

## 3. Edge Domination on $S-$Valued Graphs

In this section, we introduce the notion of edge domination in $S-$valued graph, analogous to the notion in crisp graph theory, and prove some simple results.

**Definition 3.1.** *An edge $e$ in $G^S$ is said to be a weight dominating edge if $\psi(e_i) \preceq \psi(e)\ \forall e_i \in N_S[e]$.*

**Example 3.2.** Let $(S = \{0, a, b, c\}, +, \cdot)$ be a semiring with the following Cayley Tables:

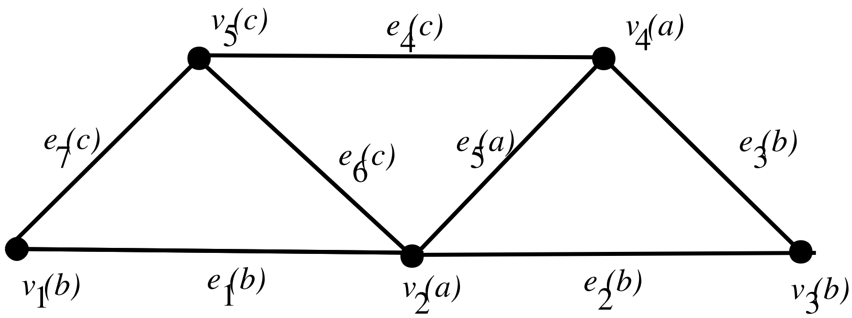
| $+$ | 0 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 0 | 0 | $a$ | $b$ | $c$ |
| $a$ | $a$ | $a$ | $a$ | $a$ |
| $b$ | $b$ | $a$ | $b$ | $b$ |
| $c$ | $c$ | $a$ | $b$ | $c$ |

| $\cdot$ | 0 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| $a$ | 0 | $a$ | $a$ | $a$ |
| $b$ | 0 | $b$ | $b$ | $b$ |
| $c$ | 0 | $b$ | $b$ | $b$ |

Let $\preceq$ be a canonical pre-order in $S$, given by

$$0 \preceq 0,\ 0 \preceq a,\ 0 \preceq b,\ 0 \preceq c,\ a \preceq a,\ b \preceq b,\ b \preceq a,\ c \preceq c,\ c \preceq a,\ c \preceq b$$

Consider the $S-$ graph $G^S$,




Define $\sigma : V \to S$ by

$$\sigma(v_1) = b, \sigma(v_2) = a, \sigma(v_3) = b, \sigma(v_4) = a, \sigma(v_5) = c$$

and $\psi : E \to S$ by

$$\psi(e_1) = \psi(e_2) = \psi(e_3) = b, \psi(e_4) = \psi(e_6) = \psi(e_7) = c, \psi(e_5) = a$$

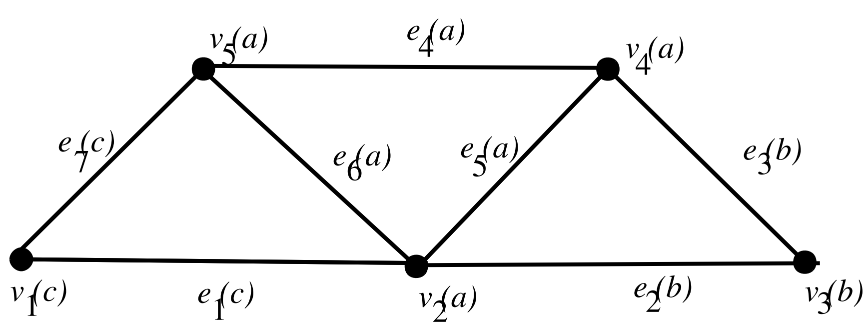
Clearly the edge $e_5$ $of$ $G^S$ is a weight dominating edge of $G^S$.

