Available online at http://scik.org
J. Math. Comput. Sci. 7 (2017), No. 2, 292-306
ISSN: 1927-5307

BAYESIAN ESTIMATION FOR A MULTI STEP-STRESS ACCELERATED GENERALIZED EXPONENTIAL MODEL WITH TYPE-II CENSORED DATA

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Abstract. We consider multiple step-stress accelerated life tests (SS-ALTs) assuming that the lifetime follows a generalized exponential distribution. Based on Type-II censored data, we calculate the maximum likelihood and Baysian estimates using MCMC. A Monte Carlo simulation study is carried out to examine the performance of the maximum likelihood and Baysin estimators through their mean squared error.

Keywords: multi-stress accelerated life tests; generalized exponential distribution; type-II censoring; maximum likelihood estimation; MCMC estimation; Monte Carlo simulation.

2010 AMS Subject Classification: 62N05.

1. Introduction

Many modern electro-mechanical materials and items tend to have a long life under normal-use operating conditions. Hence it is difficult to test their failure times since standard testing procedures are far too lengthy and expensive to be useful. However, accelerated life tests (ALTs)

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Received December 6, 2016; Published March 1, 2017

offer an alternative that manufacturing industries prefer due to the ability to obtain enough failure data in a short period of time.

In an ALT the test items are subjected to higher than usual levels of stress to induce early failures. The stress can be applied in different ways including the constant stress and step stress techniques. A step stress ALT is often preferred to a constant stress ALT because it reduces overall test time and enables quicker failures see [5-8]. We consider here m-step stress ALTs where n identical units are placed on a life-test with an initial stress level x_1 which is changed to x_2 at a fixed time τ_1 and the successive failure times are recorded. Then, at the fixed time τ_2, the stress is increased to x_3. Thus the resulting failure times are observed in a naturally ordered manner.

Cumulative exposure models are often useful in the analysis of step-stress experiments. These models relate the life distribution of the test units at one stress level to the distributions at preceding stress levels by assuming that the residual lives of the experimental units depend only on the cumulative exposure that the units have experienced, with no memory of how the stress was accumulated. Moreover, the surviving units will fail according to the cumulative distribution at the same stress level that is currently being tested at, but starting at the previous accumulated stress level. For more discussion see [1].

We develop a model for 3-step stress ALTs based on the lifetime distribution following a generalized exponential distribution see [2]. We then show how the observed ordered failure times can be used to do maximum likelihood estimation and Baysian estimation of the parameters of the distribution of failure times under normal operating conditions see [9,10]. Finally, we study the performance of these methods in a simulation study under Type-II censoring.

2. Model description

We assume that the lifetime T follows a two-parameter generalized exponential distribution, denoted GE(α,λ) where λ is a scale parameter and α is a shape parameter. The two-parameter $GE(\alpha,\lambda)$ distribution can be used quite effectively in analyzing lifetime data, particularly in place of two-parameter gamma or Weibull distributions see [3,4].

The GE(α,λ) probability density function (pdf) and cumulative distribution function (cdf) are given, respectively, by

$$f(t;\alpha,\lambda)=\alpha\lambda(1-e^{-\lambda t})^{\alpha-1}e^{-\lambda t}, \qquad \alpha,\lambda>0\,. \tag{2.1}$$

and

$$F(t;\alpha,\lambda)=(1-e^{-\lambda t})^{\alpha}. \tag{2.2}$$

The survival (sf) and hazard rate functions (hrf) are

$$\bar{F}(t;\alpha,\lambda)=1-(1-e^{-\lambda t})^{\alpha}, \tag{2.3}$$

and

$$h(t;\alpha,\lambda)=\frac{\alpha\lambda(1-e^{-\lambda t})^{\alpha-1}e^{-\lambda t}}{1-(1-e^{-\lambda t})^{\alpha}}. \tag{2.4}$$

respectively. For any λ the GE distribution has an increasing hrf if $\alpha>1$, while the hrf is decreasing if $\alpha<1$. Of course, if $\alpha=1$, then the hrf is constant.

