Available online at http://scik.org
J. Math. Comput. Sci. 9 (2019), No. 4, 372-384
https://doi.org/10.28919/jmcs/3030
ISSN: 1927-5307

ON CERTAIN CLASS OF σ-CONVERGENT SEQUENCE SPACES DEFINED BY ORLICZ FUNCTION

KHALID EBADULLAH*

College of Science and Theoretical Studies, Saudi Electronic University, Kingdom of Saudi Arabia

Abstract. In this article we introduce the sequence space $V_\sigma(M,p,r,\triangle_v^u)$, where $u \in N$, M is an Orlicz function, $p = (p_m)$ is any sequence of strictly positive real numbers, $v = (v_k)$ is any fixed sequence of non zero complex numbers and $r \geq 0$. We study some of the properties and inclusion relations that arise on the said space.

Keywords: invariant mean; paranorm; Orlicz function and difference sequences.

2010 AMS Subject Classification: 40F05, 40C05, 46A45.

1. Introduction

Let N, R and C be the sets of all natural, real and complex numbers respectively.

We write

$$\omega = \{x = (x_k) : x_k \in R \; or \; C\},$$

the space of all real or complex sequences.

*Corresponding author

E-mail address: khalidebadullah@gmail.com

Received November 5, 2016

Let ℓ_∞, c and c_0 denote the Banach spaces of bounded, convergent and null sequences respectively.

The following subspaces of ω were first introduced and discussed by Maddox [15-16].

$\ell(p) = \{x \in \omega : \sum_k |x_k|^{p_k} < \infty\}$,

$\ell_\infty(p) = \{x \in \omega : \sup_k |x_k|^{p_k} < \infty\}$,

$c(p) = \{x \in \omega : \lim_k |x_k - l|^{p_k} = 0, \text{ for some } l \in C\}$,

$c_0(p) = \{x \in \omega : \lim_k |x_k|^{p_k} = 0\}$,

where $p = (p_k)$ is a sequence of striclty positive real numbers.

The concept of paranorm is closely related to linear metric spaces.It is a generalization of that of absolute value. (see [16])

Let X be a linear space. A function $g : X \longrightarrow R$ is called paranorm, if for all $x, y, z \in X$,

(PI) $g(x) = 0 \; if \; x = \theta$,

(P2) $g(-x) = g(x)$,

(P3) $g(x+y) \leq g(x) + g(y)$,

(P4) If (λ_n) is a sequence of scalars with $\lambda_n \to \lambda \; (n \to \infty)$ and $x_n, a \in X$ with $x_n \to a \; (n \to \infty)$, in the sense that $g(x_n - a) \to 0 \; (n \to \infty)$, in the sense that $g(\lambda_n x_n - \lambda a) \to 0 \; (n \to \infty)$.

An Orlicz function is a function $M : [0, \infty) \to [0, \infty)$, which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$.

Lindenstrauss and Tzafriri[13] used the idea of Orlicz functions to construct the sequence space

$$\ell_M = \{x \in \omega : \sum_{k=1}^{\infty} M(\frac{|x_k|}{\rho}) < \infty, \text{for some } \rho > 0\}$$

The space ℓ_M is a Banach space with the norm

$$||x|| = \inf\{\rho > 0 : \sum_{k=1}^{\infty} M(\frac{|x_k|}{\rho}) \leq 1\}$$

The space ℓ_M is closely related to the space ℓ_p which is an Orlicz sequence space with $M(x) = x^p$ for $1 \leq p < \infty$.

An Orlicz function M is said to satisfy $\triangle_2$ condition for all values of x if there exists a constant $K > 0$ such that $M(Lx) \leq KLM(x)$ for all values of $L > 1$.
A sequence space E is said to be solid or normal if $(x_k) \in E$ implies $(\alpha_k x_k) \in E$ for all sequence of scalars (α_k) with $|\alpha_k| < 1$ for all $k \in N$.

For Orlicz function and related results see ([2-4], [6], [22]).

