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ON (L,M)-FUZZY SOFT TOPOLOGICAL SPACES

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Abstract. In this paper, the concepts of (L,M)-fuzzy soft topological spaces, (L,M)-fuzzy soft base and (L,M)-fuzzy soft filter spaces were introduced and their properties were studied, where L be a completely distributive lattice with 0 and 1 elements and M be a strictly two-sided, commutative quantale lattice. Also, the relationships between these concepts were investigated.

Keywords: (L,M)-fuzzy soft topological spaces; (L,M)-fuzzy soft base; (L,M)-fuzzy soft filter spaces.

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1. Introduction

In 1999, D. Molodtsov [29] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties. Also, he applied this theory to several directions (see, for example, [30],[31],[32]). The soft set theory has been applied to many different fields (see, for example, [1],[2],[6],[7],[10],[11], [21],[27],[33],[44],[39],[45]). Later, some researchers (see, for

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example, [3], [8], [19], [20], [28], [34], [40], [46]) introduced and studied the notion of soft topological spaces.

Šostak introduced a new definition of fuzzy topology in 1985 [41], which we will call"fuzzy topology on Šostak sense." According to Šostak [41], these definitions, a fuzzy topology is a crisp subfamily of family of fuzzy sets and fuzziness in the concept of openness of a fuzzy set has not been considered, which seems to be a drawback in the process of fuzzification of the concept of topological spaces.

In this paper, we introduce the concepts of (L,M)-fuzzy soft topological spaces and (L,M)-fuzzy soft filter spaces in Šostak sense. We study their properties and discuss the relationships between these concepts.

2. Preliminaries

Definition 2.1 [13]. Let $(L,\leq)$ be a poset.

(1) L is called a lattice, if $a\vee b\in L, a\wedge b\in L$ for any $a,b\in L$.

(2) L is called a complete lattice, if $\vee S\in L, \wedge S\in L$ for any $S\subseteq L$.

(3) L is called distributive, if $a\vee(b\wedge c)=(a\vee b)\wedge(a\vee c)$, $a\wedge(b\vee c)=(a\wedge b)\vee(a\wedge c)$ for any $a,b,c\in L$.

(4) L is called a complete distributive lattice (resp. a distributive lattice), if L is a complete lattice (resp. a lattice) and distributive.

Definition 2.2 [13]. Let L be a lattice with top element 1_L and bottom element 0_L and let $a,b\in L$. Then b is called a complement element of a, if $a\vee b=1_L, a\wedge b=0_L$. If $a\in L$ has a complement element, then it is unique. We denote the complement element of a by a'.

Definition 2.3 [13]. Let $(L,\leq)$ be a poset. Then

(1) L is called a Boolean lattice, if (i) L is a distributive lattice; (ii) L has 0_L and 1_L; (iii) each $a\in L$ has the complement $a'\in L$.

(2) L is called a complete Boolean lattice, if (i) L is a complete distributive lattice; (ii) L has 0_L and 1_L; (iii) each $a\in L$ has the complement $a'\in L$.

Definition 2.4 [14],[15],[35],[42]. A triple $(L,\leq,\odot)$ is called a strictly two-sided commutative quantale (stsc-quantale, for short) if and only if it satisfies the following conditions:

(L1) $(L,\leq,\vee,\wedge,1,0)$ is a completely distributive lattice where 1 is the universal upper bound and 0 is the universal lower bound.

(L2) $(L,\odot)$ is a commutative semigroup.

(L3) $x = x\odot 1$ for each $x\in L$.

(L4) $\odot$ is distributive over arbitrary joins, i.e.$(\bigvee_{i\in\Gamma} a_i)\odot b = \bigvee_{i\in\Gamma}(a_i\odot b)$.

Let $(L,\leq,\odot)$ be a stsc-quantale. Then for each $x,y\in L$ we define $(x\odot y)\leq z \Longleftrightarrow x\leq (y\rightarrow z)$. The it satisfies Galois correspondence. i.e. $(x\odot y)\leq z$ if and only if $x\leq (y\rightarrow z)$.