2.1. Basic assumptions

We assume that the lifetime distribution functions at stress levels x_1, x_2 and x_3 are F_1, F_2 and F_3, respectively, and that they belong to the same family of distributions. The experiment starts with n identical units, and each unit is subjected to an initial stress x_1 with lifetimes following the CDF $F_1(t)$. The time at which a unit failed will be collected and the surviving units will continue until time τ_1 at which the stress is increased to x_2 and the units will follow the CDF $F_2(t)$, once again the time at which a unit failed will be collected and the surviving units will continue until time τ_2 at which the stress is increased to x_3 and the units will follow the CDF $F_3(t)$, but it will start at the previously accumulated fraction failed.

Thus the change in stress level from x_1 to x_2 changes the lifetime distribution at stress level x_2 from $F_2(t)$ to $F_2(t-\tau_1+\hat{\tau}_1)$, also the change in stress level from x_2 to x_3 changes the lifetime distribution at stress level x_3 from $F_3(t)$ to $F_3(t-\tau_2+\hat{\tau}_2)$ where

$$\begin{aligned} F_1(\tau_1)&=F_2(\hat{\tau}_1).\\ F_2(\tau_2)&=F_3(\hat{\tau}_2). \end{aligned} \tag{2.5}$$

Assuming that λ_1,λ_2 and λ_3 are the scale parameters associated with F_1, F_2 and F_3, respectively, and assuming absolute continuity of the cumulative distribution function of the lifetime, we find

$$\begin{aligned} \hat{\tau}_1 &= \frac{\lambda_1}{\lambda_2}\tau_1. \\ \hat{\tau}_2 &= \frac{\lambda_2}{\lambda_3}[\tau_2 - \tau_1 + \frac{\lambda_1}{\lambda_2}\tau_1]. \end{aligned} \tag{2.6}$$

Then, the cumulative distribution function of the model, in which there are three stress levels x_1, x_2 and x_3, will become

$$G(t) = \begin{cases} G_1(t) = F_1(t), & \text{for} \quad 0 < t < \tau_1 \\ G_2(t) = F_2(t - \tau_1 + \hat{\tau}_1) & \text{for} \quad \tau_1 \le t < \tau_2 \\ G_3(t) = F_3(t - \tau_2 + \hat{\tau}_2) & \text{for} \quad \tau_2 \le t < \infty \end{cases} \tag{2.7}$$

where

$$F_i(t) = (1 - e^{-\lambda_i t})^{\alpha}, i = 1,2,3.$$

The corresponding probability density function (PDF) in this case will be in the flowing form:

$$g(t) = \begin{cases} g_1(t) = \alpha\lambda_1(1 - e^{-\lambda_1 t})^{\alpha-1}e^{-\lambda_1 t}, & 0 < t < \tau_1 \\ g_2(t) = \alpha\lambda_2(1 - e^{-\lambda_2(t-\tau_1+\hat{\tau}_1)})^{\alpha-1}e^{-\lambda_2(t-\tau_1+\hat{\tau}_1)} & \tau_1 \le t < \tau_2, \\ g_3(t) = \alpha\lambda_3(1 - e^{-\lambda_3(t-\tau_2+\hat{\tau}_2)})^{\alpha-1}e^{-\lambda_3(t-\tau_2+\hat{\tau}_2)} & \tau_2 \le t < \infty. \end{cases} \tag{2.8}$$

3. Maximum likelihood estimation

In a 3-step-stress model with Type-II censoring, we start with n independent and identical units placed simultaneously on a life-test. Each unit will be subjected to an initial stress level x_1, then the experiment will run until a fixed time τ_1 when the stress level is changed to x_2, after that the experiment will run until a fixed time τ_2 when the stress level is changed to x_3. The experiment is continued until a specified number of failures r is observed.

Let n_1 be the number of units that fail before τ_1, n_2 be the number of units that fail between τ_1 and τ_2 and n_3 is the number of units that fail after τ_2, and so $r = n_1 + n_2 + n_3$. If r failures occur before τ_1 or τ_2, then the test is terminated, otherwise the experiment continues after time τ_2 until the required r failures are observed. The ordered failure times that are observed will be denoted by $(t_1 < ... < t_{n_1} < \tau \le t_{n_1+1} ... < t_{n_2} < \tau \le t_{n_2+1} < ... < t_r)$.

