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GROUP $\{1,-1,i,-i\}$ CORDIAL LABELING OF SUM OF P_n AND K_n

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Abstract. Let G be a (p,q) graph and A be a group. For $a \in A$, we denote the order of a by $o(a)$. Let $f : V(G) \to A$ be a function. For each edge uv assign the label 1 if $(o(u), o(v)) = 1$or 0 otherwise. f is called a group A Cordial labeling if $|v_f(a) - v_f(b)| \leq 1$, $\forall a, b \in A$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(x)$ and $e_f(n)$ respectively denote the number of vertices labelled with an element x and number of edges labelled with $n (n = 0, 1)$. A graph which admits a group A Cordial labeling is called a group A Cordial graph. In this paper we define group $\{1,-1,i,-i\}$ Cordial graphs and prove that $P_n + K_2$ is group $\{1,-1,i,-i\}$ Cordial for every n. We further characterize $P_n + K_3, P_n + K_4$ and $P_n + K_n (n \leq 30)$ that are group $\{1,-1,i,-i\}$ Cordial.

Keywords: cordial labeling; group A cordial labeling; group $\{1,-1,i,-i\}$ cordial labeling.

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1. Introduction

In the mathematical discipline of graph theory, a graph labeling is the assignment of labels, traditionally represented by integers, to the edges or vertices, or both, of a graph. Interest in

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graph labeling problems became prominent in the mid 1960's from a long standing conjecture of Ringel and a paper by Rosa. Most graph labelings trace their origins to labelings presented by Alex Rosa in his 1967 paper. Rosa called a function f a $\beta-$ valuation of a graph G with q edges if f is an injection from the vertices of G to the set $\{0,1,...,q\}$ such that, when each edge uv is assigned the label $|f(u)-f(v)|$, the resulting edge labels are distinct. In 1980, Golomb called such labelings graceful and this is now the popular term. Ringel conjectured more than four decades ago that " All trees are graceful" and this conjecture has been the focus of many papers related to labeling problems.

Labelled graphs have wide applications in coding theory, X-ray crystallography, radar, astronomy, circuit design and communication network addressing.

2. Preliminaries

Graphs considered here are finite, undirected and simple. Let A be a group. The order of $a \in A$ is the least positive integer n such that $a^n = e$. We denote the order of a by $o(a)$. Cahit [3] introduced the concept of Cordial labeling. Motivated by this, we defined group A cordial labeling and investigated some of its properties. We also defined group $\{1,-1,i,-i\}$ cordial labeling and discussed that labeling for some standard graphs [1] .In this paper we prove that $P_n + K_2$ is group $\{1,-1,i,-i\}$ Cordial for every n. We further characterize $P_n + K_3, P_n + K_4$ and $P_n + K_n (n \leq 30)$ that are group $\{1,-1,i,-i\}$ Cordial. Terms not defined here are used in the sense of Harary[5] and Gallian [4].

The greatest common divisor of two integers m and n is denoted by (m,n) and m and n are said to be *relatively prime* if $(m,n) = 1$. For any real number x, we denote by $\lfloor x \rfloor$, the greatest integer smaller than or equal to x and by $\lceil x \rceil$, we mean the smallest integer greater than or equal to x.

A *path* is an alternating sequence of vertices and edges, $v_1, e_1, v_2, e_2, ..., e_{n-1}, v_n$, which are distinct, such that e_i is an edge joining v_i and v_{i+1} for $1 \leq i \leq n-1$. A path on n vertices is denoted by P_n. If G is a graph on n vertices in which every vertex is adjacent to every other vertex, then G is called a complete graph and is denoted by K_n.

Given two graphs G and H, $G+H$ is the graph with vertex set $V(G) \cup V(H)$ and edge set $E(G) \cup E(H) \cup \{uv/u \in V(G), v \in V(H)\}$.

We use the following theorem:

Theorem 1.1[2] The Fan F_n is group $\{1,-1,i,-i\}$ Cordial for all $n \leq 10$ and for $n > 10$, F_n is group $\{1,-1,i,-i\}$ Cordial iff $n \equiv 0,1,2\ (mod\ 4)$.