Let σ be an injection on the set of positive integers N into itself having no finite orbits and T be the operator defined on ℓ_∞ by $T(x_k) = (x_{\sigma(k)})$.
A positive linear functional Φ, with $||\Phi|| = 1$, is called a σ-mean or an invariant mean if $\Phi(x) = \Phi(Tx)$ for all $x \in \ell_\infty$.
A sequence x is said to be σ-convergent, denoted by $x \in V_\sigma$, if $\Phi(x)$ takes the same value, called $\sigma - \lim x$, for all σ-means Φ. We have

$$V_\sigma = \{x = (x_k) : \sum_{m=1}^{\infty} t_{m,n}(x) = L \text{ uniformly in n, L = } \sigma - \lim x\},$$

where for $m \geq 0, n > 0$.

$$t_{m,n}(x) = \frac{x_k + x_{\sigma(k)} + ... + x_{\sigma^m(k)}}{m+1}, \text{and } t_{-1,n} = 0.$$

where $\sigma^m(k)$ denotes the mth iterate of σ at n. In particular, if σ is the translation, a σ-mean is often called a Banach limit and V_σ reduces to f, the set of almost convergent sequences.

Subsequently the spaces of invariant mean have been studied by various authors. See ([1], [14], [20-21], [23], [24]).

The idea of Difference sequence sets

$$X_\triangle = \{x = (x_k) \in \omega : \triangle x = (x_k - x_{k+1}) \in X\},$$

where $X = \ell_\infty,\ c$ or c_0 was introduced by Kizmaz [12].

Kizmaz [12] defined the sequence spaces,

$$\ell_\infty(\triangle) = \{x = (x_k) \in \omega : (\triangle x_k) \in \ell_\infty\},$$

$$c(\triangle) = \{x = (x_k) \in \omega : (\triangle x_k) \in c\},$$

$$c_0(\triangle) = \{x = (x_k) \in \omega : (\triangle x_k) \in c_0\},$$

where $\triangle x = (x_k - x_{k+1})$. These are Banach spaces with the norm

$$||x||_\triangle = |x_1| + ||\triangle x||_\infty.$$

After then Mikael [17] defined the sequence spaces :

$$\ell_\infty(\triangle^2) = \{x = (x_k) \in \omega : (\triangle^2 x_k) \in \ell_\infty\},$$

$$c(\triangle^2) = \{x = (x_k) \in \omega : (\triangle^2 x_k) \in c\},$$

$$c_0(\triangle^2) = \{x = (x_k) \in \omega : (\triangle^2 x_k) \in c_0\},$$

and showed that these are Banach spaces with norm

$$||x||_\triangle = |x_1| + |x_2| + ||\triangle^2 x||_\infty.$$

After then Mikael and R. Colak [18] defined the sequence spaces

$$\ell_\infty(\triangle^m) = \{x = (x_k) \in \omega : (\triangle^m x_k) \in \ell_\infty\},$$

$$c(\triangle^m) = \{x = (x_k) \in \omega : (\triangle^m x_k) \in c\},$$

$$c_0(\triangle^m) = \{x = (x_k) \in \omega : (\triangle^m x_k) \in c_0\},$$

where $m \in N$,

$$\triangle^0 x = (x_k),$$

$$\triangle x = (x_k - x_{k+1}),$$

$$\triangle^m x = (\triangle^{m-1} x_k - \triangle^{m-1} x_{k+1}),$$

and so that

$$\triangle^m x_k = \sum_{i=0}^{m} (-1)^i \begin{bmatrix} m \\ i \end{bmatrix} x_{k+i}.$$

and showed that these are Banach spaces with the norm

$$||x||_{\triangle} = \sum_{i=1}^{m} |x_i| + ||\triangle^m x||_\infty.$$

Esi and Isik [10] defined the sequence spaces

$$\ell_\infty(\triangle_v^m, s, p) = \{x = (x_k) \in \omega : \sup_k \lim k^{-s} |\triangle_v^m x_k|^{p_k} < \infty, s \geq 0\},$$

$$c(\triangle_v^m, s, p) = \{x = (x_k) \in \omega : k^{-s} |\triangle_v^m x_k - L|^{p_k} \to 0 (k \to \infty), s \geq 0, \text{for some L}\},$$

$$c_0(\triangle_v^m, s, p) = \{x = (x_k) \in \omega : k^{-s} |\triangle_v^m x_k|^{p_k} \to 0 (k \to \infty), s \geq 0\},$$

