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MULTISET CLASSIFICATIONS AND INCLUSION PARAMETERS

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Abstract: Topological structures on a set are considered generalized flexible mathematical models for non-quantities real life problems. In many real life problems, medical investigation and teaching for example, the repetition of cases affect the process of decisions making, and so the multiset is suitable theory for modeling such cases. In this paper comparison between power whole and power full multiset topologies are presented. Classes of closure and interior operators in the two multiset topologies are initiated. Properties of the suggested operators are obtained. Also, boundary operators are defined and investigated. The concepts of Yager fuzzy intersection and union are generalized to multiset context. Examples and counter examples are given. The suggested operator can help in the process of approximation in information system.

Keywords: multiset theory; topology; closure and interior multiset space.

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1. Introduction

Many fields of modern mathematics have emerged by violating a basic principle of a given theory of traditional mathematics. Traditional mathematics implicitly assumes that all mathematical objects occur without repetition. Thus there is only one number four, one field of complex numbers, etc. So the only possible relation between two mathematical objects is either they are equal or they are different. The situation in science and in ordinary life is not like this. In

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the physical world it is observed that is enormous cases for repetition [2, 5-7, 13, 16, 18, 20]. For instance, there are many hydrogen atoms, many water molecules, many strands of DNA, etc. In classical set theory, a set is a well-defined collection of distinct objects. If repeated occurrences of any object is allowed in a set, then a mathematical structure, that is known as multiset (mset or bag, for short).

A multiset is a finite set in which each element assigned by positive integer called number of repeated so, to represent a multiset we use a finite sequence of letters or objects. Some chemical terminology form by multiset as chemical soup of molecules and communication is viewed as a chemical reaction between molecules [1]. Multiset used in some type of graph transformation and used in DNA computing. if we replace tubes of molecules by rules of applications we can study DNA by using multiset concepts [12]. The important property of multiset is the repetition of its element which is not found in classical set theory, since the possible relation between elements in classical set is either they are equal or they are different which is ideal less applicable in our ordinary life.

Topology is one of the most active areas in science and engineering application since it studies the phenomena in different views. The concept of topological structure [9, 19] and there generalizations is very useful not only in theoretical studies but also in practical applications, it is one of the most powerful notions in mathematical analysis. Many works have appeared recently for example in structural analysis [15], in chemistry [17], and physics [10]. The purpose of the present work is to put a starting point for the multiset topology and the applications of abstract topology in multiset theory, granular computing [21] and fuzzy multiset theory. Also, we will integrate some ideas in terms of concepts in topological multiset theory and uncertainty. In fact, topological structure is generalized methods for measuring similarity and dissimilarity between objects in the universes. [8, 14]

This paper is organized in the following way we first begin with the introduction to multiset and *M*- topology in section 2. Moreover we discuss properties of topology on multiset and relation between interior and closure between different types of M- topology in section 3. Also we define a sub *M*- topology and finer topology and relation between closure and interior on them in section 4. In addition we define a general class of intersection and union for multiset and define parameter for degree of dependent between two submset in section 5. Several examples are given to indicate these definitions. At last, some conclusion is presented in section 6.

2. Preliminaries

Multiset is an unordered collection of objects, like a set but which may contain copies or duplicate (Brink [4] prefers to call it multiple)

In this section, we introduce a review of some basic concepts of multiset and multiset topology.

2.1. Multiset

In this subsection, a brief survey of the notion of msets introduced by Yager [20].

Definition 2.1[13] An mset M drawn from the set X is represented by a function Count M or C_M defined as $C_M : X \to N$ where N represents the set of nonnegative integers.

In Definition 2.1, $C_M(x)$ is the number of occurrences of the element x in the mset M. However those elements which are not included in the mset M have zero count.

Example 2.1: Let $X = \{a, b, c\}$ $then$ $M = \{3/a, 5/b, 1/c\}$ represents an mset drawn from X.

Definition2.2 [13] Let M and N be two msets drawn from a set X. then, the following are defined

(i) The support set $M^* = \{x \in X: C_M(x) > 0\}$, and it is also called a root set.

(ii) $M = N$ if $C_M(x) = C_N(x)$ $for\ all$ $x \in X$.