The likelihood function based on the censored data above is given by

$$L(\alpha,\lambda_1,\lambda_2,\lambda_3;\mathbf{t})=\frac{n!}{r!}\{\prod_{i=1}^{r}g(t_i)(1-G(t_r))^{n-r}\},\tag{3.1}$$

where $r=n_1+n_2+n_3$ and $\mathbf{t}$ is the vector of observed Type-II censored data. The likelihood function of α , λ_1,λ_2 and λ_3 is as follows:

(1) If $n_1=0$:

$$\begin{aligned}L(\alpha,\lambda_2,\lambda_3;\mathbf{t})&=\frac{n!}{r!}\{\prod_{i=1}^{n_2}g_2(y_i)\}\{\prod_{i=n_2+1}^{r}g_3(z_i)\}(1-G_3(z_r))^{n-r}\\&=\frac{n!}{r!}\alpha^r\lambda_2^{n_2}\lambda_3^{n_3}e^{-\lambda_2\sum_{i=1}^{n_2}y_i-\lambda_3\sum_{i=n_2+1}^{r}z_i}\\&\quad\times\{\prod_{i=1}^{n_2}(1-e^{-\lambda_2 y_i})^{\alpha-1}\}\{\prod_{i=n_2+1}^{r}(1-e^{-\lambda_3 z_i})^{\alpha-1}\}\\&\quad\times(1-(1-e^{-\lambda_3 z_r})^{\alpha})^{n-r}\end{aligned}\tag{3.2}$$

(2) If $n_2=0$:

$$\begin{aligned}L(\alpha,\lambda_1,\lambda_3;\mathbf{t})&=\frac{n!}{r!}\{\prod_{i=1}^{n_1}g_1(t_i)\}\{\prod_{i=n_1+1}^{r}g_3(z_i)\}(1-G_3(z_r))^{n-r}\\&=\frac{n!}{r!}\alpha^r\lambda_1^{n_1}\lambda_3^{n_3}e^{-\lambda_1\sum_{i=1}^{n_1}t_i-\lambda_3\sum_{i=n_1+1}^{r}z_i}\\&\quad\times\{\prod_{i=1}^{n_1}(1-e^{-\lambda_1 t_i})^{\alpha-1}\}\{\prod_{i=n_1+1}^{r}(1-e^{-\lambda_3 z_i})^{\alpha-1}\}\\&\quad\times(1-(1-e^{-\lambda_3 z_r})^{\alpha})^{n-r}\end{aligned}\tag{3.3}$$

(3) If $n_3=0$:

$$\begin{aligned}L(\alpha,\lambda_1,\lambda_2;\mathbf{t})&=\frac{n!}{r!}\{\prod_{i=1}^{n_1}g_1(t_i)\}\{\prod_{i=n_1+1}^{n_1+n_2}g_2(y_i)\}\\&=\frac{n!}{r!}\alpha^r\lambda_1^{n_1}\lambda_2^{n_2}e^{-\lambda_1\sum_{i=1}^{n_1}t_i-\lambda_2\sum_{i=n_1+1}^{n_1+n_2}y_i}\\&\quad\times\{\prod_{i=1}^{n_1}(1-e^{-\lambda_1 t_i})^{\alpha-1}\}\{\prod_{i=n_1+1}^{n_1+n_2}(1-e^{-\lambda_2 y_i})^{\alpha-1}\}\end{aligned}\tag{3.4}$$

