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SAMPLE ALLOCATION PROBLEM IN MULTI-OBJECTIVE MULTIVARIATE STRATIFIED SAMPLE SURVEYS UNDER TWO STAGE RANDOMIZED RESPONSE MODEL

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Abstract. Warner (1965) introduced the randomized response model as an alternative survey technique for socially undesirable or incriminating behaviour questions in order to reduce response error, protect a respondent's privacy, and increase response rates. In multivariate stratified surveys with multiple randomised response data the choice of optimum sample sizes from various strata may be viewed as a multi-objective nonlinear programming problem. The allocation thus obtained may be called a "compromise allocation" in sampling literature.

In this paper, we have formulated two stage stratified Warner's Randomised Response model (*RRM*) as a multi-objective integer non-liner optimization problem. In this problem of *RRM* we have minimized the square root of coefficient of variations instead of variations for different characteristics because the coefficient of variation is unit free, subject to the linear and quadratic cost constraint. The multi-objective optimization problem of *RRM* has been solved by lexicographic goal programming integrated with fixed priority D_1 - distance method. The solution obtained by lexicographic goal programming Integrated with fixed priority D_1 - distance have been compared with various existing approaches namely the value function approach, goal programming techniques, ε - constraint method and distance-based method and Khuri & Cornel distance based method. A numerical example is also been presented to illustrate the computational details.

Keywords: coefficient of variation; multi-objective optimization; compromise allocation; multivariate stratified sampling; randomized response.

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1. Introduction

In a questionnaire survey, if a question is highly sensitive or personal, the person may refuse to answer or may give evasive answer. To get response on such question the interviewer must encourage the truthful answers without revealing the identity of the person interviewed. Warner [1965] has developed his randomized response technique which is designed to eliminate evasive answers bias by reducing the rate of non-response keeping the respondents confidentiality. The Warner's model requires the interviewee to give a "Yes" or "No" answer either to a sensitive question or to its negative, depending on the outcome of a randomizing device not disclosed to the interviewer.

Mangat and Singh [1990] proposed a two-stage randomized response model in which each interviewee (who is selected using simple random sampling with replacement) is provided with two randomization devices. Applicability of this model has been illustrated by Singh and Mangat [1996]. Mangat and Singh [1994] proposed another randomized response model which has the benefit of simplicity over that of Mangat and Singh [1990]. Hong et al. [1994] suggested a stratified randomized response technique using a proportional allocation. It may be easy to derive the variance of the proposed estimator. However, it may cause a high cost in terms of time, effort and money because of the difficulty in obtaining a proportional sample from some stratum. Kim and Warde [2004] presented a stratified randomized response technique using an optimal allocation which is more efficient than that using a proportional. Ghufran *et al.* [2012] discussed the applicability of Warner's technique [1965].

Various other Randomised Response techniques that are improved warner's techniques or provide alternative procedures for more complicated situations are discussed in Chang and Huang [2001], Chaudhuri and Mukerjee [1988], Chaudhuri [2001], Chua and Tsui [2000], Franklin [1989], Greenberg et al. [1969], Horvitz [1967], Kuk [1990], Moors [1971], Padmawar [2000] and Singh [2002].

When a single sensitive question with a dichotomous response is under analysis, several randomised response models have been introduced in the literature, starting from the pioneering randomised response model introduced by Warner [1965]. Non-exhaustive list of such randomized response models is given in Chaudhuri and Saha [2005], Diana and Perri [2009], Huang [2006] and others.

In many applications of the randomized response technique more than one sensitive issues are under analysis i.e. multiple sensitive question settings are to be considered. When information on more than one characteristic is to be obtained on each unit of the selected sample, it is not feasible to use the individual optimum allocations in various strata unless there is a strong correlation between the characteristics under study Cochran [1977]. Thus one has to use an allocation that is optimum in some sense for all the characteristics.

For a population the coefficient of variation (CV) is represented by the ratio of population standard deviation to the population mean. The coefficient of variation is used to compare the precision of various estimates that are measured in different units. Ostle [1954] found that the population coefficient of variation is an ideal device for comparing the variation in two series of data that are measured in two different units.