(iii) $M \subseteq N$ if $C_M(x) \leq C_N(x)$ $for\ all$ $x \in X$.

(iv) $P = M \cup N$ if $C_P(x) = max\{C_M(x),\ C_N(x)\}$ $for\ all$ $x \in X$.

(v) $P = M \cap N$ if $C_P(x) = min\{C_M(x),\ C_N(x)\}$ $for\ all$ $x \in X$.

(vi) The cardinality of M is denoted by $Card(M)$ or $|M|$ and is given by

$$Card(M) = \sum_{x \in M} C_M(x)$$

Let M be an mset drawn from a set, the support set of M denoted by $M^*$is a subset of X and $M^* = \{x \in X: C_M(x) > 0\}$ i.e. M^* is an ordinary set and it is also called root set

The cardinality of an mset M drawn from a set X is denoted by $Card(M)$ or$|M|$ and is given by $Card\ M = \sum_{x \in X} C_M\ (X)$

An mset M is called an empty mset if $C_M(X) = 0 \forall x \in X$

Definition 2.3[13] A domain X is defined as a set of elements from which msets are constructed. The mset space $[X]^m$ is the set of all msets whose elements are in X such that no element in the mset occurs more than m times.

Definition 2.4[13] Let X be a support set and $[X]^m$ be the mset space defined over X. Then for any mset $M \in [X]^m$, the complement M^c of M in $[X]^m$ is an element of $[X]^m$ such that $C_M^c(x) = m - C_M(x)\ \forall x \in X$.

Moreover, the following types of submset of M and collection of submset from the mset space $[X]^m$ are defined.

In multisets the number of occurrences of each element is allowed to be more than one which leads to generalization of the definition of subsets in classical set theory. So, in contrast to classical set theory, there are different types of subsets in multiset theory.

The following types of submset of M and collection of submset from the mset space$[X]^m$ can be defined

Definition 2.5 (Whole submset) [3]. A submset N of M is a whole submset of M with each element in N having full multiplicity as in M, i.e.

$$C_N(x) =_. C_M(x)\ \ for\ every\ x\ in\ N^*.$$

Definition 2.6(Partial Whole submset) [3]. A submset N of M is a partial whole submset of M with at least one element in N having full multiplicity as in M .i.e.

$$C_N(x) =_. C_M(x)\ \ for\ some\ x\ in\ N^*.$$

Definition 2.7(Full submset) [3]. Submset N of M is a full submset of M if each element in M is an element in N with the same or lesser non- zero multiplicity as in M, i.e.

$$M^* = N^*\ \ with\ \ C_N(x) =\leq_. C_M(x)\ \ for\ every\ x\ in\ N^*.$$

Example 2.2. Let $M = \left\{\frac{2}{x}, \frac{3}{y}, \frac{5}{z}\right\}$ be an mset, the following are the some of the submset of M which are whole submset, partial whole submset and full submset.

- A submset $\left\{\frac{2}{x}, \frac{3}{y}\right\}$ is a whole submset and partial whole submset of M.
- A submset $\left\{\frac{1}{x}, \frac{3}{y}, \frac{5}{z}\right\}$ is a partial whole and full submset of M.
- A submset $\left\{\frac{1}{x}, \frac{3}{y}\right\}$ is a partial whole submset of M.

As various subset relations exist in multiset theory, the concept of power mset can also be generalized as follow:

Definition 2.8 (Power whole Mset)[3]. Let $M \in [X]^m$ be an mset. The power whole mset of M denoted by $PW(M)$ is defined as the set of all the whole submset of M The cardinality of $PW(M)$ is 2^n where n is the cardinality of the support set of M.

Empty set φ is a whole submset of every mset but it is neither a full submset nor a partial whole submset of any nonempty mset M and the power whole multiset of any multiset is an $M-topology$

Definition 2.9 (Power full Mset)[3]. Let $M \in [X]^m$ be an mset. The power full mset of M denoted by $PF(M)$ is defined as the set of all the full submset of M .The cardinality of $PF(M)$ is the product of the count of the element in M.

