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ON THE FOLDING OF GROUPS

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Abstract. The aim of this work is to introduce a new type of folding called a group folding of a group into a special proper subgroup which induced a graph folding into the identity graph of the group.Also we prove that the composition of such foldings is again a group folding.we find some types of groups which have a normal group folding.Finally we discuss the relation between matrices and group folding.Theorems governing these types of foldings are achieved.

Keywords: .Normal subgroup, Folding, Graph folding..

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1. Introduction

During the previous few years there has been huge progress in the folding theory. All are focusing on topology and manifolds . EL-Ghoul in [4], turns this idea to algebra's branch by giving a definition of the folding of abstract rings and studing its properties .The idea of folding on manifolds is introduced by Robertson in [5]. Following this first paper other studies on the folding of different types of manifolds introduced by many others [2,5,8,9]. Also a graph folding has been introduced by E. El-Kholy[3].Some applications on the folding of a manifold into itself was introduced by P. Di. Francesco [1].

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we will start putting down some definitions which are needed in this paper.

Definition 1.1. A graph map $f : G_1 \to G_2$ between two graphs G_1 and G_2 is a graph folding if and only if f maps vertices to vertices and edges to edges, i.e., if,

(i) for each $v \in V(G_1)$, $f(v)$ is a vertex in $V(G_2)$,

(ii) for each $e \in E(G_1)$, $f(e)$ is an edge in $E(G_2)$, [3]

Definition 1.2. In order to represent a group by a graph ,the vertices of the group are corresponding to the elements of the group, we say two elements x , y in the group are adjacent or can be joined by an edge if $x\,y = e$ (e , *identity elementof* G) . Since, in group $x\,y = y\,x = e$, we need not use the property of commutatively. It is by convention every element is adjoint with the identity of the group G . We shall call the graph associated with the group G as identity graph G_i .Hence the order of the group G correspondes to the number of the vertices in the identity graph. [7]

Example 1.3. Let $G = S_3$ be the symmetric group of degree three, $S_3 = \{ e = (1)$, $p_1 = (2\ 3)$, $p_2 = (1\ 3\ )$, $p_3 = (1\ 2)$, $p_4 = (1\ 2\ 3)$, $p_5 = (1\ 3\ 2) \}$ the identity graph associated with S_3 is showen in Figure 1.

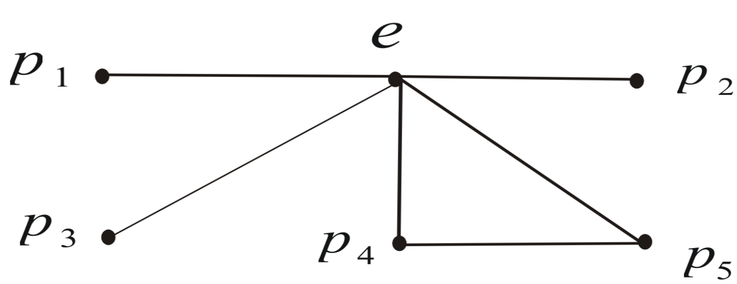

Figure 1

So every group G can be expressed as the identity graph G_i consisting of lines and triangles emerging from the identity element of G. The lines gives the number of self inversed elements in the group,i.e., $x^{-1} = x$, triangles represent elements that are not self inversed.

Theorem 1.4. A group G is isomorphic to a group H if and only if the identity graph G_i is isomorphic to the identity graph H_i .

Proof. Let $f : G \to H$ be a group isomorphism , so $O(G) = O(H)$, hence number of vertices of G_i is equal to the number of vertices of H_i. Also f maps identity in G to identity in H_i i.e., $f(e_G) = e_H$ let $x \in G$ and $f(x) = y \in H$, so $f(x^{-1}) = y^{-1}$, then we have two cases

(i) if $x = x^{-1}$ i.e., x is self inversed , then $y = y^{-1}$. Then the number of lines in G_i equal the number of lines in H_i .

(ii) if $x \neq x^{-1}$ then $f(x) \neq f(x^{-1}) \Rightarrow y \neq y^{-1}$, so in G_i we have a triangle between e_G , x , x^{-1} also we have a triangle in H_i between e_H, y , y^{-1}. Then the number of triangles in G_i equal the number of triangles in H_i .