**Definition 3.3.** *A subset $D \subseteq E$ is said to be a weight dominating edge set if for each $e \in D, \psi(e_i) \preceq \psi(e), \forall e_i \in N_S[e]$.*

**Example 3.4.** Consider the semiring $(S = \{0, a, b, c\}, +, \cdot)$ with canonical pre-order given in example 3.2

Clearly $S$ is an additively idempotent semiring,

Consider the underlying graph $G$ of example 3.2




Define $\sigma : V \to S$ by

$$\sigma(v_1) = c, \sigma(v_2) = \sigma(v_4) = \sigma(v_5) = a, \sigma(v_3) = b$$

and $\psi : E \to S$ by

$$\psi(e_1) = \psi(e_7) = c, \psi(e_2) = \psi(e_3) = b, \psi(e_4) = \psi(e_5) = \psi(e_6) = a$$

Clearly $D = \{e_4, e_5, e_6\}$ is a weight dominating edge set.

Further $D_1 = \{e_4, e_5\}, D_2 = \{e_4, e_6\}, D_3 = \{e_5, e_6\}, D_4 = \{e_4, e_5, e_6\}$ are all weight dominating edge sets.

**Definition 3.5.** *If D is weight dominating edge set of $G^S$, then the scalar cardinality of D is defined by* $|D|_S = \sum_{e \in D} \psi(e)$

In the above example 3.4 , the scalar cardinality of the weight dominating sets are respectively given by $|D_1|_S = a$; $|D_2|_S = a$; $|D_3|_S = a$; $|D_4|_S = a$.

**Definition 3.6.** *A subset $D \subseteq E$ is said to be a minimal weight dominating edge set if*

*(1) D is a weight dominating edge set.*

*(2) No proper subset of D is a weight dominating edge set.*

In the above example 3.4 ,$D_1 = \{e_4, e_5\}, D_2 = \{e_4, e_6\}, D_3 = \{e_5, e_6\}$ are all minimal weight dominating edge sets.

**Definition 3.7.** *The edge S-domination number of $G^S$ denoted by $\gamma_E^S(G^S)$ is defined by $\gamma_E^S(G^S) = (|D|_S, |D|)$,where D is the minimal weight dominating edge set.*

In the above example 3.4 ,$D_1 = \{e_4, e_5\}, D_2 = \{e_4, e_6\}, D_3 = \{e_5, e_6\}$ are all minimal weight dominating edge sets with edge $S-$ domination number

$$\gamma_E^S(G^S) = (|D_1|_S, |D_1|) = (|D_2|, |D_2|) = (|D_3|, |D_3|) = (a, 2)$$

**Remark 3.8.** Minimal weight dominating edge set in a $S-$valued graph need not be unique in general. For, in example 3.4, $D_1 = \{e_4, e_5\}, D_2 = \{e_4, e_6\}$,
$D_3 = \{e_5, e_6\}$ are all minimal weight dominating edge sets.

**Definition 3.9.** *A subset $D \subseteq E$ is said to be a maximal weight dominating edge set if*

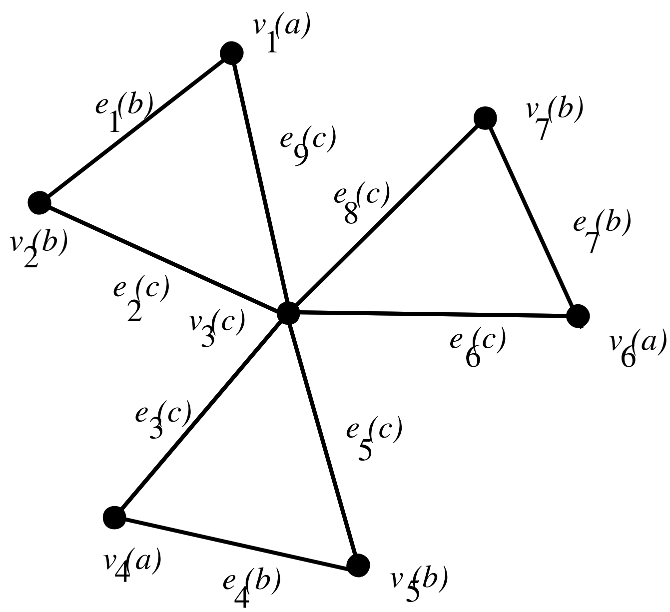
*(1) D is a weight dominating edge set.*

*(2) If there is no subset $D'$ of $E$ such that $D \subset D' \subset E$ and $D'$ is a weight dominating edge set.*

In the above example 3.4,$D_4 = \{e_4, e_5, e_6\}$ is a maximal weight $S-$ dominating edge set.