Definition 2.10 (Power Mset)[13]. Let $M \in [X]^m$ be an mset. The power mset of M denoted by $P(M)$ is defined as the set of all the submset of M, i.e.

$$N \in P(M) \;\; if\; and\; only\; if\; N \subseteq M$$

If $= \emptyset\; then\; N \in {}^1P(M); and\; if\; N \neq \emptyset, then\; N \in {}^k P(M)$, where $k = \prod_z \binom{|[M]_z|}{|[M]_z|}$, the product Π_z is taken over by distinct elements of z of the mset N and

$|[M]_z| = m\; iff\; z \in {}^mM, |[N]_z| = n\; iff\; z \in {}^nN$, then $\prod_z \binom{|[M]_z|}{|[M]_z|} = \binom{m}{n} = \frac{m!}{n!(m-n)!}$

The power set of an mset is the support set of the power mset and is denoted by$P^*(M)$.

Theorem 2.1 [13] Let $P\ (M)$ be a power mset drawn from the mset $M = \{m_1/x_1, m_2/x_2, \ldots, m_n/x_n\}$and $P^*(M)$.be the power set of an mset M. Then the card $P^*(M) = \prod_{i=1}^{n}(1+m_i)$.

Example 2.3. Let $M = \{2/x, 3/y\}$ be an mset. The collection

$PW(M) = \{\{2/x\}, \{3/y\}, M, \emptyset\}$

$PF(M) = \{\{2/x, 2/y\}, \{2/x, 1/y\}, \{1/x, 3/y\}, \{1/x, 2/y\}, \{1/x, 1/y\}, M, \}$

$$P^*(M) = \left\{ \begin{matrix} M, \emptyset, \{2/x, 1/y\}, \{2/x, 2/y\}, \{1/x, 1/y\}, \{1/x, 2/y\}, \{1/x, 3/y\}, \{2/x\} \\ , \{3/y\}, \{1/x\}, \{1/y\}, \{2/y\} \end{matrix} \right\}$$

Definition 2.11 (Multiset topology)[11]. Let $M \in [X]^m and\; \tau \subseteq P^*(M)$, then τ is called a multiset topology if τ satisfies the following properties:

1. $\emptyset\; and\; M$ are in τ.
2. The union of the elements of any sub collection of τ is in τ.

3. The intersection of the elements of any finite sub collection of τ is in τ.

Note: The collection consisting of only M and $\emptyset$ is an M-topology called indiscrete M-topology

Note: If M is any mset, then the collection $PW(M)$ is an M-topology on M

Note: The collection $PF(M)$is not an M-topology on M, because $\emptyset$ does not belong to $PF(M)$but $PF(M) \cup \emptyset$ is an M-topology on M.

Definition 2.12 [11] Closure: Given a submset A of an M-topological space M , the closure of an mset A is defined as the intersection of all closed msets containing A and is denoted by *cl(A)* .i.e. $CL(A) = \cap \{K \subseteq M: K \textit{ is closed mset and } A \subseteq K\}$ and $C_{Cl(A)}(x) = Min\{C_K(x): A \subseteq K\}$

Definition 2.13 [11] Closed mset: A sub mset N of an M-topological space M is said to be closed if the mset $M \ominus N$ is open.

Definition 2.14 [11] Interior: Given a submset A of an M-topological space M, the interior of an mset is defined as the union of all open msets contained in A and is denoted by $Int(A)$ i.e. $Int(A) = \cup \{G \subseteq M: G \textit{ is open mset and } G \subseteq A\}$ and $C_{Int(A)}(x) = Max\{C_G(x): G \subseteq A\}$

3. New view for topological properties on multiset

In this section, we introduce the concept of exterior and boundary in multiset topological space and we discuss relationships between the concepts of boundary, closure, exterior and interior of an mset and study the relation between them on power whole and power full topology.

Definition 3.1: Let A be a submset of an mset M, then $\mathrm{BND(A)} = \mathrm{Cl(A)} \cap \mathrm{Cl(M - A)}$ is called the boundary of A. The set $Int(M - A)$ is called the exterior of a submset A and is denoted by $ext(A)$

Example 3.1: let $X = \{a, b\}$ then $M = \{2/a, 3/b\}$, $\{M, \emptyset, \{1/a\}, \{2/b\}, \{1/a, 2/b\}\}$ if $A = \{1/a, 3/b\}$

$$ext(A) = \{1/a\}$$

We introduce some properties of closure, interior, boundary and exterior on multiset.