Hence the graphs are isomomorphic i.e., $G_i \cong H_i$ and vice versa.

Example 1.5. Let $D_{2.3} = \{ a, b \mid a^2 = b^3 = 1 ; bab = a \}$ be the dihedral group of order 6, i.e., $O(D_{2.3}) = 6$. The identity graph G_i associative with $D_{2.3}$ showen in Figure 2.

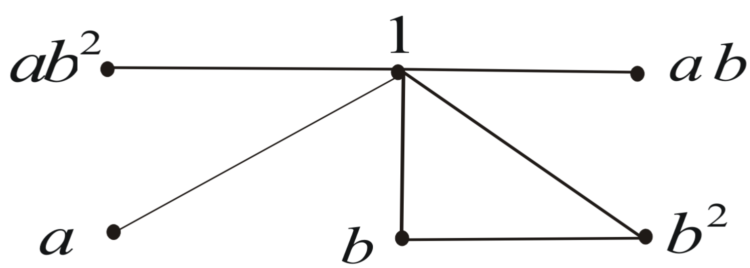

Figure 2

Since the identity graph of $D_{2.3}$ is isomophic with the identity graph of S_3 in example (1.1) . We can define an isomorphism $\theta : D_{2.3} \to S_3$ by

$$\theta(a^i b^j) = (1\ \ 2\ \ 3)^j (1\ 2)^i \quad , i = 1, 2 \ , \ j = 1, 2, 3.$$

Definition 1.6. A proper subgroup H of a group G is said to be a special proper subgroup if H contains at least one element x which is not self inverse.

Example 1.7. The proper subgroup $H_1 = \{ 1 , a \}$ is not special while the proper subgroup $H_2 = \{1 , b , b^2 \}$ is a special proper subgroup of $D_{2.3}$.

2. The Folding of Groups

Definition 2.1. The folding of a group G into a special proper subgroup H is the map $f : G \to H$ defined by

(i) for all $x \in G$ if $x = x^{-1}$ i.e., x is self inversed , then $f(x) = f(x^{-1}) = y \in H$.

(ii) for all $x \in G$ if x is not self inversed, i.e., $x \neq x^{-1}$ and $f(x) = y \in H$. then $f(x^{-1}) = y^{-1} \in H$.

In this case the folding f induces a graph folding $\overline{f}$ on the identity graphs G_i and H_i. The folding $\overline{f} : G_i \to H_i$ maps vertex to vertex and edges to edges, since there exists an edge in G_i between e and x also there exist an edge between $f\ (e)$ and $f\ (x)$ in H_i and $\overline{f}$ maps the edge $(e\ , x)$ in G_i into $(f\ (e)\ ,\ f\ (x))$ in H_i . If $x^{-1} = y$ then there exists an edge $(x\ , y\ )$ in G_i and an edge $(f\ (x), f\ (y))$ in H_i and $\overline{f} : (x\ , y\ ) = (f\ (\ x), f(y)) \in H_i$. It should be noted that the identity graph H_i of the special proper subgroup H must contains at least one triangle, so we can define a graph folding.

The folding f is said to be trivial folding in two cases

(i) if H is a trivial subgroup of G and the folding is defiend by

$$\forall\ x \in G\ , f\ (x)\ = e\ ,\ or f\ (\ x) = x,$$

(ii) if H is a proper subgroup of G but not special i.e., $H. = \{e, a\ \mid a^2 = e\}$, then the folding is defined by $\forall x \in G\ , f\ (x\ ) = a$ and $f\ (\ e\ )\ = e$.

Definition 2.2. The group folding $f : G \to H$ is called normal folding , if the special proper subgroup H is a normal subgroup of the group G.

From now on when defining a group folding or ,induced graph folding , any omitted element, vertex,will mapped onto itself.