In real life situations we face problems with multiple objectives. Generally, objectives are conflicting in nature, so simultaneous optimization of objectives is not possible. There are several approaches in the literature through which these can be converted to single objective problems. On solving this single objective problem a set of non-dominated solutions is obtained from which an optimal compromise solution is chosen. An optimal compromise solution is that feasible solution which is preferred by decision maker (DM) on all other feasible solution, taking into consideration all criteria contained in the multi-objective functions.

Charnes and Cooper [1961] introduced goal programming technique to solve multi objective problems. Haimes et al [1971] introduced $\varepsilon-$constraint technique which deals multi objective problems by selecting one of the objective functions to be optimized and the remaining objective functions are converted into constraints by setting an upper bound to each of them see Rios [1989] and Miettinen [1999]. The weighting method by Gass and Saaty (1955) and Zadeh (1963) introduced the objective function with a weighting coefficient and minimize the weighted sum of objectives. The Tchebycheff Method proposed by Steuer (1986, 1989) is an interactive weighting vector space reduction method. Value function method can be very useful if the DM could reliably express the value function see Dyer and Sarin [1981] and Miettinen [1999]. In many situations, sufficient information about a variable is not available, or it is difficult to decide most important characteristic of the survey. In such situations, the distance-based technique is very useful see Steuer [1986] and Rios [1989]. Khuri and Cornell [1986] also proposed another distance based technique. Fishburn [1974] widely examined lexicographic orders and utilities.

Lexicographic ordering technique is applied by arranging the objective function according to their importance see Panda et al. [2005] and Ali [2011].

In this paper, a lexicographic goal programming integrated with fixed priority D_1 - distances method is suggested for obtaining compromise allocation for multiple characteristics Warner's randomized response model. This problem is also solved by various existing methods namely - the value function approach, goal programming techniques, ε - constraint method, distance-based method and Khuri & Cornel distance based method. A numerical example is also presented to illustrate the computational details for all methods.

2. Formulation of the problem

Under two-stage randomized response model at stage 1, an individual respondent selected in the sample from *ith* stratum of a stratified population is instructed to use the randomization device R_{1i} which consists of the following two statements:

(i) "I belong to sensitive group" and

(ii) "Go to the randomization device R_{2i} at the second stage"

with known probabilities M_i and $(1-M_i)$ of (i) and (ii) respectively.

At stage 2 the respondents are instructed to use the randomization device R_{2i} which consists of the following two statements:

(i) "I belong to the sensitive group" and

(ii) "I do not belong to the sensitive group"

with known probabilities P_i and $(1-P_i)$ of (i) and (ii) respectively.

The probabilities of a "Yes" answer for jth characteristics; $j=1,2,...,p$ in the *ith* stratum are given by

$$Y_{ij} = M_i\,\pi_{sij} + (1-M_i)\left[P_i\,\pi_{sij} + (1-P_i)(1-\pi_{sij})\right]; \qquad i=1,2,...,L;\ j=1,2,...,p. \tag{1}$$

where π_{sij} is the proportion of respondents belonging to the sensitive group for jth characteristics in the ith stratum. The maximum likelihood estimator of π_{sij} is given as

$$\hat{\pi}_{sij} = \frac{\hat{Y}_{ij} - (1-M_i)(1-P_i)}{2P_i - 1 + 2M_i(1-P_i)};\, i=1,...,L;\ j=1,...,p. \tag{2}$$

where $\hat{Y}_{ij}$ is the estimated proportion of "Yes" answers which follows a binomial distribution $B\left(n_i, Y_{ij}\right)$ and n_i denote the sample size from ith stratum.

Expression (1) and (2) are from Mangat and Singh [1990] with suffix 'i' to denote the ith stratum; $i = 1, 2, ..., L$ and 'j' to denote the jth characteristic; $j = 1, 2, ..., p.$

It can be seen that the estimator $\hat{\pi}_{sij}$ is unbiased.