3. Main results

Let G be a (p,q)graph and consider the group $A = \{1,-1,i,-i\}$ with multiplication. Let $f : V(G) \to A$ be a function. For each edge uv assign the label 1 if $(o(u),o(v)) = 1$or 0 otherwise. f is called a group $\{1,-1,i,-i\}$ Cordial labeling if $|v_f(a) - v_f(b)| \leq 1$, $\forall a,b \in A$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(x)$ and $e_f(n)$ respectively denote the number of vertices labelled with an element x and number of edges labelled with $n(n = 0,1)$. A graph which admits a group $\{1,-1,i,-i\}$ Cordial labeling is called a group $\{1,-1,i,-i\}$ Cordial graph.

A simple example of a group $\{1,-1,i,-i\}$ Cordial graph is given in Fig. 3.1.

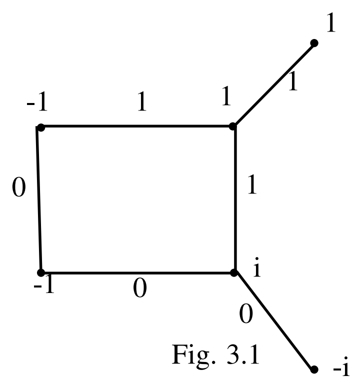

Fig. 3.1

We now investigate the group $\{1,-1,i,-i\}$ Cordial labeling of $P_n + K_m$ for $1 \leq m \leq 4$.
$P_n + K_1$ is the Fan F_n and theorem 1.1 characterizes the Fans that are group $\{1,-1,i,-i\}$ cordial.

Theorem 3.1. *$P_n + K_2$ is group $\{1,-1,i,-i\}$ cordial for every n.*

Proof. Let the vertices of P_n be $u_1, u_2, ..., u_n$ and let the vertices of K_2 be v_1, v_2 . Number of vertices of $P_n + K_2$ is n+2 and number of edges is 3n.

Case(1): $n + 2 \equiv 0(mod\ 4)$.

Let $r = \frac{n-2}{4}$. Label the vertices $v_1, u_1, u_2, ..., u_r$ with 1. Label the remaining vertices arbitrarily so that $\frac{n+2}{4}$ vertices get label -1 , $\frac{n+2}{4}$ vertices get label i and $\frac{n+2}{4}$ vertices get label $-i$. Number of edges with label 1 = $n + 2r + 1 = n + 2\left(\frac{n-2}{4}\right) + 1 = \frac{3n}{2}$. So this is a group $\{1,-1,i,-i\}$ cordial labeling.

Case(2): $n+2 \equiv 1(mod\ 4)$.

Let $r = \frac{n-3}{4}$. Label the vertices $v_1, u_1, u_2, ..., u_r$ with 1. Label the remaining vertices arbitrarily so that r+1 vertices get label -1, r+1 vertices get label i and r+2 vertices get label -i. Number of edges with label $1 = n+1+2r = \frac{3n-1}{2}$. Also number of edges with label $0 = \frac{3n+1}{2}$.

Case(3): $n+2 \equiv 2(mod\ 4)$.

If n = 4, a group $\{1,-1,i,-i\}$ cordial labeling is shown in Fig 3.2.

Suppose $n \geq 8$. Let $r = \frac{n}{4}$. Label the vertices $v_1, u_2, u_3, ..., u_r$ with 1. Label the remaining vertices arbitrarily so that $\frac{n}{4}$ vertices get label -1, $\frac{n}{4}+1$ vertices get label i and $\frac{n}{4}+1$ vertices get label -1. Number of edges with label $1 = n+1+3+2(r-2) = n+4+2(\frac{n}{4}-2) = \frac{3n}{2}$.

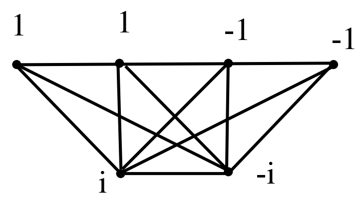

Fig. 3.2

Case (4): $n+2 \equiv 3(mod\ 4)$.

Let $r = \frac{n-1}{4}$. Label the vertices $v_1, u_1, u_2, ..., u_r$ with label 1. Label the remaining vertices arbitrarily so that $r+1$ vertices get label -1, $r+1$ vertices get label i and r vertices get label $-i$. Number of edges with label $1 = n+1+2r = n+1+2(\frac{n-1}{4}) = \frac{3n+1}{2}$. Also , number of edges with label $0 = \frac{3n-1}{2}$.

Illustration of the labeling for $n = 6$ is given in Fig.3.3.