where $p = (p_k)$ is a sequence of striclty positive real numbers, $v = (v_k)$ is any fixed sequence of non zero complex numbers, $m \in N$ is a fixed number,

$$\triangle_v^0 x_k = (v_k x_k),\ \triangle_v x_k = (v_k x_k - v_{k+1} x_{k+1})$$

and

$$\triangle_v^m x_k = (\triangle_v^{m-1} x_k - \triangle_v^{m-1} x_{k+1})$$

and so that

$$\triangle_v^m x_k = \sum_{i=0}^{m} (-1)^i \begin{bmatrix} m \\ i \end{bmatrix} v_{k+i} x_{k+i}.$$

When s=0, m=1, v=(1,1,1,.......) and $p_k = 1$ for all $k \in N$, they are just $\ell_\infty(\triangle), c(\triangle)$ and $c_0(\triangle)$ defined by Kizmaz[12].

When s=0 and $p_k = 1$ for all $k \in N$, they are the following sequence spaces defined by Mikael and Esi [19]

$$\ell_\infty(\triangle_v^m) = \{x = (x_k) \in \omega : (\triangle_v^m x_k) \in \ell_\infty\},$$

$$c(\triangle_v^m) = \{x = (x_k) \in \omega : (\triangle_v^m x_k) \in c\},$$

$$c_0(\triangle_v^m) = \{x = (x_k) \in \omega : (\triangle_v^m x_k) \in c_0\}.$$

For difference sequences see([3-12], [17], [18], [19]).

Recently Ebadullah[6] introduced and studied the sequence space

$$V_\sigma(M, p, r) = \{x = (x_k) : \sum_{m=1}^{\infty} \frac{1}{m^r} [M(\frac{|t_{m,n}(x)|}{\rho})]^{p_m} < \infty \text{ uniformly in n}, \rho > 0\}.$$

Where M is an Orlicz function, $p=(p_m)$ is any sequence of strictly positive real numbers and $r \geq 0$.

Later on Ebadullah[9] introduced and studied the difference sequence space

$$V_\sigma(M,p,r,\triangle^u)=\{x=(x_k): \sum_{m=1}^{\infty}\frac{1}{m^r}[M(\frac{|t_{m,n}(\triangle^u x)|}{\rho})]^{p_m}<\infty \text{ uniformly in n}, \rho>0\}.$$

Where M is an Orlicz function, $u \in N$, $p=(p_m)$ is any sequence of strictly positive real numbers and $r \geq 0$.

When u=1 we have the following sequence space defined in [7]

$$V_\sigma(M,p,r,\triangle)=\{x=(x_k): \sum_{m=1}^{\infty}\frac{1}{m^r}[M(\frac{|t_{m,n}(\triangle x)|}{\rho})]^{p_m}<\infty \text{ uniformly in n}, \rho>0\}.$$

When u=2 we have the following sequence space defined in[8]

$$V_\sigma(M,p,r,\triangle^2)=\{x=(x_k): \sum_{m=1}^{\infty}\frac{1}{m^r}[M(\frac{|t_{m,n}(\triangle^2 x)|}{\rho})]^{p_m}<\infty \text{ uniformly in n}, \rho>0\}.$$

2. Main results

In this article we introduce the sequence space

$$V_\sigma(M,p,r,\triangle_v^u)=\{x=(x_k): \sum_{m=1}^{\infty}\frac{1}{m^r}[M(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho})]^{p_m}<\infty \text{ uniformly in n}, \rho>0\}.$$

Where M is an Orlicz function, $u \in N$, $p=(p_m)$ is any sequence of strictly positive real numbers, $v=(v_k)$ is any fixed sequence of non zero complex numbers and $r \geq 0$.