Definition 2.5 [37]. Let E be a set of parameters, X be an initial universe. A pair (f,E) is called a fuzzy soft set over X, if f is a mapping given by $f: E\rightarrow I^X$. We also denote (f,E) by f_E. The set of all fuzzy soft set is denoted by $FS(X,E)$.

Definition 2.6 [26]. A fuzzy soft set f_E on X is called a null fuzzy soft set and denoted by $\tilde{0}$ if $f_e = \overline{0}$, for each $e\in E$.

Definition 2.7 [4]. A fuzzy soft set f_E on X is called an absolute fuzzy soft set and denoted by $\tilde{1}$ if $f_e = \overline{1}$, for each $e\in E$.

Definition 2.8 [25]. Let E be a set of parameters, X be an initial universe, L be a complete Boolean lattice and $A\subseteq E$. An L-fuzzy soft set f_A over (X,E) is a mapping $f_A: E\rightarrow L^X$ such that $f_A(e) = \overline{0}$ for all $e\notin A$. The set of all L-fuzzy soft set over (X,E) is denoted by L-$FS(X,E)$.

In other words, an L-fuzzy soft set f_E over X is a parameterized family of L-fuzzy sets in the universe X. If $L=[0,1]$, then every L-fuzzy soft set is a fuzzy soft set.

Definition 2.9 [25]. Let $f_A, g_B\in L$-$FS(X,E)$. Then

(1) f_A is said to by fuzzy soft subset of g_B, denoted by $f_A \sqsubseteq g_B$ if $f_A(e) \subseteq g_B(e)$ for all $e \in E$, that is $f_A(e)(x) \leq g_B(e)(x)$ for all $e \in E$, and for all $x \in X$.

Two L-fuzzy soft sets f_A and g_B over (X,E) are said to be equal, denoted by $f_A \cong g_B$ if $f_A \sqsubseteq g_B$ and $g_B \sqsubseteq f_A$.

(2) The union of f_A and g_B is also L-fuzzy soft set h_C, defined by $h_C(e) \cong f_A(e) \vee g_B(e)$ for all $e \in E$, where $C = A \cup B$. Here we write $h_C = f_A \sqcup g_B$.

(3) The intersection of f_A and g_B is also L-fuzzy soft set h_C, defined by $h_C(e) \cong f_A(e) \wedge g_B(e)$ for all $e \in E$, where $C = A \cap B$. Here we write $h_C = f_A \sqcap g_B$.

Definition 2.10 [38]. The fuzzy soft set $f_A \in FS(X,E)$ is called fuzzy soft point if $A = \{e\} \subseteq E$ and $f_A(e)$ is a fuzzy point in X i.e. there exists $x \in X$ such that $f_A(e)(x) = t$ $(0 < t \leq 1)$ and $f_A(e)(y) = 0$ for all $y \in X \setminus \{x\}$. We denote this fuzzy soft point $f_A = e^t_x = \{(e, x_t)\}$ and the set of all fuzzy soft point by $SP^e_t(X,E)$.

Definition 2.11 [38]. Let $e^t_x, f_A \in FS(X,E)$. we say that $e^t_x \widetilde{\in} f_A$ read as e^t_x belongs to the fuzzy soft set f_A if for the element $e \in A, t \leq f_A(e)(x)$.

Definition 2.12 [5]. Let (X,E) and (Y,E^*) be classes of fuzzy soft sets over X and Y with attributes from E and E^* respectively. Let $\rho : X \to Y$ and $\psi : E \to E^*$ be mapping. Then a fuzzy soft mapping $f = (\rho, \psi) : (X,E) \to (Y,E^*)$ would be defined as follows
For a fuzzy soft set F_A in (X,E), $f(F_A)$ is a fuzzy soft set in (Y,E^*) obtained as follows: for $\beta \in \psi(E) \subseteq E^*$ and $y \in Y$,

$$f(F_A)(\beta)(y) = \begin{cases} \vee_{x \in \rho^{-1}(y)} \big(\vee_{\alpha \in \psi^{-1}(\beta)} F_A(\alpha) \big)(x), \\ \qquad \text{if } \ \rho^{-1}(y) \neq \phi, \ \psi^{-1}(\beta) \neq \phi, \\ 0, \qquad \text{if} \qquad \text{otherwise}. \end{cases}$$

$f(F_A)$ is called fuzzy soft image of the fuzzy soft set F_A.

