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(L,M)-FUZZY SOFT QUASI- COINCIDENT NEIGHBORHOOD SPACES

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Abstract. In this paper, we introduce the concepts of (L,M)-fuzzy soft quasi-coincident neighborhood spaces and study their properties, where L be a completely distributive lattice with 0 and 1 elements and M be a strictly two-sided, commutative quantale lattice. Also, the relationships between these concepts were investigated. Furthermore, a characterization of LFS-continuous and LSN-mappings were given.

Keywords: (L,M)-fuzzy soft topological spaces; (L,M)-fuzzy soft filter spaces; (L,M)-fuzzy soft quasi-coincident neighborhood spaces

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1. Introduction

In 1999, D. Molodtsov [29] introduced the theory of soft sets as a new mathematical tool for dealing with uncertainties. The soft set theory has been applied to many different fields ([1],[2],[6],[7],[10],[11], [21],[27],[34],[45],[40],[46]). Later, few researches (see, for example,

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[3], [8], [19], [20], [28], [35], [41], [47]) introduced and studied the notion of soft topological spaces.

Höhle and Šostak [16] introduced L-fuzzy topologies with algebraic structure L(cqm, quantales, MV -algebra). Sayed[33] we introduce the concepts of (L,M)-fuzzy soft topological spaces and (L,M)-fuzzy soft filter spaces.

In this paper, we introduce the concepts of (L,M)-fuzzy soft quasi-coincident neighborhood spaces where L be a completely distributive lattice with 0 and 1 elements and M be a strictly two-sided, commutative quantale lattice. Also, the relationships between these concepts were investigated. Furthermore, a characterization of LFS-continuous and LSN-mappings were given.

2. Preliminaries

Definition 2.1 [13]. Let $(L,\leq)$ be a poset. Then

(1) L is called a Boolean lattice, if (i) L is a distributive lattice; (ii) L has 0_L and 1_L; (iii) each $a\in L$ has the complement $a'\in L$.

(2) L is called a complete Boolean lattice, if (i) L is a complete distributive lattice; (ii) L has 0_L and 1_L; (iii) each $a\in L$ has the complement $a'\in L$.

Definition 2.2 [14],[15],[36],[43]. A triple $(L,\leq,\odot)$ is called a strictly two-sided commutative quantale (stsc-quantale, for short) if and only if it satisfies the following conditions:

(L1) $(L,\leq,\vee,\wedge,1,0)$ is a completely distributive lattice where 1 is the universal upper bound and 0 is the universal lower bound.

(L2) $(L,\odot)$ is a commutative semigroup.

(L3) $x=x\odot 1$ for each $x\in L$.

(L4) $\odot$ is distributive over arbitrary joins, i.e.$(\bigvee_{i\in\Gamma}a_i)\odot b=\bigvee_{i\in\Gamma}(a_i\odot b)$.

Let $(L,\leq,\odot)$ be a stsc-quantale. Then for each $x,y\in L$ we define $(x\odot y)\leq z\Longleftrightarrow x\leq(y\to z)$. The it satisfies Galois correspondence. i.e. $(x\odot y)\leq z$ if and only if $x\leq(y\to z)$.

Definition 2.3 [38]. Let E be a set of parameters, X be an initial universe. A pair (f,E) is called a fuzzy soft set over X, if f is a mapping given by $f:E\to I^X$. We also denote (f,E) by f_E. The set of all fuzzy soft set is denoted by $FS(X,E)$.

Definition 2.4 [26]. A fuzzy soft set f_E on X is called a null fuzzy soft set and denoted by $\tilde{0}$ if $f_e = \overline{0}$, for each $e \in E$.

Definition 2.5 [4]. A fuzzy soft set f_E on X is called an absolute fuzzy soft set and denoted by $\tilde{1}$ if $f_e = \overline{1}$, for each $e \in E$.

Definition 2.6 [25]. Let E be a set of parameters, X be an initial universe, L be a complete Boolean lattice and $A \subseteq E$. An L-fuzzy soft set f_A over (X,E) is a mapping $f_A : E \to L^X$ such that $f_A(e) = \overline{0}$ for all $e \notin A$. The set of all L-fuzzy soft set over (X,E) is denoted by $L\text{-}FS(X,E)$.

In other words, an L-fuzzy soft set f_E over X is a parameterized family of L-fuzzy sets in the universe X. If $L = [0,1]$, then every L-fuzzy soft set is a fuzzy soft set.