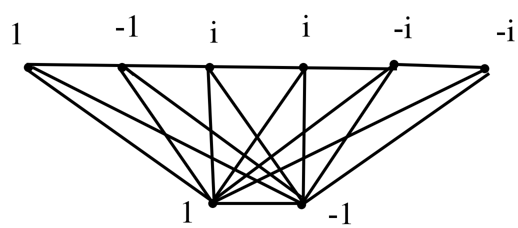

Fig 3.3

Theorem 3.2. *$P_n + K_3$ is group $\{1,-1,i,-i\}$ Cordial iff n $\neq 2,3,6$.*

Proof. Let the vertices of P_n be $u_1,u_2,...,u_n$ and let the vertices of K_3 be v_1,v_2,v_3 . Number of vertices of P_n+K_3 is $n+3$ and number of edges is $4n+2$.

If n = 2, P_2+K_3 has 5 vertices and 10 edges. If 1 vertex is labelled with 1, then 4 edges get label 1 and if 2 vertices are labelled with 1 then 7 edges get label 1. But only 5 edges have to get label 1. So P_2+K_3 is not group $\{1,-1,i,-i\}$ Cordial.

If n=3 ,P_3+K_3 has 6 vertices and 14 edges. If 1 vertex is labelled with 1 , at most 5 edges get label 1 and if 2 vertices are labelled with 1, either 8 or 9 vertices get label 1. So P_3+K_3 is not group $\{1,-1,i,-i\}$ Cordial.

If n=6 ,P_6+K_3 has 9 vertices and 26 edges. It is easy to observe that there is no choice of 2 or 3 vertices so that 13 edges get label 1. Hence P_6+K_3 is not group $\{1,-1,i,-i\}$ Cordial. Thus $n\neq 2,3,6$.

Conversely , assume $n\neq 2,3,6$.

We need to prove that P_n+K_3 is group $\{1,-1,i,-i\}$ Cordial.

Case(1): $n+3\equiv 0(mod\ 4)$.

Each vertex label occurs $\frac{n+3}{4}$ times and each edge label occurs $2n+1$ times. Let $r=\frac{n-1}{4}$. Label $v_1,u_2,u_4,....,u_{2r}$ with 1. Label the remaining vertices arbitrarily so that $r+1$ of them get label -1, $r+1$ of them get label i and $r+1$ of them get label -i.

Case(2): $n+3\equiv 1(mod\ 4)$.

By assumption , $n\geq 10$. In this case , one vertex label occurs $\lceil\frac{n+3}{4}\rceil$ times and each of the other 3 vertex labels occur $\lfloor\frac{n+3}{4}\rfloor$ times.

Let $r=\frac{n-10}{4}$. Label $v_1,u_1,u_2,u_3,u_5,u_7,...,u_{2r+3}$ with 1. Label the remaining vertices arbitrarily so that $r+3$ of them get label -1, $r+3$ of them get label i and $r+3$ of them get label -i.

Case(3): $n+3\equiv 2(mod\ 4)$.

By assumption , $n\geq 7$. In this case, 2 vertex labels occur $\lceil\frac{n+3}{4}\rceil$ times and 2 other labels occur $\lfloor\frac{n+3}{4}\rfloor$ times.

Let $r=\frac{n-7}{4}$. Label $v_1,u_1,u_2,u_4,u_6,...,u_{2r+2}$ with 1. Label the remaining vertices arbitrarily so that $r+3$ of them get label -1, $r+2$ of them get label i and $r+2$ of them get label -i.

Case(4): $n+3\equiv 3(mod\ 4)$.

In this case , 3 vertex labels occur $\lceil\frac{n+3}{4}\rceil$ times and 1 vertex label occurs $\lfloor\frac{n+3}{4}\rfloor$ times. Let

$r = \frac{n-4}{4}$. Label $v_1, u_1, u_3, u_5, ..., u_{2r+1}$ with 1. Label the remaining vertices arbitrarily so that $r+2$ vertices get label -1, $r+2$ vertices get label i and $r+1$ vertices get label -i. That $P_n + K_3$ is group $\{1, -1, i, -i\}$ Cordial for $n \neq 2, 3, 6$ follows from Table 1.