(4) If $n_i > 0$,i=1,2,3:

$$
\begin{aligned}
L(\alpha,\lambda_1,\lambda_2,\lambda_3;\mathbf{t}) &= \frac{n!}{r!}\{\prod_{i=1}^{n_1} g_1(t_i)\}\{\prod_{i=n_1+1}^{n_1+n_2} g_2(y_i)\}\{\prod_{i=n_1+n_2+1}^{r} g_3(z_i)\}(1-G_3(z_r))^{n-r} \\
&= \frac{n!}{r!}\alpha^r\lambda_1^{n_1}\lambda_2^{n_2}\lambda_3^{n_3}e^{-\lambda_1\sum_{i=1}^{n_1}t_i-\lambda_2\sum_{i=n_1+1}^{n_1+n_2}y_i-\lambda_3\sum_{i=n_1+n_2+1}^{r}z_i} \\
&\quad\times\{\prod_{i=1}^{n_1}(1-e^{-\lambda_1 t_i})^{\alpha-1}\}\{\prod_{i=n_1+1}^{n_1+n_2}(1-e^{-\lambda_2 y_i})^{\alpha-1}\} \\
&\quad\times\{\prod_{i=n_1+n_2+1}^{r}(1-e^{-\lambda_3 z_i})^{\alpha-1}\}(1-(1-e^{-\lambda_3 z_r})^{\alpha})^{n-r}
\end{aligned}
\tag{3.5}
$$

where $y_i = t_i - \tau_1 + \hat{\tau}_1$, $z_i = t_i - \tau_2 + \hat{\tau}_2$.

As we can see from (3.2)-(3.5) the three MLEs does not exist unless when $n_1, n_2, n_3 > 0$ and may be obtained by maximizing the corresponding likelihood function (3.5).

Maximizing the likelihood function for the parameters cannot be achieved analytically. The only option we have is to numerically maximize the likelihood function for the vector of parameters $(\alpha,\lambda_1,\lambda_2,\lambda_3)$. For this purpose, it is convenient to work with the log-likelihood function rather than the likelihood function in (3.5), which is given by

$$
\begin{aligned}
\ell(\alpha,\lambda_1,\lambda_2;\mathbf{t}) &= \log c + r\log\alpha + n_1\log\lambda_1 + n_2\log\lambda_2 + n_3\log\lambda_3 - \lambda_1\sum_{i=1}^{n_1}t_i \\
&\quad -\lambda_2\sum_{i=n_1+1}^{n_1+n_2}y_i - \lambda_3\sum_{i=n_1+n_2+1}^{r}z_i + (\alpha-1)\sum_{i=1}^{n_1}(1-e^{-\lambda_1 t_i}) \\
&\quad +(\alpha-1)\sum_{i=n_1+1}^{n_1+n_2}(1-e^{-\lambda_2 y_i}) + (\alpha-1)\sum_{i=n_1+n_2+1}^{r}(1-e^{-\lambda_3 z_i}) \\
&\quad +(n-r)\log(1-(1-e^{-\lambda_3 z_r})^{\alpha}).
\end{aligned}
\tag{3.6}
$$

The likelihood equations for the parameters α, λ_1,λ_2 and λ_3 are given, respectively, by

$$
\begin{aligned}
\frac{\partial\ell}{\partial\alpha} &= \frac{r}{\alpha} + \sum_{i=1}^{n_1}(1-e^{-\lambda_1 t_i}) + \sum_{i=n_1+1}^{n_1+n_2}(1-e^{-\lambda_2 y_i}) + \sum_{i=n_1+n_2+1}^{r}(1-e^{-\lambda_3 z_i}) \\
&\quad -(n-r)\frac{(1-e^{-\lambda_3 z_r})^{\alpha}\log(1-e^{-\lambda_3 z_r})}{1-(1-e^{-\lambda_3 z_r})^{\alpha}},
\end{aligned}
\tag{3.7}
$$

$$
\frac{\partial\ell}{\partial\lambda_1} = \frac{n_1}{\lambda_1} + \sum_{i=1}^{n_1}\{-t_i + \frac{(\alpha-1)t_i e^{-\lambda_1 t_i}}{1-e^{-\lambda_1 t_i}}\},
\tag{3.8}
$$

$$\frac{\partial \ell}{\partial \lambda_2} = \frac{n_2}{\lambda_2} + \sum_{i=n_1+1}^{n_1+n_2} \{-y_i + \frac{(\alpha-1)y_i e^{-\lambda_2 y_i}}{1-e^{-\lambda_2 y_i}}\}, \tag{3.9}$$

$$\frac{\partial \ell}{\partial \lambda_3} = \frac{n_3}{\lambda_3} + \sum_{i=n_1+n_2+1}^{r} \{-z_i + \frac{(\alpha-1)z_i e^{-\lambda_3 z_i}}{1-e^{-\lambda_3 z_i}}\} - (n-r)\frac{\alpha z_r e^{-\lambda_3 z_r}(1-e^{-\lambda_3 z_r})^{\alpha-1}}{1-(1-e^{-\lambda_3 z_r})^{\alpha}}. \tag{3.10}$$