Example 2.3. Let $A_4 = \{e = (1),\ p_1 = (1\ 2)(3\ 4)\ ,\ p_2 = (1\ 3)(2\ 4)\ ,\ p_3 = (1\ 4)(2\ 3)\ ,\ p_4 = (2\ 3\ 4)\ ,\ p_5 = (2\ 4\ 3)\ , p_6 = (1\ 3\ 4),\ p_7 = (1\ 3\ 4)\ ,\ p_8 = (1\ 2\ 4)\ , p_9 = (1\ 4\ 2)\ ,\ p_{10} = (1\ 2\ 3)\ ,\ p_{11} = (1\ 3\ 2)\}$ be the alternating group of S_4. The special proper subgruops of A_4 are $H_1 = \{e\ ,\ p_8\ ,p_9\}$, $H_2 = \{e\ ,\ p_4\ ,p_5\}, H_3 = \{e\ ,\ p_6\ ,p_7\}$ and $H_4 = \{e\ ,\ p_{10}\ ,p_{11}\}$. Then we can define a foldings $f_i : A_4 \to H_i$, $i = 1, 2, 3, 4,$ which are not normal foldings since all H_i are not normal subgroups . These foldings induced graph foldings on the identity graph of A_4 and the identity graphs of H_i. The group folding $f_1 : A_4 \to H_1$ can be defined.by

$$f_1\ (\ p_i\ ) = \begin{cases} p_8 & \text{if} \quad i = 1, 2, 3, 4, 6, 10 \\ p_9 & \text{if} \quad i = 5, 7, 11 \end{cases}$$

Figure 3 showes the induced graph folding $\overline{f_1}$ from the identity graph of A_4 into the identity graph of H_1.

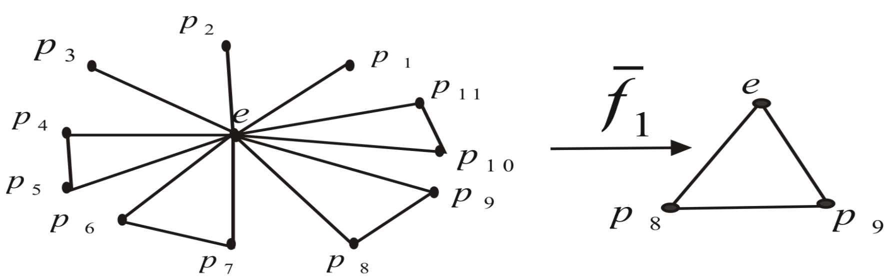

Figure 3

Example 2.4. Let $G = \langle g \mid g^8 = 1\rangle$ be the cyclic group of order 8 .The only special proper subgroup of G is $H = \{1\ , g^2\ , g^4\ , g^6\ \}$, the group folding $f : G \to H$ can be defined by

$$f(g^i) = \begin{cases} g^2 & \text{if} \quad i = 1, 3 \\ g^6 & \text{if} \quad i = 5, 7 \end{cases}$$

Since the subgroup H is a normal subgroup of G , this folding is a normal folding. This normal group folding induces a graph folding $\overline{f} : G_i \to H_i$, which showen in Figure 4.It should be noted that we may have a graph folding from the identity graph of a group G_i into a subgraph of it , but we can not define any group folding or a trivial group folding. Conversely if we have a graph folding from the identity graph G_i of the group G into an identity subgraph H_i , which is the identity graph of a special proper subgraph H of the group G . Then we can define a group folding $f : G \to H$ induced from the graph folding.

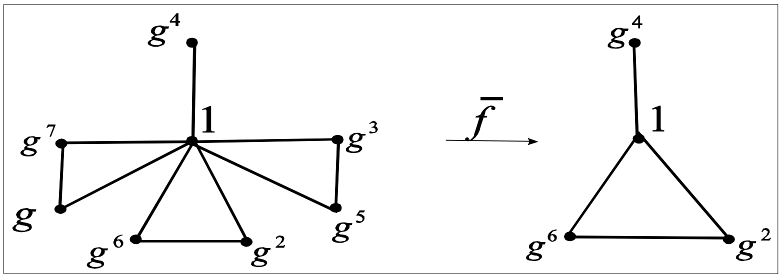

Figure 4

Example 2.5. In example (2.4) we can define a graph folding from the identity graph of $G = \langle g \mid g^8 = 1 \rangle$ into a subgroup $K = \{1,\ g^2,\ g^6\ \}$ defined by

$$\overline{f}\,(1) = 1\ ,\ \ \overline{f}\,(g^i\ ) = g^2\ ,\quad i = 1,2,3,4.,\qquad \overline{f}\,(g^i) = g^6\ ,\quad i = 5,6,7$$

But we can not define any group folding from G into K, since K is not a subgroup of G .