**Definition 3.10.** *A subset $M \subseteq E$ is an independent edge set of $G^S$ if $f, g \in M$ such that $N_S(f) \cap (g, \psi(g)) = \phi$.*

**Example 3.11.** Consider the semiring $(S = \{0, a, b, c\}, +, \cdot)$ with canonical pre-order given in example 3.2




Define $\sigma : V \to S$ by

$$\sigma(v_1) = \sigma(v_4) = \sigma(v_6) = a, \sigma(v_2) = \sigma(v_5) = \sigma(v_7) = b, \sigma(v_3) = c$$

and $\psi : E \to S$ by

$$\psi(e_1) = \psi(e_4) = \psi(e_7) = b, \psi(e_2) = \psi(e_3) = \psi(e_5) = \psi(e_6) = \psi(e_8) = \psi(e_9) = c$$

Consider the edge set $D = \{e_1, e_4, e_7\}$

Clearly $D$ is an independent edge set of $G^S$.

Further $D_1 = \{e_1, e_4\}, D_2 = \{e_1, e_7\}, D_3 = \{e_4, e_7\}, D_4 = \{e_1, e_4, e_7\}$ are all independent edge sets of $G^S$.

**Definition 3.12.** *A subset $M \subseteq E$ is said to be a minimal independent edge set if*

*(1) $M$ is an independent edge set.*

*(2) No proper subset of $M$ is an independent edge set.*

In the above example 3.12,$D_1 = \{e_1, e_4\}, D_2 = \{e_1, e_7\}, D_3 = \{e_4, e_7\}$ are all minimal independent edge sets.

**Definition 3.13.** *A subset $M \subseteq E$ is said to be a maximal independent edge set if*

*(1) $M$ is an independent edge set.*

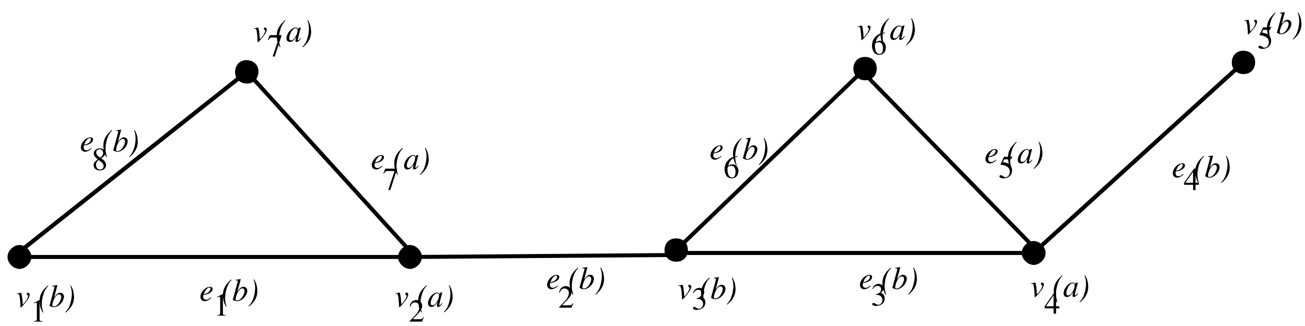
*(2) If there is no subset $M'$ of $E$ such that $M \subset M' \subset E$ and $M'$ is an independent edge set.*

In the above example 3.12,$D_4 = \{e_1, e_4, e_7\}$ is a maximal independent edge set.