Definition 2.7 [25]. Let $f_A, g_B \in L\text{-}FS(X,E)$. Then

(1) f_A is said to by fuzzy soft subset of g_B, denoted by $f_A \sqsubseteq g_B$ if $f_A(e) \subseteq g_B(e)$ for all $e \in E$, that is $f_A(e)(x) \leq g_B(e)(x)$ for all $e \in E$, and for all $x \in X$.

Two L-fuzzy soft sets f_A and g_B over (X,E) are said to be equal, denoted by $f_A \cong g_B$ if $f_A \sqsubseteq g_B$ and $g_B \sqsubseteq f_A$.

(2) The union of f_A and g_B is also L-fuzzy soft set h_C, defined by $h_C(e) \cong f_A(e) \vee g_B(e)$ for all $e \in E$, where $C = A \cup B$. Here we write $h_C = f_A \sqcup g_B$.

(3) The intersection of f_A and g_B is also L-fuzzy soft set h_C, defined by $h_C(e) \cong f_A(e) \wedge g_B(e)$ for all $e \in E$, where $C = A \cap B$. Here we write $h_C = f_A \sqcap g_B$.

Definition 2.8 [39]. The fuzzy soft set $f_A \in FS(X,E)$ is called fuzzy soft point if $A = \{e\} \subseteq E$ and $f_A(e)$ is a fuzzy point in X i.e. there exists $x \in X$ such that $f_A(e)(x) = t$ $(0 < t \leq 1)$ and $f_A(e)(y) = 0$ for all $y \in X \setminus \{x\}$. We denote this fuzzy soft point $f_A = e_x^t = \{(e, x_t)\}$ and the set of all fuzzy soft point by $SP_t^e(X,E)$.

Definition 2.9 [39]. Let $e_x^t, f_A \in FS(X,E)$. we say that $e_x^t \widetilde{\in} f_A$ read as e_x^t belongs to the fuzzy soft set f_A if for the element $e \in A, t \leq f_A(e)(x)$.

Definition 2.10 [5]. Let (X,E) and (Y,E^*) be classes of fuzzy soft sets over X and Y with attributes from E and E^* respectively. Let $\rho : X \to Y$ and $\psi : E \to E^*$ be mapping. Then a fuzzy soft mapping $f = (\rho, \psi) : (X,E) \to (Y,E^*)$ would be defined as follows
For a fuzzy soft set F_A in (X,E), $f(F_A)$ is a fuzzy soft set in (Y,E^*) obtained as follows: for $\beta \in \psi(E) \subseteq E^*$ and $y \in Y$,

$$f(F_A)(\beta)(y) = \begin{cases} \vee_{x\in\rho^{-1}(y)}\big(\vee_{\alpha\in\psi^{-1}(\beta)} F_A(\alpha)\big)(x), \\ \qquad \text{if } \rho^{-1}(y) \neq \phi,\ \psi^{-1}(\beta) \neq \phi, \\ 0, \qquad \text{if} \qquad \text{otherwise.} \end{cases}$$

$f(F_A)$ is called fuzzy soft image of the fuzzy soft set F_A.

Definition 2.11 [39]. Let $f_A, g_B \in FS(X,E)$. Then f_A is said to be soft quasi-coincident with g_B, denoted by $f_A q g_B$, if there exists $e \in E$ and $x \in X$ such that $f_A(e)(x) + g_B(e)(x) > 1$.
If f_A is not soft quasi-coincident with g_B, then we writ $f_A \overline{q} g_B$,

Definition 2.12 [5]. Let (X,E) and (Y,E^*) be classes of fuzzy soft sets over X and Y with attributes from E and E^* respectively. Let $\rho : X \to Y$, $\psi : E \to E^*$ be mappings and $f = (\rho, \psi) :$ $(X,E) \to (Y,E^*)$ a fuzzy soft mapping. Then for a fuzzy soft set g_B in (Y,E^*) $f^{-1}(g_B)$ is a fuzzy soft set in (X,E) obtained as follows: for $\alpha \in \psi^{-1}(E^*) \subseteq E$ and $x \in E$,

$$f^{-1}(g_B)(\alpha)(x) = g_B(\psi(\alpha))(\rho(x)).$$

$f^{-1}(g_B)$ is called a fuzzy soft inverse image of the fuzzy soft set g_B.

Let L be a completely distributive lattice with 0 and 1 elements and M be a strictly two-sided, commutative quantale lattice.