The maximum likelihood estimates must be derived numerically because there is no obvious solution of these four non-linear likelihood equations. We used the R software to carry out a numerical maximization on the log likelihood function and obtain the MLEs using the following algorithm:

(1) Simulate n order statistics from the uniform (0,1) distribution, $(U_1, U_2, ..., U_n)$.
(2) Find n_1 such that $U_{n_1} \leq G_1(\tau_1) \leq U_{n_1+1}$.
(3) For $i \leq n_1$ $T_i = -\frac{1}{\lambda_1}\ln(1-U_i^{\frac{1}{\alpha}})$.
(4) Find n_2 such that $U_{n_1+n_2} \leq G_2(\tau_2) \leq U_{n_1+n_2+1}$.
(5) For $i \leq n_1+n_2$ $T_i = -\frac{1}{\lambda_2}\ln(1-U_i^{\frac{1}{\alpha}}) + \tau_1 - \hat{\tau}_1$.
(6) For $n_1+n_2+1 \leq i \leq r$ set $T_i = -\frac{1}{\lambda_3}\ln(1-U_i^{\frac{1}{\alpha}}) + \tau_2 - \hat{\tau}_2$.
(7) Obtain the MLEs of $(\alpha, \lambda_1, \lambda_2, \lambda_3)$ based on $(T_1, T_2, ..., T_{n_1}, T_{n_1+1}, ..., T_{n_1+n_2}, T_{n_1+n_2+1}, ..., T_r)$ say $\hat{\alpha}, \hat{\lambda}_1, \hat{\lambda}_2$ and $\hat{\lambda}_3$.
(8) Repeat steps 2-7 1000 times.
(9) Compute the MSE of the obtained estimates.

4. Baysian Estimation

There is a fundamental difference between classical and Bayesian estimation. In classical estimation we consider the unknown parameter as a fixed value. But in Bayesian estimation we treat the parameter as a random variable. It is assumed that the parameters $\alpha, \lambda_1, \lambda_2$ and λ_3 are all independent and have the following prior distributions see [2-4]:

$$\begin{aligned}
\pi_1(\alpha) &= \frac{\mu_1^{\nu_1}}{\Gamma(\nu_1)}\alpha^{\nu_1-1}e^{-\mu_1\alpha}, \qquad \mu_1,\nu_1>0,\\
\pi_2(\lambda_1) &= \frac{\mu_2^{\nu_2}}{\Gamma(\nu_2)}\lambda_1^{\nu_2-1}e^{-\mu_2\lambda_1}, \qquad \mu_2,\nu_2>0,\\
\pi_3(\lambda_2) &= \frac{\mu_3^{\nu_3}}{\Gamma(\nu_3)}\lambda_2^{\nu_3-1}e^{-\mu_3\lambda_2}, \qquad \mu_3,\nu_3>0,\\
\pi_4(\lambda_3) &= \frac{\mu_4^{\nu_4}}{\Gamma(\nu_4)}\lambda_3^{\nu_4-1}e^{-\mu_4\lambda_3}, \qquad \mu_4,\nu_4>0.
\end{aligned} \tag{4.1}$$

Then the joint prior density function is

$$\Pi(\alpha,\lambda_1,\lambda_2,\lambda_3) = \frac{\mu_1^{\nu_1}\mu_2^{\nu_2}\mu_3^{\nu_3}\mu_4^{\nu_4}}{\Gamma(\nu_1)\Gamma(\nu_2)\Gamma(\nu_3)\Gamma(\nu_4)}\alpha^{\nu_1-1}\lambda_1^{\nu_2-1}\lambda_2^{\nu_3-1}\lambda_3^{\nu_4-1}e^{-\mu_1\alpha-\mu_2\lambda_1-\mu_3\lambda_2-\mu_4\lambda_3} \tag{4.2}$$