Since n_i are drawn independently from each stratum, the estimators for individual strata can be added to obtain the estimator for the overall population parameter. This gives the unbiased estimator of π_{sj}, which is the population proportion of respondents belonging to the sensitive group for the j^{th} characteristics, as

$$\hat{\pi}_{sj} = \sum_{i=1}^{L} W_i \hat{\pi}_{sij} = \sum_{i=1}^{L} W_i \left[\frac{\hat{Y}_{ij} - (1 - M_i)(1 - P_i)}{2P_i - 1 + 2M_i(1 - P_i)} \right], \tag{3}$$

where, $W_i; i = 1, 2, ..., L$ are the strata weights.

The sampling variance of $\hat{\pi}_{sj}$ is

$$V(\hat{\pi}_{sj}) = \sum_{i=1}^{L} W_i^2 V(\hat{\pi}_{sij}),$$

$$= \sum_{i=1}^{L} \frac{W_i^2}{n_i} \left[\pi_{sij}(1 - \pi_{sij}) + \frac{(1 - M_i)(1 - P_i)\{1 - (1 - M_i)(1 - P_i)\}}{\{2P_i - 1 + 2M_i(1 - P_i)\}^2} \right]; j = 1, 2, ..., p. \tag{4}$$

(See Mangat & Singh[1990]).

Coefficients of variation for p characteristics are written as

$$(CV)_{sj} = CV(\hat{\pi}_{sj}) = \frac{SD(\hat{\pi}_{sj})}{E(\hat{\pi}_{sj})}; j = 1, 2, ..., p \tag{5}$$

We are given that $\hat{\pi}_{sj}$ is the unbiased estimator of π_{sj}. We have

$$E(\hat{\pi}_{sj}) = \pi_{sj}$$

Thus, $(CV)_{sj}^2 = \dfrac{V(\hat{\pi}_{sj})}{\left[E(\hat{\pi}_{sj})\right]^2} = \dfrac{V(\hat{\pi}_{sj})}{\left[\pi_{sj}\right]^2}$

$$= \sum_{i=1}^{L} \frac{W_i^2 \pi_{sj}^{-2}}{n_i} \left[\pi_{sij}(1 - \pi_{sij}) + \frac{(1 - M_i)(1 - P_i)\{1 - (1 - M_i)(1 - P_i)\}}{\{2P_i - 1 + 2M_i(1 - P_i)\}^2} \right]; j = 1, 2, ..., p.$$

$$= \sum_{i=1}^{L} \frac{W_i^2 \, \pi_{sj}^{-2}}{n_i} \left[\pi_{sij} \left(1 - \pi_{sij} \right) + A_i \right]; \; j = 1, 2, ..., p. \tag{6}$$

where $$A_i = \frac{\left(1 - M_i\right)\left(1 - P_i\right)\left\{1 - \left(1 - M_i\right)\left(1 - P_i\right)\right\}}{\left\{2P_i - 1 + 2M_i \left(1 - P_i\right)\right\}^2}; i = 1, 2, \ldots, L. \tag{7}$$

The Multi-objective Integer Nonlinear Programming Problem (MINLPP) with linear cost constraint is given as (see Ghufran *et al*. (2013, 2014) :

Problem 8:

$$\text{Minimize} \quad \begin{pmatrix} (CV)_{s1}^2 \\ . \\ . \\ . \\ (CV)_{sp}^2 \end{pmatrix}$$

Subject to

$$\sum_{i=1}^{L} c_i n_i + c_0 \leq C,$$

$$\text{and} \quad 2 \leq n_i \leq N_i, \; n_i \text{ are integers}; \; i = 1, 2, \ldots, L.$$

When the travel cost between the units is substantial, the cost constraint also becomes non-linear then problem (8) define as problem (9)

Problem 9:

$$\text{Minimize} \quad \begin{pmatrix} (CV)_{s1}^{'2} \\ . \\ . \\ . \\ (CV)_{sp}^{'2} \end{pmatrix}$$

Subject to

$$\sum_{i=1}^{L} c_i^{'} n_i^{'} + \sum_{i=1}^{L} t_i^{'} \sqrt{n_i^{'}} + c_0 \leq C,$$

$$\text{and} \quad 2 \leq n_i \leq N_i, n_i \text{ are integers}; \; i = 1, 2, \ldots, L.$$

3. Solution Methods for *RR* Model

In this section we are given the procedures for solving the *RR* model problem by using various approaches of multi-objective optimization namely, Goal programming, Lexicographic goal programming, D_1- Distances and the proposed approach.