Now we define the sequence spaces as follows;

For $M(x)=x$ we get

$$V_\sigma(p,r,\triangle_v^u)=\{x=(x_k): \sum_{m=1}^{\infty}\frac{1}{m^r}|t_{m,n}(\triangle_v^u x)|^{p_m}<\infty \text{ uniformly in n}\}$$

For $p_m = 1$, for all m, we get

$$V_\sigma(M,r,\triangle_v^u) = \{x=(x_k) : \sum_{m=1}^{\infty} \frac{1}{m^r}[M(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho})] < \infty \text{ uniformly in n}, \rho > 0\}$$

For r = 0 we get

$$V_\sigma(M,p,\triangle_v^u) = \{x=(x_k) : \sum_{m=1}^{\infty} [M(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho})]^{p_m} < \infty \text{ uniformly in n}, \rho > 0\}$$

For $M(x) = x$ and r=0 we get

$$V_\sigma(p,\triangle_v^u) = \{x=(x_k) : \sum_{m=1}^{\infty} |t_{m,n}(\triangle_v^u x)|^{p_m} < \infty \text{ uniformly in n}, \rho > 0\}$$

For $p_k = 1$, for all m and r=0, we get

$$V_\sigma(M,\triangle_v^u) = \{x=(x_k) : \sum_{m=1}^{\infty} [M(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho})] < \infty \text{ uniformly in n}, \rho > 0\}$$

For $M(x) = x$, $p_m = 1$, for all m and r=0, we get

$$V_\sigma(\triangle_v^u x) = \{x=(x_k) : \sum_{m=1}^{\infty} |t_{m,n}(\triangle_v^u x)| < \infty \text{ uniformly in n}\}.$$

Theorem 2.1. The sequence space $V_\sigma(M,p,r,\triangle_v^u)$ is a linear space over the field C of complex numbers.

Proof. Let $x,y \in V_\sigma(M,p,r,\triangle_v^u)$ and $\alpha,\beta \in C$ then there exists positive numbers ρ_1 and ρ_2 such that

$$\sum_{m=1}^{\infty} \frac{1}{m^r}[M(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho_1})]^{p_m} < \infty,$$

and

$$\sum_{m=1}^{\infty} \frac{1}{m^r}[M(\frac{|t_{m,n}(\triangle_v^u y)|}{\rho_2})]^{p_m} < \infty$$

uniformly in n.

Define ρ_3 = max($2|\alpha|\rho_1$, $2|\beta|\rho_2$).

Since M is non decreasing and convex we have

$$\sum_{m=1}^{\infty}\frac{1}{m^r}[M(\frac{|\alpha t_{m,n}(\triangle_v^u x)+\beta t_{m,n}(\triangle_v^u y)|}{\rho_3})]^{p_m}$$

$$\leq \sum_{m=1}^{\infty}\frac{1}{m^r}[M(\frac{|\alpha t_{m,n}(\triangle_v^u x)|}{\rho_3}+\frac{|\beta t_{m,n}(\triangle_v^u y)|}{\rho_3})]^{p_m}$$
$$\leq \sum_{m=1}^{\infty}\frac{1}{m^r}\frac{1}{2}[M(\frac{t_{m,n}(\triangle_v^u x)}{\rho_1})+M(\frac{t_{m,n}(\triangle_v^u y)}{\rho_2})]<\infty$$

uniformly in n.

This proves that $V_\sigma(M,p,r,\triangle_v^u)$ is a linear space over the field C of complex numbers.

Theorem 2.2. For any Orlicz function M and a bounded sequence $p=(p_m)$ of strictly positive real numbers, $V_\sigma(M,p,r,\triangle_v^u)$ is a paranormed space with

$$g(x)=\inf_{n\geq 1}\{\rho^{\frac{p_n}{H}}:(\sum_{m=1}^{\infty}\frac{1}{m^r}[M(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho})]^{p_m})^{\frac{1}{H}}\leq 1,\text{ uniformly in n}\}$$

where H = max(1, supp_m).

Proof. It is clear that $g(\triangle_v^u x)=g(-\triangle_v^u x)$.

Since $M(0)=0$, we get

inf$\{\rho^{\frac{p_m}{H}}\}$ = 0, for $x=0$

Now for α=β=1, we get

$g(\triangle_v^u x+\triangle_v^u y)\leq g(\triangle_v^u x)+g(\triangle_v^u y)$.