Definition 2.13.[33] A map $\mathscr{T} : L\text{-}FS(X,E) \longrightarrow M$ is called an (L,M)-fuzzy soft topology on (X,E) if it satisfies the following conditions:

(LSO1) $\mathscr{T}(\widetilde{0}) = \mathscr{T}(\widetilde{1}) = 1$.

(LSO2) $\mathscr{T}(f_{A_1} \sqcap f_{A_2}) \geq \mathscr{T}(f_{A_1}) \odot \mathscr{T}(f_{A_2})$, for all $f_{A_1}, f_{A_2} \in L\text{-}FS(X,E)$.

(LSO3) $\mathscr{T}(\bigsqcup_{i\in\Lambda} f_{A_i} \geq \bigwedge_{i\in\Lambda})\mathscr{T}(f_{A_i})$, for all $f_{A_i} \in L\text{-}FS(X,E)$.

The triple $(X,E,\mathscr{T})$ is called (L,M)-fuzzy soft topological space.

Let $\mathscr{T}_1$ and $\mathscr{T}_2$ be (L,M)-fuzzy soft topologies on (X,E). We say that $\mathscr{T}_1$ is finer than $\mathscr{T}_2$ ($\mathscr{T}_2$ is coarser than $\mathscr{T}_1$), denoted by $\mathscr{T}_2 \sqsubseteq \mathscr{T}_1$, if $\mathscr{T}_2(f_A) \leq \mathscr{T}_1(f_A)$, for all $f_A \in L\text{-}FS(X,E)$.

Let $(X,E,\mathscr{T}_1)$ and $(Y,E^*,\mathscr{T}_2)$ be (L,M)-fuzzy soft topological spaces. A soft map ϕ : $(X,E,\mathscr{T}_1) \to (Y,E^*,\mathscr{T}_2)$ is called LFS-continuous if and only if $\mathscr{T}_2(f_A) \leq \mathscr{T}_1(\phi^{\leftarrow}(f_A))$, for all $f_A \in L\text{-}FS(Y,E^*)$.

Definition 2.14. [33] A map $\mathscr{F} : L\text{-}FS(X,E) \longrightarrow M$ is called an (L,M)-fuzzy soft filter on (X,E) if it satisfies the following conditions:

(LSF1) $\mathscr{F}(\widetilde{0}) = 0$ and $\mathscr{F}(\widetilde{1}) = 1$.

(LSF2) $\mathscr{F}(f_{A_1} \sqcap f_{A_2}) \geq \mathscr{F}(f_{A_1}) \odot \mathscr{F}(f_{A_2})$, for all $f_{A_1}, f_{A_2} \in L\text{-}FS(X,E)$.

(LSF3) If $f_{A_1} \sqsubseteq f_{A_2}$ we have $\mathscr{F}(f_{A_1}) \leq \mathscr{F}(f_{A_2})$.

The triple $(X,E,\mathscr{F})$ is called an (L,M)-fuzzy soft filter space.

3. (L,M)-fuzzy soft quasi- coincident neighborhood spaces

Definition 3.1. An (L,M)-fuzzy soft quasi-coincident neighborhood system on (X,E) is a set $\mathscr{Q} = \{\mathscr{Q}_{e_x^t} : e_x^t \in SP_t^e(X,E)\}$ of maps $\mathscr{Q}_{e_x^t} : L\text{-}FS(X,E) \to M$ such that for each $f_A, g_B \in L\text{-}FS(X,E)$, we have

(LSN1) $\mathscr{Q}_{e_x^t}$ is an (L,M)-fuzzy soft filter on (X,E).

(LSN2) $\mathscr{Q}_{e_x^t}(f_A) > 0$ implies $e_x^t q f_A$.

(LSN3) $\mathscr{Q}_{e_x^t}(f_A) = \bigvee_{e_x^t q g_B \sqsubseteq f_A} \left(\bigwedge_{e_y^t q g_B} \mathscr{Q}_{e_y^t)}(g_B) \right)$.

The triple $(X,E,\mathscr{Q})$ is called an (L,M)-fuzzy soft quasi-coincident neighborhood space. $\mathscr{Q}_{e_x^t}(f_A)$ can be interpreted as the degree to which f_A is a soft quasi-coincident neighborhood of e_x^t.