3.1 Goal Programming

The goal programming is based on the basic idea to determine a feasible solution that minimizes the deviations from the goals. This optimisation programming technique is used to handle multiple, normally conflicting objectives. The use of the goal programming for decision making problems with several conflicting objectives was first introduced by Charnes and Cooper in 1961. Thereafter various versions of goal programming have been proposed in the literature. Here we use goal programming technique to solve the Randomise Response problem (Problem 8). For this purpose we first solve separately the following *p* given objectives of problem 8 subject to the given set of constraints of Problem (8) to obtain the individual optimum solution.

Let $n_i^* = (n_1^*, n_2^*, ..., n_L^*)$ denote the individual best solution to the Problem (8) with CV_j^* as the best individual objective function value where $j=1, 2,..,p$. Further let $n_c^* = (n_{1c}^*, n_{2c}^*, ..., n_{Lc}^*)$ denote the optimal compromise allocation with objective functions values CV_{jc}^* under compromise allocation.

Obviously, $CV_{jc}^* \geq CV_j^*$ or $CV_{jc}^* - CV_j^* \geq 0 \forall\ j = 1,2,..., p$

Here we define a deviational variables $\delta_j = CV_{jc}^* - CV_j^*$, where δ_j is the deviational between CV_{jc}^* *and* CV_j^*.

The goal is to find the compromise ordered quantity such that the deviations in the net price, rejected units and late delivered units due to the use compromise quantity ordered should not exceed $\delta_j \geq 0,\ j = 1,2,..., p$ and $\sum_{j=1}^{p} \delta_j$ is minimum.

Finally, the Problem (8) can be written in the form of goal programming problem as

Problem 10:

$$Minimize \ \sum_{j=1}^{P} \delta_j$$

$$\begin{aligned} Subject\ to;\ & CV_1 - \delta_1 \le CV_1^* \\ & \vdots \\ & CV_{j-1} - \delta_{j-1} \le CV_{j-1}^* \\ & CV_j - \delta_j \le CV_j^* \\ & \sum_{i=1}^{L} c_i n_i + c_0 \le C, \\ and\ \ & 2 \le n_i \le N_i,\ n_i\ are \text{ integers}; i = 1, 2, \ldots, L \end{aligned}$$

(Same procedure will be followed for solving the Problem 9).

3.2 Lexicographic Goal Programming

Lexicographic goal programming is a special case of goal programming, in which the most important goals are optimised before the least important goals. Since the different objectives have different importance, we arrange them in lexicographic order according to their importance. Here we consider the Randomise Response problem of sampling (Problem 8) with P objectives functions those having different priority levels. Here $P!$ priorities structure can be made.

Let suppose if highest priority is given to the characteristic which has the maximum coefficient of variation $i.e. \left(CV_{j1}, CV_{j2}, \ldots CV_{jG}\right)$, be in decreasing order of magnitude. Lexicographic goal programming approach requires solving first

Problem 11:

$$Minimize \quad CV_{j1}$$

$$\begin{aligned} Subject\ to;\ & \sum_{i=1}^{L} c_i n_i + c_0 \le C,\ , \\ and\ \ & 2 \le n_i \le N_i,\ n_i\ are\ \mathrm{int}\, egers; i = 1, 2, \ldots, L \end{aligned}$$

If the minimum of problem (26) is CV_{j1}^*, then in the next stages the problem must be solved for obtaining the minimum values $CV_{j2}^*, .., CV_{jG-1}^*$. At the stage G, δ_{jG-1} denotes the deviational variable and the problem to be solved is

Problem 12:

$$\begin{aligned}
&\text{Minimize}\quad CV_{jG} + \sum_{j=1}^{P} \delta_{jG-1}\\
&\text{Subject to};\ CV_{j1} - \delta_{j} \le CV_{j1}^{*}\\
&\qquad\qquad \ldots\\
&\qquad CV_{jG-1} - \delta_{jG-1} \le CV_{jG-1}^{*}\\
&\qquad \sum_{i=1}^{L} c_i n_i + c_0 \le C, \ ,\\
&\text{and}\quad 2 \le n_i \le N_i,\ n_i \ \text{are integers};\ i = 1, 2, \ldots, L
\end{aligned}$$