Proposition 3.1: Let M be an mset space and $A, B \subset M$, then the following properties are defined:

i. $A \subseteq B \rightarrow Cl(A) \subseteq Cl(B)$
ii. $Cl(M - A) = M - Int(A)$
iii. $BND(A) = Cl(A) - Int(A)$
iv. $BND(A) \cap Int(A) = \emptyset$

v. $Cl(A) = BND(A) \cup Int(A)$

vi. $Cl(A \cup B) = Cl(A) \cup Cl(B)$

vii. $\{Int(A), BND(A), ext(A)\}\ is\ a\ partition\ of\ M$

Proof:

$(ii) since\ E \subset A\ iff\ M - A \subset M - E\ and\ E$ be open mset and the complement of open mset is closed mset, $K = M - E \supset M - A\ thus\ Cl(M - A) = \cap \{M - E: E\ open \wedge E \subset A\} = M - \cup \{E: E\ open \wedge E \subset A\}$ then $Cl(M - A) = M - Int(A)$

$(iii) BND(A) = Cl(A) \cap Cl(M - A) = Cl(A) \cap \big(M - Int(A)\big)$

$$= (Cl(A) \cap M) - \big(Cl(A) - Int(A)\big) = BND(A) = Cl(A) - Int(A)$$

Example 3.2:

Let $X = \{a, b, c, d\}$ and $M = \{5/a, 3/b, 5/c, 5/d\}$ we consider the topology $\tau = \{M, \emptyset, \{1/a, 2/b, 3/c, 2/d\}, \{1/a, 3/c\}, \{2/b, 5/d\}, \{1/a, 2/b, 3/c, 5/d\}, \{2/b, 2/d\}\}$ on M then for any set $A = \{3/a, 3/b, 3/c, 3/d\}$ we have $Cl(A) = M$ and $Cl(M - A) = \{4/a, 1/b, 2/c, 3/d\}$, $BND(A) = \{4/a, 1/b, 2/c, 3/d\}$

The next example shows the interior of power whole and power full and the relation between them:

Example 3.3: Let $M = \{2/a, 3/b\}$ be an mset, the collection $\tau_{PW}(M) = \{M, \emptyset, \{2/a\}, \{3/b\}\}$ $\tau_{PF}(M) = \{M, \emptyset, \{2/a, 1/b\}, \{2/a, 2/b\}, \{1/a, 3/b\}, \{1/a, 2/b\}, \{1/a, 1/b\}\}$

i. If $A = \{2/a\}$ $Int_{PF}(A) = \emptyset$

ii. If $A = \{1/a\}$ $Int_{PW}(A) = \emptyset$

iii. If $A = \{2/a, 1/b\}$ $Int_{PF}(A) = \{2/a, 1/b\}$ $Int_{PW}(A) = \{2/a\}$

Proposition 3.2: Let τ_{PF} and τ_{PW} be a power full and power whole topology respectively then the following properties are satisfied:

i. Let τ_{PF} be a power full topology on a mset M , then if the support set A not equal any support of open set in τ_{PF} then $Int_{PF}(A) = \emptyset$ i.e. if $A^* \neq G^*, G \in \tau_{PF}$ then $Int_{PF}(A) = \emptyset$

ii. Let τ_{PW} be a power whole topology on a mset M, then $Int_{PW}(A) = \emptyset$ if the submset A not contain at least one element has the same count on M

iii. If the submset A satisfies two conditions above then $Int_{PW}(A) \subset Int_{PF}(A)$

Proof:

i. Let $x \in Int_{PF}(A) \to x \in G, G$ is open power full mset, $C_G(x) \leq C_A(x) \forall x \in Int_{PF}(A)$, $G^* = M^* = A^*$ this is contradiction

ii. Let $x \in Int_{PW}(A), G$ open power whole mset, $C_G(x) = C_M(x) = C_A(x) \forall x$ so if all element in A contain element such that $C_A(x) < C_M(x)$ then $Int_{PW}(A) = \emptyset$

iii. Let $x \in Int_{PW}(A) \exists G_{\tau_{PW}}, x \in G_{PW}, C_G(x) = C_M(x) = C_A(x)$ and $G^*_{PW} \subset M^* \subset A^*$ in $\tau_{PF} \exists G_{PF}, x \in G_{PF}, G^*_{PF} = M^* = A^*$ so $G^*_{PW} \subset G^*_{PF}$ and $Int_{PW}(A) \subset Int_{PF}(A)$