Definition 2.13 [5]. Let (X,E) and (Y,E^*) be classes of fuzzy soft sets over X and Y with attributes from E and E^* respectively. Let $\rho : X \to Y$, $\psi : E \to E^*$ be mappings and $f = (\rho, \psi) :$

$(X,E) \to (Y,E^*)$ a fuzzy soft mapping. Then for a fuzzy soft set g_B in (Y,E^*) $f^{-1}(g_B)$ is a fuzzy soft set in (X,E) obtained as follows: for $\alpha \in \psi^{-1}(E^*) \subseteq E$ and $x \in E$,

$$f^{-1}(g_B)(\alpha)(x) = g_B(\psi(\alpha))(\rho(x)).$$

$f^{-1}(g_B)$ is called a fuzzy soft inverse image of the fuzzy soft set g_B.

3. (L,M)-fuzzy soft topological spaces

Let L be a completely distributive lattice with 0 and 1 elements and M be a strictly two-sided, commutative quantale lattice.

Definition 3.1. A map $\mathscr{T} : L\text{-}FS(X,E) \longrightarrow M$ is called an (L,M)-fuzzy soft topology on (X,E) if it satisfies the following conditions:

(LSO1) $\mathscr{T}(\widetilde{0}) = \mathscr{T}(\widetilde{1}) = 1$.

(LSO2) $\mathscr{T}(f_{A_1} \sqcap f_{A_2}) \geq \mathscr{T}(f_{A_1}) \odot \mathscr{T}(f_{A_2})$, for all $f_{A_1}, f_{A_2} \in L\text{-}FS(X,E)$.

(LSO3) $\mathscr{T}(\bigsqcup_{i\in\Lambda} f_{A_i} \geq \bigwedge_{i\in\Lambda})\mathscr{T}(f_{A_i})$, for all $f_{A_i} \in L\text{-}FS(X,E)$.

The triple $(X,E,\mathscr{T})$ is called (L,M)-fuzzy soft topological space.

Let $\mathscr{T}_1$ and $\mathscr{T}_2$ be (L,M)-fuzzy soft topologies on (X,E). We say that $\mathscr{T}_1$ is finer than $\mathscr{T}_2$ ($\mathscr{T}_2$ is coarser than $\mathscr{T}_1$), denoted by $\mathscr{T}_2 \sqsubseteq \mathscr{T}_1$, if $\mathscr{T}_2(f_A) \leq \mathscr{T}_1(f_A)$, for all $f_A \in L\text{-}FS(X,E)$.

Let $(X,E,\mathscr{T}_1)$ and $(Y,E^*,\mathscr{T}_2)$ be (L,M)-fuzzy soft topological spaces. A soft map $\phi : (X,E,\mathscr{T}_1) \to (Y,E^*,\mathscr{T}_2)$ is called LFS-continuous if and only if $\mathscr{T}_2(f_A) \leq \mathscr{T}_1(\phi^{\leftarrow}(f_A))$, for all $f_A \in L\text{-}FS(Y,E^*)$.

Remark 3.2. (1) If $(L = [0,1], \wedge)$ and $M = \{0,1\}, (L,M)$-fuzzy soft topological space is fuzzy soft topological space [37].

(2) If $(L = M = [0,1], \odot = \wedge)$ then (L,M)-fuzzy soft topological space is fuzzy soft topological space [4].

Definition 3.3. A map $\mathscr{F} : L\text{-}FS(X,E) \longrightarrow M$ is called an (L,M)-fuzzy soft filter on (X,E) if it satisfies the following conditions:

(LSF1) $\mathscr{F}(\widetilde{0}) = 0$ and $\mathscr{F}(\widetilde{1}) = 1$.