Nature of n	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$	$e_f(1)$	$e_f(0)$
$n+3 \equiv 0(mod\ 4)$	$\frac{n+3}{4}$	$\frac{n+3}{4}$	$\frac{n+3}{4}$	$\frac{n+3}{4}$	$2n+1$	$2n+1$
$n+3 \equiv 1(mod\ 4)$	$\lceil\frac{n+3}{4}\rceil$	$\lfloor\frac{n+3}{4}\rfloor$	$\lfloor\frac{n+3}{4}\rfloor$	$\lfloor\frac{n+3}{4}\rfloor$	$2n+1$	$2n+1$
$n+3 \equiv 2(mod\ 4)$	$\lceil\frac{n+3}{4}\rceil$	$\lceil\frac{n+3}{4}\rceil$	$\lfloor\frac{n+3}{4}\rfloor$	$\lfloor\frac{n+3}{4}\rfloor$	$2n+1$	$2n+1$
$n+3 \equiv 3(mod\ 4)$	$\lceil\frac{n+3}{4}\rceil$	$\lceil\frac{n+3}{4}\rceil$	$\lceil\frac{n+3}{4}\rceil$	$\lfloor\frac{n+3}{4}\rfloor$	$2n+1$	$2n+1$

Table 1

Theorem 3.3. *$P_n + K_4$ is group $\{1, -1, i, -i\}$ Cordial iff $n \in \{3, 4, 5, 6, 7, 9, 10, 11, 13, 14, 17\}$.*

Proof. Let the vertices of P_n be $u_1, u_2, ..., u_n$ and let the vertices of K_4 be v_1, v_2, v_3, v_4. Number of vertices of $P_n + K_4$ is $n+4$.

Number of edges = $\binom{4}{2} + (n-1) + 4n = 5n+5$.

Case(1): Exactly one vertex, say v_1 is given label 1.

Now , $deg\ v_i = n+3 (1 \leq i \leq 4)$, $deg u_1 = 5$, $deg u_n = 5$ and $deg\ u_i = 6 (2 \leq i \leq n-1)$.

Subcase(i): $n = 4k (k \in \mathbb{Z}_+)$.

Now total number of vertices is $4k+4$ and so each vertex label should

occur $k+1$ times. Total number of edges is $5(4k)+5 = 20k+5$. So one edge label should occur $10k+3$ times and another should occur $10k+2$ times. Maximum number of edges that can receive label 1 by taking $k+1$ vertices is $(4k+3)+5k$. So, to get a group $\{1, -1, i, -i\}$ Cordial labeling , we need to have $4k+3+5k = 9k+3 \geq 10k+2$ and so $k \leq 1$. In this case $n = 4$. A group $\{1, -1, i, -i\}$ Cordial labeling of $P_4 + K_4$ is given in Table 2 .

Subcase(ii): $n = 4k+1 (k \geq 0)$.

Now, number of vertices in $P_n + K_4$ is $4k+5$ and so one vertex label should occur $k+2$ times and each of the other three vertex labels should occur $k+1$ times. Total number of edges is $5(4k+1)+5 = 20k+10$. So each of the edge labels should occur $10k+5$ times. Maximum number of edges that can receive label 1 by taking $k+2$ vertices is $(4k+4)+5(k+1)$. So, a necessary condition to get a group $\{1, -1, i, -i\}$ Cordial labeling is $9k+9 \geq 10k+5$ and so

$k \leq 4$. When $k=0$, $P_1+K_4 \approx K_5$ and there is no choice of 1 vertex or 2 vertices so that 5 edges get label 1. So $k \neq 0$. Group $\{1,-1,i,-i\}$ Cordial labelings of P_n+K_4 when $n=5,9,13$ and $n=17$ are given in Tables 2 and 3.

Subcase(iii):$n=4k+2(k \geq 0)$.

Now, number of vertices in P_n+K_4 is $4k+6$ and so 2 vertex labels should occur $k+2$ times and 2 vertex labels should occur $k+1$ times. Total number of edges is $5(4k+2)+5=20k+15$. So , one edge label should occur $10k+7$ times and another edge label should occur $10k+8$ times. Maximum number of edges that can receive label 1 by taking $k+2$ vertices in a group $\{1,-1,i,-i\}$ Cordial labeling is $4k+6+5(k+1)$. So , the necessary condition is , $9k+11 \geq 10k+8$ and so $k \leq 3$. When $k=0, n=2$. $P_2+K_4 \approx K_6$ and 2 vertex labels should occur 2 times and 2 labels should occur 1 time. One edge label should occur 7 times and another edge label should occur 8 times. There is no choice of 1 vertex or 2 vertices so that 7 or 8 edges get label 1. So , $n \neq 2$. Group $\{1,-1,i,-i\}$ Cordial labeling of P_n+K_4 when $n=6,10,14$ are given in Tables 2 and 3.