An LSN-map between (L,M)-fuzzy soft quasi-coincident neighborhood spaces $(X,E,\mathscr{Q}_1)$ and
$(Y,E^*,\mathscr{Q}_2)$ is a soft map $\phi:(X,E,\mathscr{Q}_1)\to(Y,E^*,\mathscr{Q}_2)$ such that $(\mathscr{Q}_1)_{e_x^t}(\phi^{\leftarrow}(f_A))\geq(\mathscr{Q}_2)_{\phi^{\rightarrow}(e_x^t)}(f_A)$ for all $f_A\in L\text{-}FS(Y,E^*)$ and for all $e_x^t\in SP_t^e(X,E)$.

Theorem 3.2. Let $(X,E,\mathscr{T})$ be an (L,M)-fuzzy soft topological space and $e_x^t\in SP_t^e(X,E)$. Define a map $\mathscr{Q}_{e_x^t}^{\mathscr{T}}:L\text{-}FS(X,E)\to M$ as:

$$\mathscr{Q}_{e_x^t}^{\mathscr{T}}(f_A)=\begin{cases}\bigvee\{\mathscr{T}(g_B):e_x^tqg_B\sqsubseteq f_A\} & \text{if } e_x^tqf_A,\\ 0 & \text{if } e_x^t\overline{q}f_A.\end{cases}$$

Then:

(1) $\mathscr{Q}^{\mathscr{T}}=\{\mathscr{Q}_{e_x^t}^{\mathscr{T}}:e_x^t\in SP_t^e(X,E)\}$ is an (L,M)-fuzzy soft-coincident neighborhood system on (X,E).

(2) If $t<s$ for $t,s\in L$ then $\mathscr{Q}_{e_x^t}^{\mathscr{T}}(f_A)\leq\mathscr{Q}_{e_x^s}^{\mathscr{T}}(f_A)$.

Proof. (1) (LSN1) (LSF1) and (LSF3) are easily proved.

(LSF2) Suppose there exist $f_A,g_B\in L\text{-}FS(X,E)$ such that

$$\mathscr{Q}_{e_x^t}^{\mathscr{T}}(f_A\sqcap g_B)\not\geq\mathscr{Q}_{e_x^t}^{\mathscr{T}}(f_A)\odot\mathscr{Q}_{e_x^t}^{\mathscr{T}}(g_B).$$

By the definition of $\mathscr{Q}_{e_x^t}^{\mathscr{T}}(f_A)$ and (L4) of Definition 1.4, there exist $f_{A1}\in L\text{-}FS(X,E)$ with $e_x^tqf_{A1}\sqsubseteq f_A$ such that

$$\mathscr{Q}_{e_x^t}^{\mathscr{T}}(f_A\sqcap g_B)\not\geq\mathscr{T}(f_{A1})\odot\mathscr{Q}_{e_x^t}^{\mathscr{T}}(g_B).$$

Again, by the definition of $\mathscr{Q}_{e_x^t}^{\mathscr{T}}(g_B)$ and (L4) of Definition 1.4, there exist $g_{B1}\in L\text{-}FS(X,E)$ with $e_x^tqg_{B1}\sqsubseteq g_B$ such that

$$\mathscr{Q}_{e_x^t}^{\mathscr{T}}(f_A\sqcap g_B)\not\geq\mathscr{T}(f_{A1})\odot\mathscr{T}(g_{B1}).$$

Since $e_x^tq(f_{A1}\sqcap g_{B1})\sqsubseteq f_A\sqcap g_B$ we have

$$\mathscr{Q}_{e_x^t}^{\mathscr{T}}(f_A\sqcap g_B)\geq\mathscr{T}(f_{A1}\sqcap g_{B2})\geq\mathscr{T}(f_{A1})\odot\mathscr{T}(g_{B1}).$$

It is a contradiction. Hence, for all $f_A, g_B \in L\text{-}FS(X,E)$,

$$\mathscr{Q}^{\mathscr{T}}_{e^t_x}(f_A \sqcap g_B) \geq \mathscr{Q}^{\mathscr{T}}_{e^t_x}(f_A) \odot \mathscr{Q}^{\mathscr{T}}_{e^t_x}(g_B).$$

So, $\mathscr{Q}_{e^t_x}$ is an (L,M)-fuzzy soft filter on (X,E).

(LSN2) It is easy from the definition of $\mathscr{Q}^{\mathscr{T}}_{e^t_x}$.