Example 3.4 let $M = \{2/x, 2/y, 2/z\}$ the collection $\tau_{PW}(M) =$
$\{M, \emptyset, \{2/x\}, \{2/y\}, \{2/z\}, \{2/x, 2/y\}, \{2/x, 2/z\}, \{2/y, 2/z\}\}$
$\tau_{PF}(M) = \{M, \emptyset, \{2/x, 2/y, 1/z\}, \{2/x, 1/y, 2/z\}, \{1/x, 2/y, 2/z\}, \{1/x, 1/y, 2/z\},$
$\{1/x, 2/y, 1/z\}, \{2/x, 1/y, 1/z\}, \{1/x, 1/y, 1/z\}\}$

i. If $A = \{1/x, 1/y, 1/z\}$ then $Cl_{PW}(A) = M$

ii. If $A = \{1/x, 2/y\}$ then $Cl_{PF}(A) = M$

iii. If $A = \{1/x, 1/z\}$ then $Cl_{PF}(A) = \{1/x, 1/z\}, Cl_{PW}(A) = \{2/x, 2/z\}$

Proposition 3.3: Let τ_{PF} and τ_{PW} be a power full and power whole topology respectively then the following properties are satisfied:

i. Let τ_{PW} be a power whole topology on a mset M, then if the support of a submset A equal the support of a mset M then $Cl_{PW}(A) = M$i.e. if $M^* = A^*$then $Cl_{PW}(A) = M$.

ii. Let τ_{PF} be a power full topology on an mset M, then if A contain any element with the same count in M then $Cl_{PF}(A) = M$.

iii. If a submset A not satisfies two conditions above then $Cl_{PF}(A) \subset Cl_{PW}(A)$

Proof:

i. Let $G^* \neq M^*, x \epsilon Cl_{PW}(A) \exists K \in \tau^C_{PW}$, $A \subseteq K, C_A(x) \leq C_K(x)$, $C_K(x) = C_M(x) \forall \boldsymbol{x} \in K$ sinceτ is a power whole topology and quasi-discrete topology, $K^* \neq M^* \neq A^*$so if $A^* = M^*$ there is no closed set contain A except the set M.

ii. Let A be a submset, $x \in Cl_{PF}(A) \exists K \in \tau^C_{PF}, A \subseteq K, C_K(x) = m - C_G(x)$ where m is the count of x in the mset M and $\forall x \in G_{PF}, C_G(x) \leq C_M(x), G^* = M^*$ so each closed set not contain any element with the same count in M .

iii. Let $x \in Cl_{PF}(A) \exists K \in \tau_{PF}^{C}, x \in K, C_A(x) \leq C_K(x) \forall \boldsymbol{x} \in A$ and , $C_{K_{PF}}(x) \subset C_M(x) \forall \boldsymbol{x} \in K_{PF}$ but in τ_{PW}^{C} $C_{K_{PW}}(x) = C_M(x) \forall \boldsymbol{x} \in K_{PW}, \exists K_{PW} \in \tau_{PW}^{C}, \boldsymbol{x} \in K_{PW}$ so $x \epsilon Cl_{PW}(A)$ and $Cl_{PF}(A) \subset Cl_{PW}(A)$

Note: The power whole topology is quasi discrete topology but the power full topology isn't quasi discrete topology except some open set is also closed set and it called closed power full (CPF)

Proposition 3.4: The closure of a Power Full set A is the multiset M or the set A

Proof:

Let A be a CPF set i.e. A is closed $Cl(A) = A$

If A isn't CPF i.e. $C_A(x) \leq C_M(x), \forall K \in \tau_{PF}^{C} C_K(x) < C_M(x)$ and there is no closed mset contain the power full set A except M and $Cl_{PF}(A) = M$