(LSF2) $\mathscr{F}(f_{A_1} \sqcap f_{A_2}) \geq \mathscr{F}(f_{A_1}) \odot \mathscr{F}(f_{A_2})$, for all $f_{A_1}, f_{A_2} \in L\text{-}FS(X,E)$.

(LSF3) If $f_{A_1} \sqsubseteq f_{A_2}$ we have $\mathscr{F}(f_{A_1}) \leq \mathscr{F}(f_{A_2})$.

The triple $(X,E,\mathscr{F})$ is called an (L,M)-fuzzy soft filter space.

Theorem 3.4. Let $(X,E,\mathscr{F})$ be an (L,M)-fuzzy soft filter space. We define a mapping $\mathscr{T}_{\mathscr{F}} : L\text{-}FS(X,E) \longrightarrow M$ as follows:

$$\mathscr{T}_{\mathscr{F}}(f_A) = \begin{cases} \mathscr{F}(f_A), & \text{if } f_A \ncong \widetilde{0}, \\ 1, & \text{if } f_A \cong \widetilde{0}. \end{cases}$$

Then $(X,E,\mathscr{T}_{\mathscr{F}})$ is an (L,M)-fuzzy soft topological space.

Proof. We show the condition (LSO3). For $f_{A_i} \in L\text{-}FS(X,E)$, since $f_{A_i} \sqsubseteq \bigsqcup_{i\in\Gamma} f_{A_i}$ for all $i \in \Gamma$, we have $\mathscr{F}(f_{A_i}) \leq \mathscr{F}(\bigsqcup_{i\in\Gamma} f_{A_i})$, so

$$\bigwedge_{i\in\Gamma} \mathscr{T}_{\mathscr{F}}(f_{A_i}) \leq \mathscr{T}_{\mathscr{F}}(\bigsqcup_{i\in\Gamma} f_{A_i}).$$

Definition 3.5. A map $\mathscr{B} : L\text{-}FS(X,E) \to M$ is called an (L,M)-fuzzy soft base on (X,E) if it satisfies the following conditions:

(LSB1) $\mathscr{B}(\widetilde{0}) = \mathscr{B}(\widetilde{1}) = 1$.

(LSB2) $\mathscr{B}(f_{A_1} \sqcap f_{A_2}) \geq \mathscr{B}(f_{A_1}) \odot \mathscr{B}(f_{A_2})$, for all $f_{A_1}, f_{A_2} \in L\text{-}FS(X,E)$.

Theorem 3.6. Let $\mathscr{B}$ be an (L,M)-fuzzy soft base on (X,E). Define a map $\mathscr{T}_{\mathscr{B}} : L\text{-}FS(X,E) \to M$ as follows:

$$\mathscr{T}_{\mathscr{B}}(f_A) = \bigvee \{ \bigwedge_{i\in\Gamma} \mathscr{B}(f_{A_i}) : f_A = \bigsqcup_{i\in\Gamma} f_{A_i} \}.$$

Then $\mathscr{T}_{\mathscr{B}}$ is the coarsest (L,M)-fuzzy soft topology on (X,E) such that $\mathscr{T}_{\mathscr{B}}(f_A) \geq \mathscr{B}(f_A)$ for all $f_A \in L\text{-}FS(X,E)$.

Proof. (1) It is trivial from the definition of $\mathscr{T}_{\mathscr{B}}$.