(LSN3) For all $f_A \in L\text{-}FS(X,E)$ with $e^t_x q g_B \sqsubseteq f_A$ we have

$$\mathscr{T}(g_B) \leq \bigwedge\{\mathscr{Q}^{\mathscr{T}}_{e^s_y}(g_B) : e^s_y q g_B\} \leq \mathscr{Q}^{\mathscr{T}}_{e^t_x}(g_B) \leq \mathscr{Q}^{\mathscr{T}}_{e^t_x}(f_A).$$

Therefore,

$$\mathscr{Q}^{\mathscr{T}}_{e^t_x}(f_A) = \bigvee_{e^t_x q g_B \sqsubseteq f_A} \mathscr{T}(g_B) \leq \bigvee_{e^t_x q g_B} \Big(\bigwedge_{e^s_y q g_B} \mathscr{Q}^{\mathscr{T}}_{e^s_y}(g_B)\Big) \leq \mathscr{Q}^{\mathscr{T}}_{e^t_x}(f_A).$$

This means that

$$\mathscr{Q}^{\mathscr{T}}_{e^t_x}(f_A) = \bigvee_{e^t_x q g_B \sqsubseteq f_A} \Big(\bigwedge_{e^s_y q g_B} \mathscr{Q}^{\mathscr{T}}_{e^s_y}(g_B)\Big).$$

(2) For $t < s$ with $t,s \in L$ and for all $f_A \in L\text{-}FS(X,E)$ since

$$\{g_B \in L\text{-}FS(X,E) : e^t_x q g_B \sqsubseteq f_A\} \subset \{h_C \in L\text{-}FS(X,E) : e^s_x q h_C \sqsubseteq f_A\},$$

we have $\mathscr{Q}^{\mathscr{T}}_{e^t_x}(f_A) \leq \mathscr{Q}^{\mathscr{T}}_{e^s_x}(f_A)$.

Example 3.3. Let $X = \{x,y\}$ be a set, $E = \{e_1,e_2,e_3\}$ be a set of parameters and $L = M = [0,1]$ a completely distributive lattice. Define a binary operation $\odot$ on $M = [0,1]$ by $x \odot y = \max\{0, x+y-1\}$. Then $([0,1], \leq, \odot)$ is a stsc-quantale. Let $g_B, h_C \in L\text{-}FS(X,E)$ be defined as follows:

$$g_B = \{g(e_1) = \{(x,0.6),(y,0.3)\},\ g(e_2) = \overline{0},\ g(e_2) = \overline{0}\}$$

$$h_C = \{h(e_1) = \{(x,0.5),(y,0.7)\},\ h(e_2) = \overline{0},\ h(e_2) = \overline{0}\}.$$

Then we have

$$\begin{aligned} g_B \sqcap h_C =& \{(g_B \sqcap h_C)(e_1) = \{(x,0.5),(y,0.3)\}, \\ & (g_B \sqcap h_C)(e_2) = \overline{0}, (g_B \sqcap h_C)(e_2) = \overline{0}\} \\ g_B \sqcup h_C =& \{(g_B \sqcup h_C)(e_1) = \{(x,0.6),(y,0.7)\}, \\ & (g_B \sqcup h_C)(e_2) = \overline{0}, (g_B \sqcup h_C)(e_2) = \overline{0}\}. \end{aligned}$$

We define an (L,M)-fuzzy soft topology $\mathscr{T} : L\text{-}FS(X,E) \to [0,1]$ as follows:

$$\mathscr{T}(f_A) = \begin{cases} 1, & \text{if } f_A \cong \widetilde{0} \text{ or } \widetilde{1}, \\ 0.8, & \text{if } f_A \cong g_B, \\ 0.4, & \text{if } f_A \cong h_C, \\ 0.6, & \text{if } f_A \cong g_B \sqcup h_C, \\ 0.2, & \text{if } f_A \cong g_B \sqcap h_C, \\ 0, & \text{otherwise.} \end{cases}$$

We obtain $\mathscr{Q}^{\mathscr{T}}_{(e_1)^{0.5}_x} : L\text{-}FS(X,E) \to [0,1]$ as follows:

$$\mathscr{Q}^{\mathscr{T}}_{(e_1)^{0.5}_x}(f_A) = \begin{cases} 1, & \text{if } f_A \cong \widetilde{1}, \\ 0.8, & \text{if } g_B \sqsubseteq f_A, \\ 0, & \text{otherwise.} \end{cases}$$

Theorem 3.4. Let $\mathscr{Q} = \{\mathscr{Q}_{e^t_x} : e^t_x \in SP^e_t(X,E)\}$ be a family of $\mathscr{Q}_{e^t_x} : L\text{-}FS(X,E) \to M$ satisfying (LSN1) and (LSN2) of definition 3.1.