In the above Problem (12) the highest priority goals and constraints are considered first. If more than one solution is found for Problem (12), another goal programming problem is then formulated which takes into account the second priority goals and so on. This procedure is repeated until a unique solution is found gradually considering decreasing priority levels. If the minimum of problem (12) is CV_{jG}^{*}, then in the next stage the problem must be solved for minimum values. Thus, final next problem to be solved is

Problem (13):

$$\begin{aligned}
&\text{Minimize}\quad \sum_{j=1}^{P} \delta_{jG}\\
&\text{Subject to};\ CV_{j1} - \delta_{j} \le CV_{j1}^{*}\\
&\qquad\qquad \vdots\\
&\qquad CV_{jG-1} - \delta_{jG-1} \le CV_{jG-1}^{*}\\
&\qquad CV_{jG} - \delta_{jG} \le CV_{jG}^{*}\\
&\qquad \sum_{i=1}^{L} c_i n_i + c_0 \le C,\\
&\text{and}\quad 2 \le n_i \le N_i,\ n_i \ \text{are integers};\ i = 1, 2, \ldots, L
\end{aligned}$$

For the other different priority structure same procedure will be followed.

Same procedure will be followed for the Problem (9).

3.3 D_1 Distance Algorithm

This method is an extension of lexicographic goal programming. In this method the objectives functions are arranged in order of their priorities in different manners to generate set of priorities structures. An idle solution is then identified from these set of priorities structure.

The stepwise procedure of D_1- Distance method for solving *RR* Model (Problem 8) with *P* objective functions is as follows:

Step 1: Let us we have *P* objective functions then *P!* set of problems of different priority structure will generate and hence *P!* different solutions are obtained after solving *P!* problems.

Table 1 Calculations for ideal solutions

Priority Structure	n_1	n_2	…	n_i
$CV^{(1)}$	$n_1^{(1)}$	$n_2^{(1)}$	…	$n_i^{(1)}$
$CV^{(2)}$	$n_1^{(2)}$	$n_2^{(2)}$	…	$n_i^{(2)}$
…	…	…	…	…
$CV^{(r)}$	$n_1^{(r)}$	$n_2^{(r)}$	…	$n_i^{(r)}$
Ideal Solution	n_1^*	n_2^*	…	n_i^*

Step 2: Let $n_i^{(r)} = (n_1^{(r)}, n_2^{(r)}, ..., n_L^{(r)}),\ 1 \le r \le P!$ be the *P!* number of solutions obtained in step 1. Out of these solutions an idle is identified as follows:-

$n_i^* = \left\{ Max(n_1^{(1)}, n_1^{(2)} ..., n_1^{(r)}), Max(n_2^{(1)}, n_2^{(2)} ..., n_2^{(r)}),..., Max(n_i^{(1)}, n_i^{(2)} ..., n_i^{(r)}) \right\} = \left\{ n_1^*, n_2^*, ..., n_i^* \right\}$ But in practice ideal solution can never be achieved. The solution, which is closest to the ideal solution is acceptable as the best compromise solution, and the corresponding priority structure in the planning context.

Step 3: To obtain the best compromise solution, the following procedure is to be followed -

First we define a distance function to obtain the distances of solutions from ideal solution and the solution with minimum distance is considered as optimal solution. Let the D_1-distance from the ideal solution $(n_1^*, n_2^*, ..., n_L^*)$, to the $r-th$ solution $\left\{ n_1^{(r)}, n_2^{(r)}, ..., n_L^{(r)} \right\}$, $1 \le r \le P!$ is defined as

$$(D_1)^r = \sum_{i=1}^{L} \left| n_i^* - n_i^{(r)} \right|$$

Therefore, the optimal D_1- distance from the ideal solution is given as

$$(D_1)_{opt} = \underset{1 \le r \le P!}{Min} (D_1)^r = \underset{P!}{Min} \sum_{i=1}^{L} \left| n_i^* - n_i^{(r)} \right|$$