Theorem 2.6. The composition of two group foldings is a group folding.

Proof. Let K be a special proper subgroup of a group H , also H is a special proper subgroup of a group G, so K and H contanis at least one element which is not self inversed. Let $f : G \to H$ and $\Phi : H \to K$ be group foldings, then the map $\Phi \circ f : G \to K$ can be defined as followes : let x be an element of G , we have two cases

(i) if x is self inversed element of G , the folding $\Phi \circ f$ is defined by

$$(\ \Phi \circ\ f\ )(x)\ = \Phi[\ f\ (\ x\ )\ ] =\ \Phi(\ y\ )\ =\ z\ ,\qquad z \in K$$

($i\,i$) if x is not self inversed element of G , so let $f\ (\ x\ ) =\ y$, $y \in\ H$, since f is a group folding , then $f\ (\ x^{-1}\ ) =\ y^{-1}$, $y^{-1} \in H$, again Φ is a group folding ,i.e., if $\Phi(\ y\ ) =\ z$ then $\Phi(\ y^{-1}\ ) =\ z^{-1}$, $z^{-1} \in K$. Then the folding $\Phi \circ\ f$ is dffiend by

$$(\Phi \circ f)\,(x) \;=\; \Phi\,[f\ (x)] = \Phi(y) = z\ ,\qquad z \in K$$

$$also\quad (\Phi \circ f)\left(x^{-1}\right) \;=\; \Phi\left[f\ (x^{-1})\right] = \Phi(y^{-1}) = z^{-1}\ ,\qquad z^{-1} \in K$$

Thus $\Phi \circ f$ is a group folding .

Example 2.7. Let $G = D_{2.8} = \{ a, b \mid a^2 = b^8 = 1 ; bab = a \}$ be the dihedral group of order 16 , and $H = \langle b \mid b^8 = 1 \rangle$ be the cyclic group of order 8 and $K = \{1 , b^2 , b^4 , b^6 \}$. Since K is a special proper subgroup of H , we can define a folding $\Phi : H \to K$ as followes: $\Phi (b, b^3, b^5, b^7) = (b^2, b^2, b^6, b^6)$.Also H is a special proper subgroup of G , we can define a group folding $f : G \to H$ as followes: $f(a, ab, ab^2, ab^{3,} ab^4, ab^5 , ab^6, ab^7) = (b^4)$. Then the composition folding $\Phi \circ f : G \to K$ can be defined by

$$(\Phi \circ f)(ab^i) = (b^4) , \quad 0 \leq i \leq 8 \qquad and$$

$$(\Phi \circ f)(b^i) = \begin{cases} b^2 & \text{if } i = 1, 2, 3 \\ b^6 & \text{if } i = 5, 6, 7 \end{cases}$$

also the induced graph folding $\overline{\Phi \circ f}$ on the identity graphs G_i , H_i, and K_i can be defiend.

Theorem 2.8. If a group G is isomorphic with a group H and H can be folded into a ptoper subgroup K of it . Then the group G is foldable into a proper subgroup M which is isomorphic with K.

Proof. Let $\theta : (G , *) \to (H , \circ)$ be an isomorphism from a group G into a group H , then from Theorem 1 the identity graph G_i of G is isomorphic with the identity graph H_i of H .So any self inversed element in G has an image in H which is self inversed and any not self inversed elements in G has an image in H which is not self inversed . Let $f : H \to K$ be a group folding from H into a prober subgroup K . Then the map $(f \circ \theta) : G \to K$ is well defiend and also it is a group folding from G into K , becaus K_i is a subgrsph of G_i and $\overline{f \circ \theta}$ is a graph folding which maps vertex to vertex and edge to edge from G_i into K_i . Since K is a proper subgroup of H , so K can be embededd in G , then we can fined a proper subgroup M of G which is isomorphic to H i.e., $M \cong K$ so $\Phi : (M, *) \to (K, \circ)$ is an isomorphism . Then we can defiend a group folding $g : G \to M$ which defiend in such away that $\Phi \circ g = f \circ \theta$, so the following diagram is comutative

$$\begin{array}{ccc} (G, *) & \xrightarrow{\theta} & (H, \circ) \\ g \downarrow & & \downarrow f \\ (M, *) & \underset{\Phi}{\to} & (K, \circ) \end{array}$$