**Definition 3.14.** *A subset $M \subseteq E$ is said to be an independent weight dominating edge set if $M$ is both independent edge set and a weight dominating edge set.*

**Example 3.15.** Consider the semiring $(S = \{0, a, b, c\}, +, \cdot)$ with canonical pre-order given in example 3.2

Consider the $S-$graph $G^S$,




Define $\sigma : V \to S$ by

$$\sigma(v_1) = \sigma(v_3) = \sigma(v_5) = b, \sigma(v_2) = \sigma(v_4) = \sigma(v_6) = \sigma(v_7) = a$$

and $\psi : E \to S$ by

$$\psi(e_1) = \psi(e_2) = \psi(e_3) = \psi(e_4) = \psi(e_6) = \psi(e_8) = b, \psi(e_5) = \psi(e_7) = a$$

Consider the edge set $D = \{e_5, e_7\}$

Clearly $D = \{e_5, e_7\}$ is an independent weight dominating edge set.

**Theorem 3.16.** *A weight dominating edge set D of a graph $G^S$ is a minimal weight dominating edge set of G iff every edge $e \in D$ satisfies atleast one of the following properties:*

*(1) there exist an edge $f \in E - D$, such that $N_S(f) \cap (D \times S) = \{(e, \psi(e))\}$*

*(2) e is adjacent to no edge of D.*

**Proof :** *Let $e \in D$. Assume that e is adjacent to no edge of D, then $D - \{e\}$ cannot be a weight dominating edge set. $\Rightarrow$ D is a minimal weight dominating edge set.*

*On the other hand, if for any $e \in D$ there exist a $f \in E - D$ such that $N_S(f) \cap (D \times S) = \{(e, \psi(e))\}$*

*Then f is adjacent to $e \in D$ and no other edge of D.*

*In this case also, $D - \{e\}$ cannot be a weight dominating edge set of $G^S$.*

*Hence D is a minimal weight dominating edge set.*

**Conversely,** *assume that D is a minimal weight dominating edge set of $G^S$.*

*Then for each $e \in D$, $D - \{e\}$ is not a weight dominating edge set of $G^S$.*

*$\therefore$ there exist an edge, $f \in E - (D - \{e\})$ that is adjacent to no edge of $(D - \{e\})$.*

*If $f = e$, then e is adjacent to no edge of D.*

*If $f \neq e$, then D is a weight dominating edge set and $f \notin D \Rightarrow f$ is adjacent to atleast one edge of D. However f is not adjacent to any edge of $D - \{e\}$.*

*$\Rightarrow N_S(f) \cap D \times S = \{(e, \psi(e))\}$.*

**Remark 3.17.** The above theorem can be rephrased as follows:

A weight dominating edge set $D$ of a graph $G^S$ is a minimal weight dominating edge set of $G^S$ iff for every edge $e \in D$,

(1) either the edge $e$, dominates some edge of $E - D$ such that no other edge of $D$ dominates.

(2) or no other edge of $D$,dominates $e$.

**Theorem 3.18.** *A set $D \subseteq E$ of $G^S$ is an independent weight dominating edge set iff D is a maximal independent edge set in $G^S$.*

**Proof:** *Clearly every maximal independent edge set D in $G^S$ is a weight dominating independent edge set.*

**Conversely,** *assume that D is an independent weight dominating edge set.*

*Then D is independent and every edge not in D is adjacent to a edge of D and therefore D is a maximal independent edge set in $G^S$.*

**Theorem 3.19.** *Every maximal independent edge set of edges D in $G^S$ is a minimal weight dominating edge set.*

**Proof :** *Let D be a maximal independent edge set of edges in $G^S$. Then by theorem* 3.18 *, D is a weight dominating edge set.*

*Since D is independent, every edge of D is adjacent to no edge of D.*

*Thus, every edge of D satisfies the second condition of theorem* 3.16. *Hence D is a minimal weight dominating edge set in $G^S$.*

## Conflict of Interests

The authors declare that there is no conflict of interests.

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