And hence the posterior function will be as the following

$$\begin{aligned}
F(\alpha,\lambda_1,\lambda_2,\lambda_3;\mathbf{t}) &\propto \Pi(\alpha,\lambda_1,\lambda_2,\lambda_3)L(\alpha,\lambda_1,\lambda_2,\lambda_3;\mathbf{t})\\
&\propto \frac{\mu_1^{\nu_1}\mu_2^{\nu_2}\mu_3^{\nu_3}\mu_4^{\nu_4}}{\Gamma(\nu_1)\Gamma(\nu_2)\Gamma(\nu_3)\Gamma(\nu_4)}\alpha^{\nu_1-1}\lambda_1^{\nu_2-1}\lambda_2^{\nu_3-1}\lambda_3^{\nu_4-1}e^{-\mu_1\alpha-\mu_2\lambda_1-\mu_3\lambda_2-\mu_4\lambda_3}\\
&\quad\times\frac{n!}{r!}\alpha^r\lambda_1^{n_1}\lambda_2^{n_2}\lambda_3^{n_3}e^{-\lambda_1\sum_{i=1}^{n_1}t_i-\lambda_2\sum_{i=n_1+1}^{n_1+n_2}y_i-\lambda_3\sum_{i=n_1+n_2+1}^{r}z_i}\\
&\quad\times\{\prod_{i=1}^{n_1}(1-e^{-\lambda_1 t_i})^{\alpha-1}\}\{\prod_{i=n_1+1}^{n_1+n_2}(1-e^{-\lambda_2 y_i})^{\alpha-1}\}\\
&\quad\times\{\prod_{i=n_1+n_2+1}^{r}(1-e^{-\lambda_3 z_i})^{\alpha-1}\}(1-(1-e^{-\lambda_3 z_r})^{\alpha})^{n-r}
\end{aligned} \tag{4.3}$$

It is obvious from the posterior function that we are not going to be able to estimate the parameters by the traditional Bayesian methods with integration, so we are going to use one of the MCMC methods which attempt to simulate direct draws from some complex distribution of interest. MCMC approaches are so-named because one uses the previous sample values to randomly generate the next sample value. Here we are going to use the Metropolis algorithm.

Suppose you want to obtain M samples from a univariate distribution with probability density function $f(\theta,t)$. Suppose θ_i is the $i-th$ sample from f. To use the Metropolis algorithm, you need to have an initial value θ^o and a symmetric proposal density $q(\theta^{i+1}|\theta^i)$. For the $(i+1)-th$ iteration, the algorithm generates a sample from $q(.|.)$ based on the current sample i, and it makes a decision to either accept or reject the new sample. If the new sample is accepted, the algorithm repeats itself by starting at the new sample. If the new sample is rejected, the algorithm starts at the current point and repeats. The algorithm is self-repeating, so it can be carried out as long as required.The most common choice of the proposal distribution is the normal distribution $N(\theta^i, \sigma)$ with a fixed σ. The Metropolis algorithm can be summarized as follows:

(1) Set $i = 0$. Choose a starting point θ^o. This can be an arbitrary point as long as $f(\theta^o, t) > 0$.
(2) Generate a new sample, θ_{new}, by using the proposal distribution $q(.|\theta^i)$.
(3) Calculate the following quantity $w = min[\frac{f(\theta_{new}|t)}{f(\theta^i|t)}, 1]$.
(4) Sample u from the uniform distribution $U(0,1)$.
(5) Set $\theta^{i+1} = \theta_{new}$ if $u < w$; otherwise set $\theta^{i+1} = \theta^i$.
(6) Set $i = i+1$. If $i < M$, the number of desired samples, return to step 2. Otherwise, stop.

The number of iterations used to calculate the MCMC estimates is 50000.
The performance of the MLEs and the Bayes estimates are evaluated using a simulation study in the next section.