Example 2 .9. Let $\theta : D_{2.3} \to S_3$ be an isomorphism in example (1.5) , and $f : S_3 \to K = \{1, p_4, p_5\}$ be a graph folding defiend by $f(p_1) = p_4$, $f(p_2) = p_5$, $f(p_3) = p_4$. Since the group K can be embededd in the group $D_{2.3}$ i.e., the group K is isomorphic with a proper subgroup $M = \{1, b, b^2\}$ of $D_{2.3}$ by the isomorphism $\Phi : M \to H$, defined by $\Phi(1) = 1$, $\Phi(b) = p_4$, $\Phi(b^2) = p_5$. Then there exists a graph folding $g : D_{2.3} \to M$, which defined by $g(a) = b$, $g(ab) = b^2$, $g(ab^2) = b$. Then the following diagram is comutative

$$\begin{array}{ccc} D_{2.3} & \xrightarrow{\theta} & S_3 \\ g \downarrow & & \downarrow f \\ M & \underset{\Phi}{\to} & K \end{array}$$

3. Folding of some types of Groups

In this section we discuss the folding of some different types of groups

Lemma 3.1 Let $G = \langle g \mid g^p = 1 \rangle$ be a cyclic group of order p where p is a prime . Then G has no any non-trivial group folding

Proof. Since G be a cyclic group of a prime order , so it has only a trivial subgroup. Then there is not exists any secial proper or normal subgroups, so it has only trivial folding.

Lemma 3.2 The symmetric group S_n has a normal group folding over one normal subgroup and many group foldings

Proof. Since the symmetric group S_n is divided into two types of permutations , the odd permutation and the even permutation which forms normal subgroup called the alternating subgroup A_n . Then we always have normal folding $f : S_n \to A_n$. Also the symmetric group S_n has many special proper subgroup of the form $H = \{ e , p_i , p_i^{-1} \}$ such that $p_i^2 = p_i^{-1}$ so we have many group folding of the form $f : S_n \to H$.

Lemma 3.3 The minimal group folding $f : G \to H$ of the group G which induced a graph folding is when the special proper subgroup H is on the form $H = \{ 1 , g , g^{-1} \}$where $g^2 = g^{-1}$.

Theorem 3.4. Let $G = \left\langle g \mid g^{p^2} = 1 \right\rangle$ be a cyclic group of order p^2 where p is a prime. Then G has a normal group folding over one normal subgroup.

Proof. Let $G = \left\langle g \mid g^{p^2} = 1 \right\rangle = \{ 1 , g , g^2 , ..., g^{p^2-1} \}$ be a cyclic group of order p^2 where p is a prime .The only subgroup of G is $H = \{1 , g^p , g^{2p} , ..., g^{(p-1)p} \}$, clearly H is a special proper subgroup of order P , since any proper subgroup of a cyclic group is normal subgroup , so it is a normal subgroup. The identity graph G_i of G is formed by only $\frac{(p^2-1)}{2}$ triangles centered around 1 , and the identity graph H_i of H is formed by $\frac{(p-1)}{2}$ triangles . Then we can define a normal folding $f : G \to H$ defied by if $f(g^i) = g^{np}$, then $f(g^{-i}) = g^{-np}$, for all $0 \le i \le p^2 - 1$, ..., $0 \le n \le p - 1$ and the induced graph folding $\bar{f} : G_i \to H_i$ also defiend which maps triangles into triangles.

Example 3.5. Let $G = \left\langle g \mid g^{25} = g^{5^2} = 1 \right\rangle$ be a cyclic group of order 25, so $p = 5$. The only subgroup of G is $H = \{1, g^5, g^{10}, g^{15}, g^{20} \}$ which is normal , the identity graph G_i is formed by 12 triangles centered at 1 , and H_i is formed by two triangles centered at 1 . Then we can defined a normal folding $f : G \to H$ by

$$f(g^i) = \begin{cases} g^5 & \text{if} \quad 0 \le i \le 5 \\ g^{10} & \text{if} \quad 6 \le i \le 12 \\ g^{15} & if \quad 13 \le i \le 19 \\ g^{20} & if \quad 20 \le i \le 24 \end{cases}$$

Theorem 3.6. Let $G = \langle g \mid g^n = 1 \rangle$, where $n = pq$ with p and q two distinct primes ,be a cyclic group of order n . Then G has two normal group foldings over normal subgroups .