Theorem 3.1: Let M be a mset and $C_M(x)$ is the number of occurrences of the element x in the mset M , $C_1 = min_{x \in M} C_i(x), C_2 = max_{x \in M} C_i(x), C = \frac{C_1 + C_2}{2}$ and $M_C = \{x/m : x \in M, m > C\}$ then

i. $M_C \subset M_{C-1} \subset M_{C-2} \subset \cdots \subset M.$

ii. $\tau_{PW}(M_C) \subset \tau_{PW}(M_{C-1}) \subset \tau_{PW}(M_{C-2}) \subset \cdots \subset \tau_{PW}(M).$

iii. $\tau_{RPW}(M_C) \subset \tau_{RPW}(M_{C-1}) \subset \tau_{RPW}(M_{C-2}) \subset \cdots \subset \tau_{RPW}(M).$

Proof:$(\boldsymbol{iii})$We obtain $\tau_{RPW}(M)$ by $M_C \cap O, O \in \tau_{PW}(M_{C-1}), M_C \subset M_{C-1}$ and we obtain $\tau_{PW}(M_{C-1}) by$ $M_{C-1} \cap U, U \in \tau_{PW}(M_{C-2}), M_{C-1} \subset M_{C-2}$ so $O \subset$ U,

$\tau_{RPW}(M_C) \subset \tau_{RPW}(M_{C-1}) \subset \tau_{RPW}(M_{C-2}) \subset \cdots \subset \tau_{RPW}(M).$

Example 3.5: Let $M = \{1/x, 2/y, 3/z\}$ then $M_C = \{3/z\}, M_{C-1} = \{2/y, 3/z\}$ then $\tau_{PW}(M_C) \subset \tau_{PW}(M_{C-1}) \subset \tau_{PW}(M), \tau_{RPW}(M_C) \subset \tau_{RPW}(M_{C-1}).$

4. New result on M-topology

In this section we define a submset topology and study the relation between the closure and interior on it and study the finer and coarser topology and their impact on closure and interior.

Definition 4.1: The topology τ' called submset topology of τ if $\forall G'_M \in \tau' \exists G_M \in \tau \ni G'_M \subset G_M$

Proposition 4.1: Let τ' be a submset topology of τ then the following properties are defined:

i. $Cl_{\tau}(A) \subseteq Cl_{\tau'}(A)$

ii. $Int_{\tau'}(A) \subseteq Int_{\tau}(A)$

Proof:

i. Let $m/x \in Cl_{\tau}(A) \exists K \in \tau^{C}, m/x \in K$ and $\exists K' \in {\tau'}^{C}, K \subset K'$ so $m/x \in K', m/x \in Cl_{\tau'}(A)$, $Cl_{\tau}(A) \subseteq Cl_{\tau'}(A)$.

ii. Let $m/x \in Int_{\tau'}(A) \exists G' \in \tau', m/x \in \tau'$ and $G' \subset G, G \in \tau$ so $m/x \in Int_{\tau}(A), Int_{\tau'}(A) \subseteq Int_{\tau}(A)$

Proposition 4.2: Let τ' and τ be two $M-$ topologies on an mset M such that τ' is contained in τ then the following properties are defined:

i. $Cl_{\tau}(A) \subseteq Cl_{\tau'}(A)$.

ii. $Int_{\tau'}(A) \subseteq Int_{\tau}(A)$

Proof:

i. Let $m/x \in Cl_{\tau}(A) \exists K \in \tau^{C}, m/x \in K$ and ${\tau'}^{C} \subset \tau^{C}$ there are two cases: $K \in {\tau'}^{C}$ so $m/x \in Cl_{\tau'}(A), Cl_{\tau}(A) \subseteq Cl_{\tau'}(A)$ or there is exist another closed set $K' \in {\tau'}^{C}$ contain A and

$K \subset K', m/x \in Cl_{\tau'}(A)$, $Cl_{\tau}(A) \subseteq Cl_{\tau'}(A)$.

ii. Let $m/x \in Int_{\tau'}(A) \exists G' \subset \tau', m/x \in G', \tau' \subset \tau \therefore G' \subset \tau, m/x \in Int_{\tau}(A)$ and $Int_{\tau'}(A) \subseteq Int_{\tau}(A)$

5. New classes of multi set:

In this section we use Yager fuzzy intersection and union to define multi intersection and union to multi set and obtain M-basis and M-topologies.