For the continuity of scalar multiplication let $l\neq 0$ be any complex number. Then by the definition we have

$$g(l\triangle_v^u x)=\inf_{n\geq 1}\{\rho^{\frac{p_n}{H}}:(\sum_{m=1}^{\infty}\frac{1}{m^r}[M(\frac{|t_{m,n}(l\triangle_v^u x)|}{\rho})]^{p_m})^{\frac{1}{H}}\leq 1,\text{ uniformly in n}\}$$
$$g(l\triangle_v^u x)=\inf_{n\geq 1}\{(|l|s)^{\frac{p_n}{H}}:(\sum_{m=1}^{\infty}\frac{1}{m^r}[M(\frac{|t_{m,n}(l\triangle_v^u x)|}{(|l|s)})]^{p_m})^{\frac{1}{H}}\leq 1,\text{ uniformly in n}\}$$

where $s = \frac{\rho}{|l|}$.

Since $|l|^{p_m} \leq$ max(1,$|l|^H$), we have

$$g(l\triangle_v^u x) \leq max(1,|l|^H)\inf_{n\geq 1}\{s^{\frac{p_n}{H}} : (\sum_{m=1}^{\infty}\frac{1}{m^r}[M(\frac{|t_{m,n}(\triangle_v^u x)|}{(|l|s)})]^{p_m})^{\frac{1}{H}} \leq 1, \text{ uniformly in n}\}$$

$$g(\triangle_v^u lx) \leq max(1,|l|^H)g(\triangle_v^u x)$$

Therefore $g(\triangle_v^u x)$ converges to zero when $g(\triangle_v^u x)$ converges to zero in $V_\sigma(M,p,r,\triangle_v^u)$.

Now let x be fixed element in $V_\sigma(M,p,r,\triangle_v^u)$. There exists $\rho > 0$ such that

$$g(\triangle_v^u x) = \inf_{n\geq 1}\{\rho^{\frac{p_n}{H}} : (\sum_{m=1}^{\infty}\frac{1}{m^r}[M(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho})]^{p_m})^{\frac{1}{H}} \leq 1, \text{ uniformly in n}\}.$$

Now

$$g(l\triangle_v^u x) = \inf_{n\geq 1}\{\rho^{\frac{p_n}{H}} : (\sum_{m=1}^{\infty}\frac{1}{m^r}[M(\frac{|t_{m,n}(l\triangle_v^u x)|}{\rho})]^{p_m})^{\frac{1}{H}} \leq 1, \text{ uniformly in n}\} \to 0 \text{ as } l \to 0.$$

This completes the proof.

Theorem 2.3. Suppose that $0 < p_m < t_m < \infty$ for each $m \in N$ and $r > 0$. Then

(a) $V_\sigma(M,p,\triangle_v^u) \subseteq V_\sigma(M,t,\triangle_v^u)$.

(b) $V_\sigma(M,\triangle_v^u) \subseteq V_\sigma(M,r,\triangle_v^u)$

Proof.(a) Suppose that $x \in V_\sigma(M,p,\triangle_v^u)$.

This implies that $[M(\frac{|t_{i,n}(\triangle_v^u x)|}{\rho})]^{p_m}) \leq 1$

for sufficiently large value of i, say $i \geq m_0$ for some fixed $m_0 \in N$.

Since M is non decreasing, we have

$$\sum_{m=m_0}^{\infty}[M(\frac{|t_{i,n}(\triangle_v^u x)|}{\rho})]^{t_m} \leq \sum_{m=m_0}^{\infty}[M(\frac{|t_{i,n}(\triangle_v^u x)|}{\rho})]^{p_m} < \infty.$$

Hence $x \in V_\sigma(M,t,\triangle_v^u)$.

(b) The proof is trivial.

Corollary 2.4. $0 < p_m \leq 1$ for each m, then $V_\sigma(M,p,\triangle_v^u) \subseteq V_\sigma(M,\triangle_v^u)$

If $p_m \geq 1$ for all m , then $V_\sigma(M,\triangle_v^u) \subseteq V_\sigma(M,p,\triangle_v^u)$.

Theorem 2.5. The sequence space $V_\sigma(M,p,r,\triangle_v^u)$ is solid.