Proof. Let $G = \{ 1, g, g^2, \ldots, g^{n-1} \}$ be the cyclic group of order n. Since $n = pq$ and $g^n = g^{pq} = (g^p)^q = (g^q)^p = 1$,then we have two maximal normal subgroups of order p and q which are $H_1 = \{ 1, g^p, g^{2p}, \ldots, g^{(q-1)p} \}$ and $H_2 = \{1, g^q, g^{2q}, \ldots, g^{(p-1)q} \}$ .Then H_1 is a group of prime order q so the identity graph of it consists of $\frac{(q-1)}{2}$ triangels only and H_2 is a group of prime order p so the identity graph of it consists of $\frac{(p-1)}{2}$ triangels only , so each H_1 and H_2 contains an element and its inverse. Hence we can define normal group foldings $f_i : G \to H_i$, $i = 1, 2$ which inducess graph foldings on the identity graph of them .

Example 3.7. Let $G = \langle g \mid g^{21} = 1 \rangle$ be a cyclic group of order 21 .The two maximal normal subgroups are $H_1 = \{1, g^7, g^{14} \}$ and $H_2 = \{1, g^3, g^6, g^9, g^{12}, g^{15}, g^{18} \}$ so the identity graph of H_1 consists of a triangle . Then we can defined a group folding $f_1 : G \to H_1$, which is the minimal folding ,by

$$f_1(g^i) = \begin{cases} g^7 & \text{if } 1 \leq i \leq 10 \\ g^{14} & \text{if } 11 \leq i \leq 20 \end{cases}$$

this group folding induced a graph folding fron G_i into H_1 by mapping all triangels of G_i into the unique triangle in the identity graph of H_1, similarly we can define $f_2 : G \to H_2$.

Theorem 3.8. Let $D_{2.p} = \{1, a, b \mid a^2 = b^p = 1, bab = a \}$ be the dihedral group of order $2p$, p is prime . Then $D_{2.p}$ has a normal group folding over one normal subgroup.

Proof. Let $D_{2.p} = \{1, a, b, b^2, \ldots, b^{p-1}, ab, ab^2, \ldots, ab^{p-1} \}$ be the dihedral group of order $2p$, p is prime, since $D_{2.p}$ has one and only one subgroup $H = \{1, a, b, b^2, \ldots, b^{p-1} \}$ of order p where b is the genaratore of $D_{2.p}$ such that $b^p = 1$, then H is normal . The identity graph of $D_{2.p}$ consists of p lines and $\frac{p}{2}$ triangels and the identity graph of H is formed by $\frac{p}{2}$ triangels. Then we can defined the normal folding $f : D_{2.p} \to H$ by $f(ab^i) = b^i$, $\forall \; 1 \leq i \leq p-1$ and the induced graph folding is defined by mapping the lines in the identity graph of $D_{2.p}$ into any edge of triangels in H_i .

4. Matrices and Group Folding

In this section we will describe the group folding by using the identity graph matrices of the groups.

Definition 4.1. Let G be a group with elements e , g_1 , g_2 , ... , g_n . Clearly the order of G is $n+1$. Let G_i be the identity graph of G . The adjacent matrix of G_i is $(n+1) \times (n+1)$ matrix $M = (x_{ij})$ in which the diagonal terms are zero i.e., $x_{ii} = 0$ for $i = 1, 2, ..., n+1$, the first row and first column are one except the diagonal element ,$x_{ij} = 1$ if the element g_i is the inverse of g_j in which case $x_{ij} = x_{ji} = 1$ if $i \neq j$. We call the matrix $M = (x_{ij})$ the identity graph matrix of the group G. [7]

It should be noted that the matrix $M = (x_{ij})$ is a symmetric matrix with diagonals entiers to be zero, further if the row g_i has only one 1 at the 1^{st} column then $g_i \in G$ is such that $g_i^2 = 1$. Also a row g_i has two ones and the rest zero then for g_i we have a g_k row that has two ones and $g_i\ g_k = g_k\ g_i = 1$

Now let G be a finite group and $f : G \to H$ is a group folding of G into a proper subgroup H of G , this suggests that the identity graph matrix M^* of the subgroup H is a submstrix of M , possibly after rearranging it's rows and columns.