A general class of intersection and union are presented for multi sets. Example of this class is explained in comparison to ordinary intersection and union.

Definition 5.1: Let M be a multi set such that no element in the mset occurs more than m times and a, b two elements in M has count $C(a)$ and $C(b)$ respectively then multi union can be obtained by

$$S_W^M\left(\frac{C(a)}{a}, \frac{C(b)}{b}\right) = \min\left[m, (C(a)^w + C(b)^W)^{1/w}\right], 0 < W < \infty$$

Example 5.1: Let $M = \{3/a, 4/b, 2/c, 5/d\}$, $m = 5$ and sub M-basis $S = \{\{3/a, 2/d\}, \{2/a\}, \{2/b, 2/c\}\}$ the M-topology by ordinary intersection and union is $\tau = \{M, \emptyset, \{2/a\}, \{3/a, 2/d\}, \{2/b, 2/c\}, \{2/a, 2/b, 2/c\}, \{3/a, 2/b, 2/c, 2/d\}\}$ we can obtain the M- topology at $w = 2$ by

$$S_W^M\left(\frac{C(a)}{a},\frac{C(b)}{b}\right)=\min\left[m,(C(a)^w+C(b)^W)^{1/w}\right],0<W<\infty$$

$\beta=\left\{M,\emptyset,\{2/a\},\{3/a\,,2/d\},\{2/b\,,2/c\}\right\}$ then $\tau=$

$$\left\{\begin{matrix}M,\emptyset,\{2/a\},\{3/a\,,2/d\},\{2/b\,,2/c\},\{4/a\,,2/d\},\{2/a\,,2/b\,,2/c\},\{3/a\,,2/b\,,2/c\,,2/d\},\\ \{4/a\,,2/b\,,2/c\,,2/d\}\end{matrix}\right\}$$

At $w=3$ we can also obtain the M- topology

$\tau=\left\{M,\emptyset,\{2/a\},\{3/a\,,2/d\},\{2/b\,,2/c\},\{2/a\,,2/b\,,2/c\},\{3/a\,,2/b\,,2/c\,,2/d\}\right\}$

The topology varies depending on w

Definition 5.2: Let M be a multi set such that no element in the mset occurs more than m times and a,b two elements in M has count $C(a)$ and $C(b)$ respectively then multi intersection can be obtained by

$$t_W^M\left(\frac{C(a)}{a},\frac{C(b)}{b}\right)=m-\min\left[m,((m-a)^w+(m-b)^W)^{1/w}\right],0<W<\infty$$

Example 5.2: Let $M=\{3/a\,,4/b\,,2/c\,,5/d\}$, $m=5$ and sub M-basis $S=\left\{\{3/a\,,2/d\},\{2/a\},\{2/b\,,2/c\}\right\}$ the M-topology by ordinary intersection and union is $\tau=\left\{M,\emptyset,\{2/a\},\{3/a\,,2/d\},\{2/b\,,2/c\},\{2/a\,,2/b\,,2/c\},\{3/a\,,2/b\,,2/c\,,2/d\}\right\}$ we can obtain the M- topology at $w=2$ by

$$t_W^M\left(\frac{C(a)}{a},\frac{C(b)}{b}\right)=m-\min\left[m,((m-a)^w+(m-b)^W)^{1/w}\right],0<W<\infty$$

$\beta=\left\{M,\emptyset,\{2/a\},\{3/a\,,2/d\},\{2/b\,,2/c\},\{1/a\}\right\}$ then $\tau=$

$\left\{M,\emptyset,\{2/a\},\{1/a\},\{3/a\,,2/d\},\{2/b\,,2/c\},\{2/a\,,2/b,2/c\},\{1/a\,,2/b\,,2/c\},\{3/a\,,2/b\,,2/c\,,2/d\}\right\}$

At $w=3$ we can also obtain the M- topology

$\tau=\left\{M,\emptyset,\{2/a\},\{3/a\,,2/d\},\{2/b\,,2/c\},\{2/a\,,2/b\,,2/c\},\{3/a\,,2/b\,,2/c\,,2/d\}\right\}$

The topology varies depending on w

Proposition 5.1: Let β be M-basis and we obtain M-topology by $S_W^M\left(\frac{C(a)}{a},\frac{C(b)}{b}\right)=\min\left[m,(C(a)^w+C(b)^W)^{1/w}\right],0<W<\infty$ if $w_1<w_2$ then $\tau_2\subseteq\tau_1$ where τ_1 and τ_2 are the M- topology obtained by$w_1 and\ w_2$ respectively.