Proof. Let $x \in V_\sigma(M,p,r,\triangle_v^u)$. This implies that

$$\sum_{m=1}^{\infty} \frac{1}{m^r}[M(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho})]^{p_m} < \infty.$$

Let α_m be a sequence of scalars such that $|\alpha_m| \leq 1$ for all $m \in N$. Then the result follows from the following inequality.

$$\sum_{m=1}^{\infty} \frac{1}{m^r}[M(\frac{|\alpha_m t_{i,n}(\triangle_v^u x)|}{\rho})]^{p_m} \leq \sum_{m=1}^{\infty} \frac{1}{m^r}[M(\frac{|t_{i,n}(\triangle_v^u x)|}{\rho})]^{p_m} < \infty.$$

Hence $\alpha x \in V_\sigma(M,p,r,\triangle_v^u)$ for all sequence of scalars (α_m) with $|\alpha_m| \leq 1$ for all $m \in N$ whenever $x \in V_\sigma(M,p,r,\triangle_v^u)$.

Corollary 2.6. The sequence space $V_\sigma(M,p,r,\triangle_v^u)$ is monotone.

Theorem 2.7. Let M_1, M_2 be Orlicz function satisfying $\triangle_2$ condition and $r, r_1, r_2 \geq 0$. Then we have

(a) If $r > 1$ then $V_\sigma(M_1,p,r,\triangle_v^u) \subseteq V_\sigma(M0M_1,p,r,\triangle_v^u)$,

(b) $V_\sigma(M_1,p,r,\triangle_v^u) \cap V_\sigma(M_2,p,r,\triangle_v^u) \subseteq V_\sigma(M_1+M_2,p,r,\triangle_v^u)$,

(c) If $r_1 \leq r_2$ then $V_\sigma(M,p,r_1,\triangle_v^u) \subseteq V_\sigma(M,p,r_2,\triangle_v^u)$.

Proof. (a) Since M is continuous at 0 from right, for $\varepsilon > 0$ there exists $0 < \delta < 1$ such that $0 \leq c \leq \delta$ implies $M(c) < \varepsilon$.

If we define

$$I_1 = \{m \in N : M_1(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho}) \leq \delta \text{ for some } \rho > 0\},$$

$$I_2 = \{m \in N : M_1(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho}) > \delta \text{ for some } \rho > 0\},$$

when

$$M_1(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho}) > \delta$$

we get

$$M(M_1(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho})) \leq \{\frac{2M(1)}{\delta}\} M_1(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho})$$

Hence for $x \in V_\sigma(M_1, p, r, \triangle_v^u)$ and $r > 1$

$$\sum_{m=1}^{\infty} \frac{1}{m^r}[M0M_1(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho})]^{p_m} = \sum_{m \in I_1} \frac{1}{m^r}[M0M_1(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho})]^{p_m} + \sum_{m \in I_2} \frac{1}{m^r}[M0M_1(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho})]^{p_m}.$$

$$\sum_{m=1}^{\infty} \frac{1}{m^r}[M0M_1(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho})]^{p_m} \leq max(\varepsilon^h, \varepsilon^H) \sum_{m=1}^{\infty} \frac{1}{m^r} + max(\{\frac{2M_1}{\delta}\}^h, \{\frac{2M_1}{\delta}\}^H)$$

where $0 < h = \inf p_m \leq p_m \leq H = \sup_m p_m < \infty$

(b)The proof follows from the following inequality

$$\frac{1}{m^r}[(M_1 + M_2)(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho})]^{p_m} \leq C\frac{1}{m^r}[M_1(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho})]^{p_m} + C\frac{1}{m^r}[M_2(\frac{|t_{m,n}(\triangle_v^u x)|}{\rho})]^{p_m}$$

(c)The proof is straightforward.

Corollary 2.8. Let M be an Orlicz function satisfying $\triangle_2$ condition. Then we have

(a) If $r > 1$ then $V_\sigma(p, r, \triangle_v^u) \subseteq V_\sigma(M, p, r, \triangle_v^u)$,

(b) $V_\sigma(M, p, \triangle_v^u) \subseteq V_\sigma(M, p, r, \triangle_v^u)$,

(c) $V_\sigma(p, \triangle_v^u) \subseteq V_\sigma(p, r, \triangle_v^u)$,

(d) $V_\sigma(M,\triangle_v^u) \subseteq V_\sigma(M,r,\triangle_v^u)$.

Proof. The proof is straightforward.

Conflict of Interests

The author declare that there is no conflict of interests.

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