We claim that after deleting (neglecting) the row and column of the identity element, the matrix M can be partationed into four blocks, such that M^* appears in the upper left corner block and a zero matrix O in the upper right one. The matrix M^* will be a submatrix of M which is the complement of M^*. The zero matrix O is due to the fact that non of the elements , g_{k+1} , g_{k+2} , ... , g_{n+1} is adjacent (inverse) of any elements , g_1 , g_2 , ··· , g_k

$$M = \begin{array}{c|c|c} & e, g_1, ..., g_k & g_{k+1}, ..., g_{n+1} \\ \hline \begin{array}{c} e \\ g_1 \\ . \\ . \\ g_k \end{array} & M^* & O \\ \hline \begin{array}{c} g_{k+1} \\ . \\ . \\ g_{n+1} \end{array} & Q & R \\ \hline \end{array}$$

Conversely, if the identity graph matrix M of a group G can be partationed into four blocks with a zero matrix at the right hand corner block. Then a group folding may be defined ,if there is any, as a map $f : G \rightarrow H$ charactrized by the identity group matrix M^* which lie in the upper left corner of M provided that the elements in H which also in M^* forming a special proper subgroup.This map can be defined by mapping

(i) the elements $g_i,\ i = k+1, ... ,n+1$,which is self inversed will have zeros in its columns , mapped to any elements g_j , $j = 1, 2, ..., k$

(ii) the elements $g_i,\ i = k+1, ... ,n+1$,which is not self inversed, mapped to the elements g_j , $j = 1, 2, ..., k$ if the columns of g_i , g_j contains the 1 element above(below)the diagonal and number of zeros between the diagonal and the 1 element is equal.

Example 4.2. Let $Z_{10} = \{\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9\ \}$ be the group under addition modulo 10 .The identity graph matrix of Z_{10} is 10×10 matrix M

$$M = \begin{array}{c|cccccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 4 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 7 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

Now, we can partation M into the following form

$$M = \begin{array}{c|ccccc|ccccc} & 0 & 2 & 4 & 6 & 8 & 1 & 3 & 7 & 9 & 5 \\ \hline 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 7 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 9 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

Thus, we can define a graph folding $f : Z_{10} \to H$ by $f\ (1,\ 3,\ 7,\ 9,\ 5\ ) = (\ 2,\ 4,\ 6,\ 8,\ 8\ )$ such that $f\ (G\ ) = H\ = \{0,\ 2,\ 4,\ 6,\ 8\ \}$. Since H is a special proper subgroup and also a normal subgroup with group matrix M^* which lie in the upper left corner of M

,i.e.

$M^* =$

	0	2	4	6	8
0	0	1	1	1	1
2	1	0	0	0	1
4	1	0	0	1	0
6	1	0	1	0	0
8	1	1	0	0	0

Then this group folding is also a normal group folding.

Example 4.3. Let $G = \langle g \mid g^7 = 1 \rangle$ be a cyclic group of order 7 . The identity graph matrix of G is 7×7 matrix M.

$M =$

	1	g	g^2	g^3	g^4	g^5	g^6
1	0	1	1	1	1	1	1
g	1	0	0	0	0	0	1
g^2	1	0	0	0	0	1	0
g^3	1	0	0	0	1	0	0
g^4	1	0	0	1	0	0	0
g^5	1	0	1	0	0	0	0
g^6	1	1	0	0	0	0	0

Now , we can partation M into the folowing form

$M =$

	1	g	g^6	g^3	g^4	g^2	g^5
1	0	1	1	1	1	1	1
g	1	0	1	0	0	0	0
g^6	1	1	0	0	0	0	0
g^3	1	0	0	0	1	0	0
g^4	1	0	0	1	0	0	0
g^2	1	0	0	0	0	0	1
g^5	1	0	0	0	0	1	0

Thus , we can define a graph folding $f : G \to H$ by $f(g^2, g^3, g^4, g^5) = (g, g, g^6, g^6)$ such that $f(G) = H = \{1, g, g^6\}$, H is subgraph with identity graph matrix M^*

which lie in the upper left corner of M . But since H is not a special proper subgroup of G .Then $f : G \to H$ is not a group folding.

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