Subcase(iv): $n=4k+3(k \geq 0)$.

Now, number of vertices in P_n+K_4 is $4k+7$. So 3 vertex labels should occur $k+2$ times and 1 vertex label should occur $k+1$ times. Total number of edges is $5(4k+3)+5=20k+20$. So, each edge label should occur $10k+10$ times. Maximum number of edges that can receive label 1 by taking $k+2$ vertices in a group $\{1,-1,i,-i\}$ cordial labeling is $(4k+7)+5(k+1)=9k+12$. So , the necessary condition is , $9k+12 \geq 10k+10$ i.e., $k \leq 2$. Group $\{1,-1,i,-i\}$ Cordial labelings of P_n+K_4 when $n=3,7$ or $n=11$ are given in Tables 2 and 3.

Case(2): At least two vertices $v_i (1 \leq i \leq 4)$ are given label 1.

Suppose v_1 and v_2 are given label 1. Now, $(n+3)+(n+2)=2n+5$ edges get label 1.

Subcase(i): $n=4k(k \geq 1)$.

Number of edges that has to receive label 1 is $10k+2$. Minimum number of edges that can receive label 1 is $8k+5+(k-1)3=11k+2$. Therefore, $10k+2 \geq 11k+2 => k \leq 0$.

n	v_1	v_2	v_3	v_4	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}
3	1	-1	-1	i	1	i	$-i$							
4	1	-1	i	$-i$	-1	1	i	$-i$						
5	1	1	-1	-1	i	i	i	$-i$	$-i$					
6	1	1	-1	-1	i	i	i	$-i$	$-i$	$-i$				
7	1	-1	-1	-1	i	1	i	1	i	$-i$	$-i$			
9	1	-1	-1	-1	1	1	i	1	i	i	$-i$	$-i$	$-i$	
10	1	-1	-1	-1	-1	1	i	1	1	i	i	$-i$	$-i$	$-i$
11	1	1	-1	-1	1	-1	-1	i	i	i	i	$-i$	$-i$	$-i$
13	1	-1	-1	-1	1	1	i	1	i	1	-1	i	i	$-i$
14	1	-1	-1	-1	-1	1	-1	1	i	1	i	1	i	i
17	1	-1	-1	-1	-1	1	-1	1	i	1	i	1	i	1

Table 2

n	u_{11}	u_{12}	u_{13}	u_{14}	u_{15}	u_{16}	u_{17}
11	$-i$						
13	$-i$	$-i$	$-i$				
14	$-i$	$-i$	$-i$	$-i$			
17	i	i	$-i$	$-i$	$-i$	$-i$	$-i$

Table 3

Subcase(ii): $n = 4k+1 (k \geq 0)$.

Number of edges that has to receive label 1 is $10k+5$. Minimum number of edges that can receive label 1 is $8k+5+(k-1)3 = 11k+2$. Therefore, $10k+5 \geq 11k+2 => k \leq 3$.

Subcase(iii) $n = 4k+2 (k \geq 0)$.

Number of edges that has to receive label 1 is $10k+8$. Minimum number of edges that can receive label 1 is $8k+5+(k-1)3 = 11k+2$. So $10k+8 \geq 11k+2 => k \leq 6$. It is easy to observe that $P_{18}+K_4$, $P_{22}+K_4$ and $P_{26}+K_4$ are not group $\{1,-1,i,-i\}$ Cordial.

Subcase(iv): $n = 4k+3 (k \geq 0)$.

Number of edges that has to receive label 1 is $10k+10$. Minimum number of edges that can receive label 1 is $(8k+5)+(k-1)=11k+2$. Hence $10k+10 \geq 11k+2 => k \leq 8$. It is easy to observe that P_n+K_4 is not group $\{1,-1,i,-i\}$ Cordial for $3 \leq k \leq 8$.

Case(3): No vertex v_i is given label 1.

Subcase(i):$n=4k(k>0)$.

Maximum number of edges that can receive label 1 is $(k+1)6=6k+6$.

So, $6k+6 \geq 10k+2 => 4k \leq 4 => k \leq 1$.

Subcase(ii): $n=4k+1(k \geq 0)$.