(2) For all families $\{f_{A_i} : f_A = \bigsqcup_{i\in\Delta} f_{A_i}\}$ and $\{g_{B_j} : g_B = \bigsqcup_{j\in\Gamma} g_{B_j}\}$ there exists a family $\{f_{A_i} \sqcap g_{B_j}\}$ such that:

$$f_A \sqcap g_B = (\bigsqcup_{i\in\Delta} f_{A_i}) \sqcap (\bigsqcup_{j\in\Gamma} g_{B_i}) = \bigsqcup_{i\in\Delta, j\in\Gamma} (f_{A_i} \sqcap g_{B_i}).$$

It implies

$$\mathscr{T}_{\mathscr{B}}(f_A \sqcap g_B) \geq \bigwedge_{i\in\Delta, j\in\Gamma} \mathscr{B}(f_{A_i} \sqcap g_{B_i})$$

$$\geq \bigwedge_{i\in\Delta, j\in\Gamma} (\mathscr{B}(f_{A_i}) \odot \mathscr{B}(g_{B_j})) \text{ (by Definition 3.5 (LSB2))}$$

$$\geq (\bigwedge_{i\in\Delta} \mathscr{B}(f_{A_i})) \odot (\bigwedge_{j\in\Gamma} \mathscr{B}(g_{B_j})).$$

By definition 2.4 (L4) we have $\mathscr{T}_{\mathscr{B}}(f_A \sqcap g_B) \geq \mathscr{T}_{\mathscr{B}}(f_A) \odot \mathscr{T}_{\mathscr{B}}(g_B)$.

(3) Let $\mathscr{J}_i$ be the collection of all index sets K_i such that $\{f_{A_{i_k}} \in L\text{-}FS(X,E) : f_{A_i} = \bigsqcup_{k\in K_i} f_{A_{i_k}}\}$ with $f_A = \bigsqcup_{i\in\Gamma} f_{A_i} = \bigsqcup_{i\in\Gamma} \bigsqcup_{k\in K_i} f_{A_{i_k}}$. For each $i \in \Gamma$ and each $\psi \in \Pi_{i\in\Gamma} \mathscr{J}_i$ with $\psi(i) = K_i$ we have

$$\mathscr{T}_{\mathscr{B}}(f_A) \geq \bigwedge_{i\in\Gamma} (\bigwedge_{k\in K_i} \mathscr{B}(f_{A_{i_k}})). \tag{1}$$

Put $a_{i,\psi(i)} = \bigwedge_{k\in K_i} \mathscr{B}(f_{A_{i_k}})$. From (3.1) we have

$$\mathscr{T}_{\mathscr{B}}(f_A) \geq \bigvee_{\psi\in\Pi_{i\in\Gamma}} (\bigwedge_{i\in\Gamma} a_{i,\psi(i)})$$

(Since L is a completely distributive lattice,)

$$= \bigwedge_{i\in\Gamma} (\bigvee_{M_i\in\mathscr{J}_i} a_{i,M_i}) = \bigwedge_{i\in\Gamma} (\bigvee_{M_i\in\mathscr{J}_i} (\bigwedge_{m\in M_i} \mathscr{B}(f_{A_{i_m}})))$$

$$= \bigwedge_{i\in\Gamma} \mathscr{T}_{\mathscr{B}}(f_{A_i}).$$

Thus $\mathscr{T}_{\mathscr{B}}$ is a (L,M)-fuzzy soft topology on X.

If $\mathscr{T} \geq \mathscr{B}$ for every $f_A = \bigsqcup_{i\in\Delta} f_{A_i}$ we have

$$\mathscr{T}(f_A) \geq \bigwedge_{i\in\Delta} \mathscr{T}(f_{A_i}) \geq \bigwedge_{i\in\Delta} \mathscr{B}(f_{A_i}).$$

Thus $\mathscr{T} \sqsupseteq \mathscr{T}_{\mathscr{B}}$.

From Theorem 3.6, we can easily prove the following lemma.

Lemma 3.7. Let $\mathscr{T}$ be an (L,M)-fuzzy soft topology on (X,E) and $\mathscr{B}$ be an (L,M)-fuzzy soft base on (Y,E^*). Then a map $\phi : (X,E,\mathscr{T}) \to (Y,E^*,\mathscr{T}_{\mathscr{B}})$ is LFS-continuous if and only if $\mathscr{T}(\phi^{\leftarrow}(f_A)) \geq \mathscr{B}(f_A)$ for each $f_A \in L\text{-}FS(Y,E^*)$.