Table 2: D_1-Distances from the ideal solution

P.S.	n_1	…	n_i	$(D_1)^r$
$CV^{(1)}$	$\vert n_1^* - n_1^{(1)} \vert$	…	$\vert n_i^* - n_i^{(1)} \vert$	$\sum_{i=1}^{k} \vert n_i^* - n_i^{(1)} \vert$
$CV^{(2)}$	$\vert n_1^* - n_1^{(2)} \vert$	…	$\vert n_i^* - n_i^{(2)} \vert$	$\sum_{i=1}^{k} \vert n_i^* - n_i^{(2)} \vert$
…	…	…	…	…
$CV^{(r)}$	$\vert n_1^* - n_1^{(r)} \vert$	…	$\vert n_i^* - n_i^{(r)} \vert$	$\sum_{i=1}^{k} \vert n_i^* - n_i^{(r)} \vert$

Let the minimum be attained for $r = p$. Then $\{n_1^{(p)}, n_2^{(p)}, ..., n_i^{(p)}\}$ is the best compromise allocation for the given problem.

(Same procedure will be followed for solving the Problem 9).

4. Proposed Method (Fixed priority Ideal D_1 - Distances Method)

In this method the priorities of extremes are fixed *i.e.* in our case we put the first priority to the objective which has worst value (maximum value) and give last priority to the objective which has best objective value (minimum value) and the rest (*P-2*) objectives are solved under all possible combination in between these two fixed extreme priorities. Out of these solutions, an ideal solution is identified. Let $n_i^{(r)} = \{n_1^{(r)}, n_2^{(r)}, ..., n_L^{(r)}\}, 1 \le r \le (P-2)!$ be the $(P-2)!$ number of solutions obtained by giving priorities to $(P-2)$ objective functions. As explained above, we will obtain $(P-2)!$ different solutions by solving the $(P-2)!$ problems arising for $(P-2)!$ different priority structures. The stepwise procedure for solving the problem of *RR* model is given below -

Step1: Solve all the objectives for the given set of constraints ignoring other objectives.

Step2: Fix the first priority to the objective having worst value and last priority to the objective having best value.

Step3: Rest priorities are given to other objectives subsequently.

Step4: Obtain $(P-2)!$ different solutions by solving the $(P-2)!$ problems.

Step5: Obtain Ideal solution

Step6: Calculate D_1-distances of different solutions from the ideal solution.

Step7: The best compromise solution can be achieved after solving the following goal programming problem defined as

$$\underset{1\le r\le (P-2)!}{Min} \quad \sum_{i=1}^{k}\left(d_{ir}^{+}, d_{ir}^{-}\right)$$

$$\text{subject to } n_i^{*} - n_i^{(r)} + d_{ir}^{-} - d_{ir}^{+} = 0,$$

$$and \;\; d_{ir}^{+} \ge 0, d_{ir}^{-} \ge 0,$$

$$1 \le r \le (P-2)!, \;\; i = 1, \ldots, L \;\; and \, \text{int}\, eger.$$

where d_{ir}^{+} and d_{ir}^{-} are the under and over deviational variable respectively,

(Same procedure will be followed for solving the Problem 9).

5. Numerical illustrations

The following data is taken from Ghufran *et al.* (2012). The population size N is assumed to be 1,000 and divided into four strata. This gives $N_1 = 81$, $N_2 = 343$, $N_3 = 455$ and $N_4 = 121$. Let the mean population proportion of respondents belonging to the sensitive group for the four characteristics is assumed to be $\pi_{s1} = 0.842$, $\pi_{s2} = 0.924$, $\pi_{s3} = 0.654$ and $\pi_{s4} = 0.825$. It is also assumed that the total budget of the survey $C = 4{,}500$ units with an overhead cost $c_0 = 500$ units. Thus $C_0 = (C - c_0) = 4{,}500 - 500 = 4{,}000$ units are available for measurements or measurements and travelling as the case may be. Table 3 presents the relevant information. A_i *'s* are calculated by equation (7) using the values given in Table 3 as $A_1 = 0.08387864, A_2 = 0.08387864, A_3 = 0.08387864$ and $A_4 = 0.08387864.$