Maximum number of edges that can receive label 1 is $6k+6$. So $6k+6 \geq 10k+5 => 4k \leq 1 => k=0$.

Subcase(iii): $n=4k+2(k \geq 0)$.

Now, $6k+6 \geq 10k+8 => 4k \leq -2$ which is impossible.

Subcase(iv): $n=4k+3(k \geq 0)$.

Now, $6k+6 \geq 10k+10 => 4k \leq -4$ which is impossible.

Thus, P_n+K_4 is group $\{1,-1,i,-i\}$ Cordial iff $n \in \{3,4,5,6,7,9,10,11,13,14,17\}$.

Theorem 3.4. *For $n \leq 30$,P_n+K_n is group $\{1,-1,i,-i\}$ Cordial iff* $n \in \{1,2,4,5,7,9,10,11,16,25,26,27\}$.

Proof.

Let the vertices of P_n be labelled as $u_1,u_2,.....,u_n$ and the vertices of K_n be labelled as $v_1,v_2,.....,v_n$. Number of vertices in P_n+K_n is $2n$ and the number of edges is $\binom{n}{2}+n^2+n-1=\frac{(3n-2)(n+1)}{2}$.

For $1 \leq i \leq n$, deg $v_i=2n-1$, $deg\ u_1=deg\ u_n=n+1$ and deg $u_i(1<i<n)=n+2$.

For $n=1$, the two vertices of $P_1+K_1 \approx K_2$ be labelled as 1,-1 respectively and this is a group $\{1,-1,i,-i\}$ Cordial labeling.

For $n=2$, $P_2+K_2 \approx K_4$ which is group $\{1,-1,i,-i\}$ Cordial.

For $n=3$, P_3+K_3 has 6 vertices and 14 edges. Two vertex labels should appear 2 times and 2 other vertex labels should appear once. Each edge label should appear 7 times. There is no choice of 1 vertex or 2 vertices so that 7 edges get label 1.

For $n=4$,f_1 defined by $f_1(v_1)=f_1(v_2)=1$, $f_1(v_3)=f_1(v_4)=-1$, $f_1(u_1)=f_1(u_2)=i$, $f_1(u_3)=$

$f_1(u_4) = -i$ is a group $\{1,-1,i,-i\}$ Cordial labeling.

For $n = 5$, f_2 defined by $f_2(v_1) = f_2(u_1) = f_2(u_3) = 1, f_2(v_2) = f_2(v_3) = f_2(v_4) = -1, f_2(v_5) = f_2(u_2) = i$, $f_2(u_4) = f_2(u_5) = -i$ is a group $\{1,-1,i,-i\}$ Cordial labeling.

For $n = 6$, $P_6 + K_6$ has 12 vertices and 56 edges. Each vertex label should occur 3 times. Each edge label should appear 28 times. There is no choice of 3 vertices so that 28 edges get label 1.

For $n = 7$, f_3 defined by, $f_3(v_1) = f_3(v_2) = f_3(u_1) = f_3(u_3) = 1$, $f_3(v_3) = f_3(v_4) = f_3(v_5) = f_3(v_6) = -1$, $f_3(v_7) = f_3(u_2) = f_3(u_4) = i$, $f_3(u_5) = f_3(u_6) = f_3(u_7) = -i$ is a group $\{1,-1,i,-i\}$ Cordial labeling.

For $n = 8$, $P_8 + K_8$ has 16 vertices and 99 edges. There is no choice of 4 vertices so that 49 or 50 edges get label 1.

For $n = 9$, f_4 defined by, $f_4(v_1) = f_4(v_2) = f_4(v_3) = f_4(u_1) = f_4(u_2) = 1$, $f_4(v_4) = f_4(v_5) = f_4(v_6) = f_4(v_7) = f_4(v_8) = -1$, $f_4(v_9) = f_4(u_3) = f_4(u_4) = f_4(u_5) = i$, $f_4(u_j) = -i$ for $6 \le j \le 9$ is a group $\{1,-1,i,-i\}$ Cordial labeling.

For $n = 10$, f_5 defined by, $f_5(v_j) = 1 (1 \le j \le 4), f_5(u_1) = 1$, $f_5(v_j) = -1$, for $5 \le j \le 9$, $f_5(v_{10}) = i, f_5(u_j) = i$ for $2 \le j \le 5$, $f_5(u_j) = -i$ for $6 \le j \le 10$, is a group $\{1,-1,i,-i\}$ Cordial labeling.