We define a map $\mathscr{T}^{\mathscr{Q}} : L\text{-}FS(X,E) \to M$ as follows:

$$\mathscr{T}^{\mathscr{Q}}(f_A) = \begin{cases} \bigwedge\{\mathscr{Q}_{e^t_x}(f_A) : e^t_x q f_A\}, & \text{if } f_A \ncong \widetilde{0}, \\ 1, & \text{if } f_A \cong \widetilde{0}. \end{cases}$$

Then we have the following properties.

(1) $\mathscr{T}^{\mathscr{Q}}$ is an (L,M)-fuzzy soft topology on (X,E).

(2) If $\mathscr{Q} = \{\mathscr{Q}_{e^t_x} : e^t_x \in SP^e_t(X,E)\}$ is an (L,M)-fuzzy soft quasi-coincident neighborhood system on (X,E) then $\mathscr{Q}^{\mathscr{T}^{\mathscr{Q}}}_{e^t_x} = \mathscr{Q}_{e^t_x}$ for all $(e,x_t) \in SP^e_t(X,E)\}$.

(3) If $\mathscr{Q}_1$ and $\mathscr{Q}_2$ are (L,M)-fuzzy soft quasi- coincident neighborhood systems on (X,E) such that $\mathscr{T}^{\mathscr{Q}_1} = \mathscr{T}^{\mathscr{Q}_2}$ then $\mathscr{Q}_1 = \mathscr{Q}_2$.

Proof. (1) (LSO1) is trivial.

(LSO2) For all $f_A, g_B \in L\text{-}FS(X,E)$ we have

$$\begin{aligned}
\mathscr{T}^{\mathscr{Q}}(f_A \sqcap g_B) &= \bigwedge\{\mathscr{Q}_{e_x^t}(f_A \sqcap g_B) : e_x^t q(f_A \sqcap g_B)\} \\
&\geq \bigwedge\{\mathscr{Q}_{e_x^t}(f_A) \odot \mathscr{Q}_{e_x^t}(g_B) : e_x^t q(f_A \sqcap g_B)\} \\
&\geq \big(\bigwedge \mathscr{Q}_{e_x^t}(f_A) : e_x^t q(f_A \sqcap g_B)\big) \odot \big(\bigwedge \mathscr{Q}_{e_x^t}(g_B) : e_x^t q(f_A \sqcap g_B)\big) \\
&\geq \big(\bigwedge \mathscr{Q}_{e_x^t}(f_A) : e_x^t q f_A\big) \odot \big(\bigwedge \mathscr{Q}_{e_x^t}(g_B) : e_x^t q g_B\big) \\
&= \mathscr{T}^{\mathscr{Q}}(f_A) \odot \mathscr{T}^{\mathscr{Q}}(g_B).
\end{aligned}$$

(LSO3) Since $\mathscr{Q}_{e_x^t}(\bigvee_{i\in\Gamma} f_{A_i}) \geq \bigwedge_{i\in\Gamma} \mathscr{Q}_{e_x^t}(f_{A_i})$.

$$\begin{aligned}
\mathscr{T}^{\mathscr{Q}}(\textstyle\bigvee_{i\in\Gamma} f_{A_i}) &= \bigwedge\{\mathscr{Q}_{e_x^t}(\bigvee_{i\in\Gamma} f_{A_i}) : e_x^t q(\bigvee_{i\in\Gamma} f_{A_i})\} \\
&\geq \bigwedge\{\bigwedge_{i\in\Gamma} \mathscr{Q}_{e_x^t}(f_{A_i}) : e_x^t q(f_{A_i})\} \\
&\geq \bigwedge_{i\in\Gamma}\{\bigwedge \mathscr{Q}_{e_x^t}(f_{A_i}) : e_x^t q(f_{A_i})\} \\
&= \bigwedge_{i\in\Gamma} \mathscr{T}^{\mathscr{Q}}(f_{A_i}).
\end{aligned}$$

(2)

$$\begin{aligned}
\mathscr{Q}_{e_x^t}^{\mathscr{T}^{\mathscr{Q}}}(f_A) &= \bigvee\{\mathscr{T}^{\mathscr{Q}}(g_B) : e_x^t q g_B \sqsubseteq f_A\} \\
&= \bigvee\big\{\bigwedge\{\mathscr{Q}_{e_y^s}(g_B) : e_y^s q g_B\} : e_x^t q g_B \sqsubseteq f_A\big\} \\
&= \mathscr{Q}_{e_x^t}(f_A) \quad \text{by (LSN3)}.
\end{aligned}$$