Proof: a class β is M-basis for τ_1 and τ_2 so, each open set $G_i, i\in I$ (i.e. member of τ_1 and τ_2) is the union of β, $G=\bigcup_j B_j$ and we obtain the union by $S_W^M\left(\frac{C(a)}{a},\frac{C(b)}{b}\right)=\min\left[m,(C(a)^w+\right.$

$C(b)^W)^{1/w}\big], 0 < W < \infty$ then the count of element by using $w_2$is less than or equal the count of element at $w_1 \therefore \tau_2 \subseteq \tau_1$

Example 5.3: Let $M = \{3/a, 4/b, 2/c, 5/d\}$, $m = 5$ and sub M-basis $S = \big\{\{3/a, 3/b\}, \{4/d\}, \{2/a\}\big\}$ then $\beta = \big\{M, \emptyset, \{3/a, 3/b\}, \{4/d\}, \{2/a\}\big\}$ we can obtain the M-topology at $w = 2$ by

$$S_W^M\left(\frac{C(a)}{a}, \frac{C(b)}{b}\right) = \min\big[m, (C(a)^w + C(b)^W)^{1/w}\big], 0 < W < \infty$$

then $\tau_1 = \big\{M, \emptyset, \{2/a\}, \{4/d\}, \{3/a, 3/b\}, \{3/a, 3/b, 4/d\}, \{4/a, 3/b\}, \{2/a, 4/d\}, \{4/a, 3/b, 4/d\}\big\}$

At $w = 5$ we can also obtain the M- topology

$\tau_2 = \big\{M, \emptyset, \{2/a\}, \{4/d\}, \{3/a, 3/b\}, \{3/a, 3/b, 4/d\}, \{2/a, 4/d\}\big\}$ we observe that $\tau_2 \subset \tau_1$

Proposition 5.2

Let β be M-basis and we obtain M-topology by

$S_W^M\left(\frac{C(a)}{a}, \frac{C(b)}{b}\right) = \min\big[m, (C(a)^w + C(b)^W)^{1/w}\big], 0 < W < \infty$ and $\tau_2 \subseteq \tau_1$ where τ_1 and τ_2 are the M- topology obtained by $w_1 and\ w_2$ then

i. $Cl_{\tau_1}(A) \subseteq Cl_{\tau_2}(A)$

ii. $Int_{\tau_1}(A) \subseteq Int_{\tau_2}(A)$

Proof: Is obvious

Definition 5.3: Let M be a multiset and A, B be two submset from M then we can define degree of dependent by $D_{W_i} = \frac{|Int_{\tau_i}(A) \cap Int_{\tau_i}(B)|}{|Cl_{\tau_i}(A) \cup Cl_{\tau_i}(B)|}$ where τ_i is a M-topology generated by S_W^M at w_i

In the following proposition we discuss the properties of this parameter

Proposition 5.2:

Let $A\ and\ B$ be two submset from M and D_{W_i} be the degree of dependent then the following properties satisfy

i. $0 \leq D \leq 1$

ii. If $w_1 < w_2\ then\ D_{W_1} \geq D_{W_2}$

iii. $if\ A \cap B = \emptyset\ then\ D = 0$

iv. $D = 1\ if\ A = B$ and be open mset and the topology is power whole topology

Proof: Is obvious.

6. Conclusion:

The operator suggested for power whole and power full multiset topologies can be used in the approximation of uncertain concepts in information systems in which repetition of case is significant such as cases of food preparation and medical investigation. Considering the accounts of objects in the process of measuring approximation and accuracy depending on the new operators will help in decision making process. Also these results can be applied in the modification of topological graph theory.

Conflict of Interests

The authors declare that there is no conflict of interests.

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