For $n = 11$, f_6 defined by, $f_6(v_j) = 1$ for $1 \le j \le 4, f_6(u_1) = f_6(u_2) = 1$, $f_6(v_j) = -1$, for $5 \le j \le 10$, $f_6(v_{11}) = i, f_6(u_j) = i$ for $3 \le j \le 6$, $f_6(u_j) = -i$ for $7 \le j \le 11$ is a group $\{1,-1,i,-i\}$ Cordial labeling.

For $n = 12$, number of vertices of $P_{12} + K_{12}$ is 24 and number of edges is 221. There is no choice of 6 vertices so that 110 or 111 edges get label 1.

For $n = 13$, number of vertices of $P_{13} + K_{13}$ is 26 and number of edges is 259. There is no choice of 6 or 7 vertices so that 129 or 130 edges get label 1.

For $n = 14$, number of vertices of $P_{14} + K_{14}$ is 28 and number of edges is 300. There is no choice of 7 vertices so that 150 edges get label 1.

For $n = 15$, number of vertices of $P_{15} + K_{15}$ is 30 and number of edges is 344. There is no choice of 7 or 8 vertices so that 172 edges get label 1.

For $n = 16$, f_7 defined by, $f_7(v_j) = 1 (1 \le j \le 6), f_7(u_2) = 1, f_7(u_4) = 1$, $f_7(v_j) = -1 (7 \le j \le 14)$, $f_7(v_{15}) = f_7(v_{16}) = i$, $f_7(u_1) = f_7(u_3) = i, f_7(u_j) = i (5 \le j \le 8)$ $f_7(u_j) = -i (9 \le j \le 16)$

is a group $\{1,-1,i,-i\}$ Cordial labeling.

For $n=17$, $P_{17}+K_{17}$ has 34 vertices and 441 edges. There is no choice of 8 or 9 vertices so that 220 or 221 edges get label 1.

For $n=18$, there is no choice of 9 vertices so that 247 edges get label 1.

For $n=19$, there is no choice of 9 or 10 vertices so that 225 or 226 edges get label 1.

For $n=20$, there is no choice of 10 vertices so that 305 or 306 edges get label 1.

For $n=21$, there is no choice of 10 or 11 vertices so that 337 edges get label 1.

For $n=22$, there is no choice of 11 vertices so that 370 edges get label 1.

For $n=23$, there is no choice of 11 or 12 vertices so that 404 or 405 edges get label 1.

For $n=24$, there is no choice of 12 vertices so that 440 or 441 edges get label 1.

For $n=25$, f_8 defined by, $f_8(v_j)=1(1\le j\le 10), f_8(u_1)=f_8(u_3)=1$, $f_8(v_j)=-1(11\le j\le 23), f_8(v_{24})=f_8(v_{25})=i$, $f_8(u_2)=i, f_8(u_j)=i(4\le j\le 13)$, $f_8(u_j)=i(14\le j\le 25)$ is a group $\{1,-1,i,-i\}$ Cordial labeling.

For $n=26$, f_9 defined by, $f_9(v_j)=1(1\le j\le 10), f_9(u_1)=f_9(u_2)=f_9(u_4)=1$, $f_9(v_j)=-1(11\le j\le 23), f_9(v_j)=i(24\le j\le 26)$, $f_9(u_3)=i, f_9(u_j)=i(5\le j\le 13), f_9(u_j)=-i(14\le j\le 26)$ is a group $\{1,-1,i,-i\}$ Cordial labeling.

For $n=27$, f_{10} defined by, $f_{10}(v_j)=1(1\le j\le 10), f_{10}(u_1)=f_{10}(u_2)=f_{10}(u_4)=f_{10}(u_6)=1$, $f_{10}(v_j)=-1(11\le j\le 24), f_{10}(v_j)=i(25\le j\le 27)$, $f_{10}(u_3)=f_{10}(u_5)=i, f_{10}(u_j)=i(7\le j\le 14)$, $f_{10}(u_j)=-i(15\le j\le 27)$ is a group $\{1,-1,i,-i\}$ Cordial labeling.

For $n=28$, there is no choice of 14 vertices so that 599 or 600 edges get label 1.

For $n=29$, there is no choice of 14 or 15 vertices so that 643 edges get label 1.

For $n=30$, there is no choice of 15 vertices so that 688 edges get label 1.

Conflict of Interests

The authors declare that there is no conflict of interests.

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