Table 3: Data for four Strata and four characteristics

I	W_i	π_{si1}	π_{si2}	π_{si3}	π_{si4}	M_i	P_i	Travel cost is not significant	Travel cost is significant	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	c_i	c_i'	t_i'
1	0.0808	0.28	0.33	0.40	0.62	0.80	0.70	25	15	10
2	0.3434	0.48	0.53	0.35	0.22	0.80	0.70	33	20	13
3	0.4546	0.68	0.73	0.55	0.82	0.80	0.70	40	30	10
4	0.1212	0.88	0.93	0.75	0.32	0.80	0.70	30	18	12

6. Comparisons of Optimum allocation

In this section, a comparative study of the optimum allocations in table 4 and 5 have been given for the various existing approaches and proposed approach.

Table 4: Summary of Results for Linear Cost Function

Approaches	n_1	n_2	n_3	n_4
Value Function Approach	12	40	49	14
Goal Programming Technique	12	40	49	14
ε - constraint method	12	43	48	12
Distance based Method	9	42	51	11
Khuri and Cornell	12	42	48	13
Proposed Approach	12	40	49	14

Table 5: Summary of Results for Non-Linear Cost Function

Approaches	n_1	n_2	n_3	n_4
Value Function Approach	14	51	56	17
Goal Programming Technique	15	52	54	18
ε - constraint method	13	50	58	16
Distance based Method	15	57	49	19
Khuri and Cornell	18	58	45	22
Proposed Approach	11	65	63	22

The trace value of coefficient of variation for linear cost and quadratic cost function is summarised in Table 6 and Table 7 respectively.

Table 6: Summary of trace values of coefficient of variation for linear cost function

Technique	$(CV)^2_{s1}$	$(CV)^2_{s2}$	$(CV)^2_{s3}$	$(CV)^2_{s4}$	Trace Value
Value function	0.0009199204	0.0007288042	0.001622887	0.004042650	0.007314262
Goal programming	0.003679677	0.002915213	0.006491542	0.003261107	0.01634754
$\varepsilon-$ constraint	0.003667031	0.002894428	0.006520836	0.003291245	0.01637354
Distance based	0.003692815	0.002920744	0.006580085	0.003364038	0.01655768
Khuri & Cornell	0.003672579	0.002903462	0.006508626	0.003274046	0.01635871
Proposed method	0.003679677	0.002915213	0.006491542	0.003261107	0.01634754

Table 7: Summary of trace values of coefficient of variation for quadratic cost

Technique	$(CV)^{'2}_{s1}$	$(CV)^{'2}_{s2}$	$(CV)^{'2}_{s3}$	$(CV)^{'2}_{s4}$	Trace Value
Value function	0.003075535	0.002433581	0.005443718	0.001594991	0.01254782
Goal programming	0.003087475	0.002441807	0.005463369	0.001547158	0.01253966
$\varepsilon-$ constraint	0.003071491	0.002431509	0.005439352	0.001648332	0.01259068
Distance based	0.003125137	0.002468594	0.005552044	0.001478469	0.01262424
Khuri & Cornell	0.003233743	0.002550407	0.005736522	0.001385358	0.01290603
Proposed method	0.003075604	0.002433638	0.005443821	0.001595042	0.01254810

7. Conclusion

Ghufran et al. (2014) was solved the RR model problem for minimizing the characteristic variances. In this paper, instead of characteristics variances we taken into account the CV's and formulate the *RRM* as a multi-objective optimization problem. The compromise allocation is then obtained by the propose technique lexicographic goal programming approach integrated with fix priority D_1 - distance method. We also obtained compromise allocations by the various other existing methods of multi-objective optimization problem and then made comparisons of them with the lexicographic goal programming approach integrated with fix priority D_1 - distance method. We have been found that minimum trace value of coefficient of variation (CV) for linear cost is attained by the proposed fixed priority Ideal D_1 - distance method. In case of quadratic cost constraint function the minimum trace value of CV is attained by value function method.

Conflict of Interests

The authors declare that there is no conflict of interests.

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