5. Simulation study

A simulation study was carried out for different values of τ_1 and τ_2 in order to examine MSE of the ML and the Bayes estimates . For each setting we simulated 1000 data sets to fit the model and estimated the desired quantities. The results are presented in Tables 1 - 4.

Table 1: Conditional Failure Probabilities (in %) for a multi-stress model under Type-II censoring when $\alpha = 3, \lambda_1 = 1.3, \lambda_2 = 0.65, \lambda_3 = 2$ and $n = 25$.

r	τ_1	τ_2	Conditional Failure Probabilities (in%)		
			$0 < t < \tau_1$	$\tau_1 < t < \tau_2$	$\tau_2 < t < \infty$
20	0.5	1	5	5	90
		1.5	5	35	60
		2	5	85	20
	0.8	1.5	10	30	60
		2	10	55	35
		2.5	10	75	15
	1.2	2	40	15	45
		3	40	45	15
17	0.5	1	5.88	5.88	88.24
		1.5	5.88	41.18	52.94
		2	5.88	88.24	5.88
	0.8	1.5	11.77	35.29	52.95
		2	11.77	64.71	23.53
		2.5	11.77	88.24	11.77

Table 2: The MSE of the MCMC and ML estimates $\hat{\alpha}, \hat{\lambda}_1, \hat{\lambda}_2$ and $\hat{\lambda}_3$ based on 1000 simulations when $\alpha = 3, \lambda_1 = 1.3, \lambda_2 = 0.65, \lambda_3 = 2$ and $n = 25$.

			$MSE(\hat{\alpha})$		$MSE(\hat{\lambda}_1)$		$MSE(\hat{\lambda}_2)$		$MSE(\hat{\lambda}_3)$	
r	τ_1	τ_2	ML	$MCMC$	ML	$MCMC$	ML	$MCMC$	ML	$MCMC$
20	0.5	1	5.2353	1.0966	0.1580	0.3560	0.1254	0.7093	2.2618	0.3586
		1.5	5.2353	1.0966	1.0331	0.3560	0.0273	0.7093	2.2729	0.3586
		2	1.7221	1.0966	0.2235	0.3560	0.0389	0.7093	1.1282	0.3586
	0.8	1.5	5.1046	1.0966	0.7247	0.3560	0.0190	0.7093	2.3978	0.3586
		2	4.5516	1.0966	0.9467	0.3560	0.1778	0.7093	2.8297	0.3586
		2.5	16.359	1.0966	5.4831	0.3560	3.7837	0.7093	0.1366	0.3586
	1.2	2	4.9804	1.0966	0.0731	0.3560	0.2397	0.7093	2.7448	0.3586
		3	33.995	1.0966	2.4813	0.3560	2.1534	0.7093	0.1701	0.3586
17	0.5	1	5.2750	1.0966	1.7909	0.3560	0.1140	0.7093	2.1031	0.3586
		1.5	5.9015	1.0966	0.5811	0.3560	0.0054	0.7093	1.0971	0.3586
		2	1.9988	1.0966	104.40	0.3560	53.794	0.7093	79.939	0.3586
	0.8	1.5	4.5483	1.0966	0.7570	0.3560	0.0016	0.7093	1.1262	0.3586
		2	1.2824	1.0966	1.5518	0.3560	1.6582	0.7093	0.6459	0.3586
		2.5	4.3609	1.0966	6.4301	0.3560	3.1809	0.7093	0.0490	0.3586

Table 3: Conditional Failure Probabilities (in %) for a multi-stress model under Type-II censoring when $\alpha = 3, \lambda_1 = 1.3, \lambda_2 = 0.65, \lambda_3 = 2$ and $n = 150$.

r	τ_1	τ_2	Conditional Failure Probabilities (in%)		
			$0 < t < \tau_1$	$\tau_1 < t < \tau_2$	$\tau_2 < t < \infty$
120	0.5	1	6.6667	8.3333	85
		1.5	6.6667	25	68.3333
		2	6.6667	44.16667	49.16667
		2.5	6.6667	63.3333	30
	0.8	1.5	15	10.83333	74.16667
		2	15	30	55
		2.5	15	50.83333	34.16667
		3	15	65.83333	19.16667
100	0.5	1	8	10	82
		1.5	8	30	62
		2	8	53	39
	0.8	1.5	18	13	69
		2	18	36	46
		2.5	18	61	21

Table 4: The MSE of the MCMC and ML estimates $\hat{\alpha}, \hat{\lambda}_1, \hat{\lambda}_2$ and $\hat{\lambda}_3$ based on 1000 simulations when $\alpha = 3, \lambda_1 = 1.3, \lambda_2 = 0.65, \lambda_3 = 2$ and $n = 150$.