(3) Since $\mathscr{T}^{\mathscr{Q}_1} = \mathscr{T}^{\mathscr{Q}_2}$ for $f_A \in L\text{-}FS(X,E)$ and $e_x^t \in SP_t^e(X,E)$ we have

$$\begin{aligned}(\mathscr{Q}_1)_{e_x^t}(f_A) &= \bigvee\left\{\bigwedge\{(\mathscr{Q}_1)_{e_y^s}(g_B) : e_y^s q g_B\} : e_x^t q g_B \sqsubseteq f_A\right\}\\ &= \bigvee\left\{\mathscr{T}^{\mathscr{Q}_1}(g_B) : e_x^t q g_B \sqsubseteq f_A\right\}\\ &= \bigvee\left\{\mathscr{T}^{\mathscr{Q}_2}(g_B) : e_x^t q g_B \sqsubseteq f_A\right\}\\ &= \bigvee\left\{\bigwedge\{(\mathscr{Q}_2)_{e_y^s}(g_B) : e_y^s q g_B\} : e_x^t q g_B \sqsubseteq f_A\right\}\\ &= (\mathscr{Q}_2)_{e_x^t}(f_A).\end{aligned}$$

Hence $\mathscr{Q}_1 = \mathscr{Q}_2$.

Lemma 3.5. If for every $e_x^t q f_A$ there exists $(g_B)_{e_x^t} \in L\text{-}FS(X,E)$ such that $e_x^t q (g_B)_{e_x^t} \sqsubseteq f_A$ then we have $f_A = \bigvee_{e_x^t q f_A} (g_B)_{e_x^t}$.

Theorem 3.6. Let $(X,E,\mathscr{T})$ be an (L,M)-fuzzy soft topological space and $\mathscr{Q}^{\mathscr{T}}$ an (L,M)-fuzzy soft quasi-coincident neighborhood system in $(X,E,\mathscr{T})$. Then $\mathscr{T} = \mathscr{T}^{\mathscr{Q}^{\mathscr{T}}}$.

Proof. Since $\mathscr{Q}^{\mathscr{T}}_{e_x^t}(f_A) = \bigvee\{\mathscr{T}(g_B) : e_x^t q g_B \sqsubseteq f_A\} \geq \mathscr{T}(f_A)$ for all $e_x^t q f_A$ we have:

$$\bigwedge\{\mathscr{Q}_{e_x^t}(f_A) : e_x^t q f_A\} \geq \mathscr{T}(f_A).$$

So $\mathscr{T}^{\mathscr{Q}^{\mathscr{T}}} \geq \mathscr{T}$.

Conversely there exists $f_A \in L\text{-}FS(X,E)$ such that $\mathscr{T}^{\mathscr{Q}^{\mathscr{T}}}(f_A) \not\leq \mathscr{T}(f_A)$.. For each $e_x^t \in SP_t^e(X,E)$ with $e_x^t q f_A$ if $(e,x_t) q (g_B)_{(e,x_t)} \sqsubseteq f_A$ then by Lemma 3.5. we get $f_A = \bigvee_{e_x^t q f_A} (g_B)_{e_x^t}$. So,

$$\mathscr{T}(f_A) = \mathscr{T}(\bigvee (g_B)_{e_x^t}) \geq \bigwedge \mathscr{T}((g_B)_{e_x^t}).$$

Thus $\bigwedge \mathscr{T}((g_B)_{e_x^t}) \not\geq \mathscr{T}^{\mathscr{Q}^{\mathscr{T}}}(f_A) = \bigwedge\{\mathscr{Q}^{\mathscr{T}}_{e_x^t}(f_A) : e_x^t q f_A\}$. There exists $(g_B)_{e_x^t}$ with $e_x^t q (g_B)_{e_x^t} \sqsubseteq f_A$ such that:

$$\mathscr{T}((g_B)_{e_x^t}) \not\geq \bigwedge\{\mathscr{Q}^{\mathscr{T}}_{e_x^t}(f_A) : e_x^t q f_A\}.$$

It is a contradiction. Thus $\mathscr{T} \geq \mathscr{T}^{\mathscr{Q}^{\mathscr{T}}}$.