Theorem 3.8. Let $\{(X_i,E_i,\mathscr{T}_i) : i \in \Gamma\}$ be a family of (L,M)-fuzzy soft topological spaces, X a set, E be a set of parameters and for each $i \in \Gamma$, $\phi_i : (X,E) \to (X_i,E_i)$ a fuzzy soft map. Define a map $\mathscr{B} : L\text{-}FS(X,E) \to M$ on (X,E) by:

$$\mathscr{B}(f_A) = \bigvee\{\odot_{j=1}^{n} \mathscr{T}_{k_j}(g_{B_{k_j}}) : f_A = \sqcap_{j=1}^{n} \phi_{k_j}^{\leftarrow}(g_{B_{k_j}})\}$$

where $\bigvee$ is taken over all finite subsets $K = \{k_1,...,k_n\} \subset \Gamma$.

Then: (1) $\mathscr{B}$ is an (L,M)-fuzzy soft base on (X,E).

(2) The (L,M)-fuzzy soft topology $\mathscr{T}_{\mathscr{B}}$ generated by $\mathscr{B}$ is the coarsest (L,M)-fuzzy soft topology on (X,E) for which all $\phi_i, i \in \Gamma$ are LFS-continuous maps.

Proof. (1)(LSB1)Since $f_A = \phi_i^{\leftarrow}(f_A)$ for each $f_A \in \{\widetilde{0},\widetilde{1}\}$ we have $\mathscr{B}(\widetilde{0}) = \mathscr{B}(\widetilde{1}) = 1$.

(LSB2) For all finite subsets $K = \{k_1,...,k_p\}$ and $J = \{j_1,...j_q\}$ of Γ such that

$$f_A = \sqcap_{i=1}^{p} \phi_{k_i}^{\leftarrow}(f_{A_{k_i}}), \qquad g_B = \sqcap_{i=1}^{q} \phi_{j_i}^{\leftarrow}(g_{B_{j_i}}),$$

we have

$$f_A \sqcap g_B = (\sqcap_{i=1}^{p} \phi_{k_i}^{\leftarrow}(f_{A_{k_i}})) \sqcap \sqcap_{i=1}^{q} \phi_{j_i}^{\leftarrow}(g_{B_{j_i}}).$$

Furthermore, we have for each $k \in K \cap J$,

$$\phi_k^{\leftarrow}(f_{A_k}) \sqcap \phi_k^{\leftarrow}(g_{B_k}) = \phi_k^{\leftarrow}(f_{A_k} \sqcap g_{B_k}).$$

Put $f_A \sqcap g_B = \sqcap_{m_i \in K\cup J} \phi^{\leftarrow}_{m_i}(h_{C_{m_i}})$ where

$$h_{C_{m_i}} = \begin{cases} f_{A_{m_i}}, & \text{if } m_i \in K-(K\cap J), \\ g_{B_{m_i}}, & \text{if } m_i \in J-(K\cap J), \\ f_{A_{m_i}} \sqcap g_{B_{m_i}}, & \text{if } m_i \in K\cap J. \end{cases}$$

We have

$$\begin{aligned} &\mathscr{B}(f_A \sqcap g_B) \geq \odot_{j\in K\cup J} \mathscr{T}_J(h_{C_j}) \\ &\geq (\odot_{m_i\in K-K\cap J} \mathscr{T}_{m_i}(f_{A_{m_i}})) \odot (\odot_{i=1} \mathscr{T}_{m_i \in J-K\cap J}(g_{B_{m_i}})) \\ &\odot (\odot_{m_i\in K\cap J} \mathscr{T}_{m_i}(f_{A_{m_i}} \sqcap g_{B_{m_i}})) \\ &\geq (\odot_{i=1}^{p} \mathscr{T}_{j_i}(f_{A_{m_i}}) \odot (\odot_{i=1}^{q} \mathscr{T}_{j_i}(g_{B_{j_i}})). \end{aligned}$$

By Definition 2.4 (L4) we have $\mathscr{B}(f_A \sqcap g_B) \geq \mathscr{B}(f_A) \odot \mathscr{B}(g_B)$.