			$MSE(\hat{\alpha})$		$MSE(\hat{\lambda}_1)$		$MSE(\hat{\lambda}_2)$		$MSE(\hat{\lambda}_3)$	
r	τ_1	τ_2	*ML*	*MCMC*	*ML*	*MCMC*	*ML*	*MCMC*	*ML*	*MCMC*
120	0.5	1	4.0500	0.7869	0.7090	0.2501	0.1040	0.2962	2.5623	0.4036
		1.5	4.1847	0.7869	0.6675	0.2501	0.0713	0.2962	2.2004	0.4036
		2	4.2574	0.7869	0.3999	0.2501	0.0098	0.2962	1.2196	0.4036
		2.5	2.3898	0.7869	1.4883	0.2501	0.0564	0.2962	0.3534	0.4036
	0.8	1.5	4.3684	0.7869	0.0014	0.2501	0.2031	0.2962	2.5434	0.4036
		2	4.3046	0.7869	0.0190	0.2501	0.0873	0.2962	1.9579	0.4036
		2.5	2.2203	0.7869	0.3369	0.2501	0.0244	0.2962	0.5566	0.4036
		3	0.4339	0.7869	3.4879	0.2501	1.1105	0.2962	0.3788	0.4036
100	0.5	1	4.0048	0.7869	0.9461	0.2501	0.0656	0.2962	2.6447	0.4036
		1.5	4.0933	0.7869	0.7069	0.2501	0.0529	0.2962	2.1232	0.4036
		2	4.0685	0.7869	0.0152	0.2501	0.0161	0.2962	1.3705	0.4036
	0.8	1.5	4.7227	0.7869	0.1324	0.2501	0.2291	0.2962	2.6742	0.4036
		2	3.7934	0.7869	0.0039	0.2501	0.0483	0.2962	1.6252	0.4036
		2.5	1.1006	0.7869	2.8560	0.2501	1.3784	0.2962	0.4753	0.4036

6. Concluding Remarks

We have considered multiple step-stress accelerated model when the observed failure times come from a $GE(\alpha,\lambda)$ distribution under type-II censoring. A simulation study, based on two different examples, was performed to examine the performance of the mean square error of the maximum likelihood and Bayesian estimates.

In Tables 1 and 3, we can see that for a fixed τ_1 and increasing τ_2 the conditional failure probabilities occurring on the first level of stress in the interval $0 < t < \tau_1$ is the same. on the meanwhile those occurring on the second and third levels of stress change. As τ_2 increases the conditional failure probabilities in the interval $\tau_1 < t < \tau_2$ increase ,but decrease in $\tau_2 < t < \infty$. This means that as τ_2 increases, there will be more failures occurring before τ_2 and less failures occurring after it, which means more information about λ_2 and less information about λ_3. We also can see that as τ_1 increases the conditional failure probabilities occurring on the first level of stress in the interval $0 < t < \tau_1$ also increase.

In Tables 2 and 4, we can see that the MSEs of the Bayesian estimates of the parameters α,λ_1,λ_2 and λ_3 doesn't change for different values of r,$\tau_1$or τ_2, they only change for different sample sizes n. On the other hand any change in those values affects the MSEs of the maximum likelihood estimates. In general we can say that MCMC is abetter method to estimate our model parameters in either small and larg sample sizes.

Conflict of Interests

The authors declare that there is no conflict of interests.

Acknowledgements

The first author wishes to thank the Egyptian Ministry of Higher Education and Scientific Research for supporting her visiting scholar to the University of Minnesota and Galin Jones for helpful conversations about this paper.

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