Theorem 3.7. Let $(X,E,\mathscr{Q}_1)$ and $(Y,E^*,\mathscr{Q}_2)$ be two (L,M)-fuzzy soft quasi-coincident neighborhood spaces. A soft mapping $\phi : (X,E,\mathscr{Q}_1) \to (Y,E^*,\mathscr{Q}_2)$ is an *LSN*-map if and only if $\phi : (X,E,\mathscr{T}^{\mathscr{Q}_1}) \to (Y,E^*,\mathscr{T}^{\mathscr{Q}_2})$ is *LFS*-continuous.

Proof. Since for all $f_A \in L\text{-}FS(Y,E^*)$, for all $e_x^t \in SP_t^e(X,E)$, $e_x^t q\phi^{\leftarrow}(f_A)$ if and only if $(\phi^{\rightarrow}(e_x^t))qf_A$ and

$$\{(e^*)_y^t \in SP_t^{e^*}(Y,E^*) : (e^*)_y^t q f_A\}$$
$$\supset \{(\phi^{\rightarrow}(e_x^t)) \in SP_t^{e^*}(Y,E^*) : \quad e_x^t \in SP_t^e(X,E), (\phi^{\rightarrow}(e_x^t))qf_A,\}$$

we have:

$$\begin{aligned}
\mathscr{T}^{\mathscr{Q}_2}(f_A) &= \bigwedge\{(\mathscr{Q}_2)_{(e^*)_y^t}(f_A) : (e^*)_y^t q f_A\} \\
&\leq \bigwedge\{(\mathscr{Q}_2)_{\phi^{\rightarrow}(e_x^t)}(f_A) : \phi^{\rightarrow}(e_x^t) q f_A\} \\
&\leq \bigwedge\{(\mathscr{Q}_1)_{e_x^t}(\phi^{\leftarrow}(f_A)) : e_x^t q \phi^{\leftarrow}(f_A)\} \\
&= \mathscr{T}^{\mathscr{Q}_1}(\phi^{\leftarrow}(f_A)).
\end{aligned}$$

Thus, $\phi : (X,E,\mathscr{T}^{\mathscr{Q}_1}) \to (Y,E^*,\mathscr{T}^{\mathscr{Q}_2})$ is *LFS*-continuous.
Conversely since for all $f_A \in L\text{-}FS(Y,E^*), \mathscr{T}^{\mathscr{Q}_2}(f_A) \leq \mathscr{T}^{\mathscr{Q}_1}(\phi^{\leftarrow}(f_A)), \mathscr{Q}_1 = \mathscr{Q}^{\mathscr{T}^{\mathscr{Q}_1}}$ and $\mathscr{Q}_2 = \mathscr{Q}^{\mathscr{T}^{\mathscr{Q}_2}}$, we have

$$\begin{aligned}
(\mathscr{Q}_2)_{\phi^{\rightarrow}e_x^t}(f_A) &= \bigvee\{\mathscr{T}^{\mathscr{Q}_2}(g_B) : \phi^{\rightarrow}(e_x^t) q g_B \sqsubseteq f_A\} \\
&\leq \bigvee\{\mathscr{T}^{\mathscr{Q}_2}(g_B) : e_x^t q \phi^{\leftarrow}(g_B) \sqsubseteq \phi^{\leftarrow}(f_A)\} \\
&\leq \bigvee\{\mathscr{T}^{\mathscr{Q}_1}(\phi^{\leftarrow}(g_B)) : e_x^t q \phi^{\leftarrow}(g_B) \sqsubseteq \phi^{\leftarrow}(f_A)\} \\
&\leq (\mathscr{Q}_1)_{e_x^t}(\phi^{\leftarrow}(f_A)).
\end{aligned}$$

Hence the proof is complete.

From Theorems 3.6 and 3.7 we obtain the following corollary.

Corollary 3.8. Let $(X,E,\mathscr{T}_1)$ and $(Y,E^*,\mathscr{T}_2)$ be two (L,M)-fuzzy soft topological spaces. A soft map $\phi:(X,E,\mathscr{T}_1)\to(Y,E^*,\mathscr{T}_2)$ is LFS-continuous if and only if $\phi:(X,E,\mathscr{Q}^{\mathscr{T}_1})\to(Y,E^*,\mathscr{Q}^{\mathscr{T}_2})$ is an LSN-map.

Conflict of Interests

The authors declare that there is no conflict of interests.

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