(2) For each $f_{A_i} \in L\text{-}FS(X_i,E_i)$, one family $\{\phi_i^{\leftarrow}(f_{A_i})\}$ and $i\in\Gamma$ we have

$$\mathscr{T}_{\mathscr{B}}(\phi_i^{\leftarrow}(f_{A_i})) \geq \mathscr{B}(\phi_i^{\leftarrow}(f_{A_i})) \geq \mathscr{T}_i(f_{A_i}).$$

Thus, for each $i\in\Gamma, \phi_i : (X,E,\mathscr{T}_{\mathscr{B}}) \to (X_i,E_i,\mathscr{T}_i)$ is LFS-continuous. Let $\phi_i : (X,E,\mathscr{T}^0) \to (X_i,E_i,\mathscr{T}_i)$ is LFS-continuous, that is for each $i\in\Gamma$ and $f_{A_i} \in L\text{-}FS(X_i,E_i)$, $\mathscr{T}^0(\phi_i^{\leftarrow}(f_{A_i})) \geq \mathscr{T}_i(f_{A_i})$. For all finite subsets $K=\{k_1,...k_p\}$ of Γ such that $f_A = \odot_{i=1}^{p} \phi^{\leftarrow}_{k_i}(f_{A_{k_i}})$ we have

$$\mathscr{T}^0(f_A) \quad \geq \odot_{i=1}^{p} \mathscr{T}^0(\phi^{\leftarrow}_{k_i}(f_{A_{k_i}})) \geq \odot_{i=1}^{p} \mathscr{T}_{k_i}(f_{A_{k_i}}).$$

It implies $\mathscr{T}^0(f_A) \geq \mathscr{B}(f_A)$ for each $f_A \in L\text{-}FS(X,E)$. By Theorem 3.6 $\mathscr{T}^0 \geq \mathscr{T}_{\mathscr{B}}$.

Example 3.9. Let $X=\{x,y\}$ be a set, $E=\{e_1,e_2,e_3\}$ be a set of parameters and $L=M=[0,1]$ a completely distributive lattice. Define a binary operation $\odot$ on $M=[0,1]$ by $x\odot y = \max\{0,x+y-1\}$. Then $([0,1],\leq,\odot)$ is a stsc-quantale. Let $g_B, h_C \in L\text{-}FS(X,E)$ be defined as follows:

$$\begin{aligned} g_B =& \{g(e_1) = \{(x,0.6),(y,0.3)\},\ g(e_2)=\overline{0},\ g(e_2)=\overline{0}\} \\ h_C =& \{h(e_1) = \{(x,0.5),(y,0.7)\},\ h(e_2)=\overline{0},\ h(e_2)=\overline{0}\}. \end{aligned}$$

Then we have

$$g_B \sqcap h_C = \{(g_B \sqcap h_C)(e_1) = \{(x,0.5),(y,0.3)\},$$
$$(g_B \sqcap h_C)(e_2) = \overline{0}, (g_B \sqcap h_C)(e_2) = \overline{0}\}$$
$$g_B \sqcup h_C = \{(g_B \sqcup h_C)(e_1) = \{(x,0.6),(y,0.7)\},$$
$$(g_B \sqcup h_C)(e_2) = \overline{0}, (g_B \sqcup h_C)(e_2) = \overline{0}\}.$$

We define an (L,M)-fuzzy soft topology $\mathscr{T} : L\text{-}FS(X,E) \to [0,1]$ as follows:

$$\mathscr{T}(f_A) = \begin{cases} 1, & \text{if } f_A \cong \widetilde{0} \text{ or } \widetilde{1}, \\ 0.8, & \text{if } f_A \cong g_B, \\ 0.4, & \text{if } f_A \cong h_C, \\ 0.6, & \text{if } f_A \cong g_B \sqcup h_C, \\ 0.2, & \text{if } f_A \cong g_B \sqcap h_C, \\ 0, & \text{otherwise.} \end{cases}$$

Conflict of Interests

The authors declare that there is no conflict of interests.

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