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SYMMETRIC BIDIRECTIONAL QUANTUM TELEPORTATION VIA FIVE-QUBIT CLUSTER STATE

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Abstract. We propose a protocol of symmetric bidirectional quantum teleportation via five-qubit cluster state. Based on the Bell-state measurements, introduces an auxiliary particle and perform joint unitary transformation, Alice wants to transmit an arbitrary two-qubit entangled state to Bob and Bob wants to transmit an arbitrary two-qubit state to Alice.

Keywords: symmetric bidirectional quantum teleportation; five-qubit cluster state; two-qubit entangled state.

2010 AMS Subject Classification: 81P45.

1. Introduction

Bidirectional quantum teleportation is a new subject in the research of quantum information. Until now, many schemes have been proposed. In 2013, applying cluster state as a quantum channel, the first bidirectional quantum teleportation protocol was reported by Zha et al.[1]. In

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2013, Li et al.[2] introduced a bidirectional controlled teleportation scheme by using a five-qubit composite GHZ-Bell state. some bidirectional quantum teleportation protocols have been investigated by using different states as quantum channel[3-10], such as six-qubit cluster state, genuine six-qubit entangled state, five-qubit entangled state, maximally seven-qubit entangled state and nine-qubit entangled state. Up to now, many teleportation and controlled teleportation schemes have been reported[11-18], but symmetric bidirectional quantum teleportation has not yet been presented.

In this paper, we propose a scheme of symmetric bidirectional quantum teleportation via five-qubit cluster state. Suppose that Alice has two particles A and B in an unknown state, she wants to transmit the state of particles A and B to Bob; at the same time, Bob has two particles C and D in an unknown state, he wants to transmit the state of particles C and D to Alice.

2. Symmetric Bidirectional Quantum Teleportation

Our scheme can be described as follows. Suppose Alice has qubits A and B in an arbitrary entangled state,which is described by

$$|\Psi\rangle_{AB} = a_0|00\rangle + a_1|11\rangle, \tag{1}$$

where $|a_0|^2 + |a_1|^2 = 1$, and Bob has qubits C and D in the following state,

$$|\Psi\rangle_{CD} = b_0|00\rangle + b_1|11\rangle, \tag{2}$$

where $|b_0|^2 + |b_1|^2 = 1$.

Now Alice wants transmit the state of qubits A and B to Bob and Bob wants to transmit the qubits C and D to Alice. Assume that Alice and Bob share a five-qubit cluster state, which has the form

$$|C_5\rangle_{12345} = \frac{1}{2}(|00000\rangle + |00111\rangle + |11101\rangle + |11010\rangle)_{12345}, \tag{3}$$

the qubits A, B, 1 and 5 belong to Alice and qubits C, D, 2, 3 and 4 belong to Bob, respectively. The initial state of the total system can be shown as

$$
\begin{aligned}
|\Psi\rangle_{12345ABCD} &= |\Psi\rangle_{AB}\otimes|\Psi\rangle_{CD}\otimes|C_5\rangle_{12345}\\
&= \frac{1}{4}[|\Phi^{\pm}\rangle_{A1}|\Phi^{\pm}\rangle_{C4}|\chi^1\rangle_{BD235}+|\Phi^{\pm}\rangle_{A1}|\Psi^{\pm}\rangle_{C4}|\chi^2\rangle_{BD235}\\
&\quad+|\Psi^{\pm}\rangle_{A1}|\Phi^{\pm}\rangle_{C4}|\chi^3\rangle_{BD235}+|\Psi^{\pm}\rangle_{A1}|\Psi^{\pm}\rangle_{C4}|\chi^4\rangle_{BD235}]
\end{aligned}
\tag{4}
$$

where

$$|\chi^1\rangle_{BD235} = (a_0b_0|00000\rangle \pm_1 a_0b_1|01011\rangle \pm_2 a_1b_0|10111\rangle \pm_1 \pm_2 a_1b_1|11100\rangle), \tag{5}$$

$$|\chi^2\rangle_{BD235} = (a_0b_0|00011\rangle \pm_1 a_0b_1|01000\rangle \pm_2 a_1b_0|10100\rangle \pm_1 \pm_2 a_1b_1|11111\rangle), \tag{6}$$

$$|\chi^3\rangle_{BD235} = (a_0b_0|00111\rangle \pm_1 a_0b_1|01100\rangle \pm_2 a_1b_0|10000\rangle \pm_1 \pm_2 a_1b_1|11011\rangle), \tag{7}$$

$$|\chi^4\rangle_{BD235} = (a_0b_0|00100\rangle \pm_1 a_0b_1|01111\rangle \pm_2 a_1b_0|10011\rangle \pm_1 \pm_2 a_1b_1|11000\rangle), \tag{8}$$

where $|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ and $|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ denote Bell basis. In the above equations, the notes $\pm_1$ correspond to the Bell-state measurements of qubit pairs $(C,4)$ and the notes $\pm_2$ correspond to the Bell-state measurements of qubit pairs $(A,1)$ in the basis of $\{|\Phi^{\pm}\rangle, |\Psi^{\pm}\rangle\}$, respectively, and they mean multiplication of $\pm$ signs.

Alice firstly perform a Bell-state measurement on qubit pairs $(A,1)$ and Bob perform a Bell-state measurement on qubit pairs $(C,4)$, respectively. Then they announce their results to each other via classical channel.

In order to simplify our descriptions, without loss of generality, we take the outcome $|\Phi^+\rangle_{A1}$, $|\Phi^+\rangle_{C4}$ as an example to show the principle of this symmetric bidirectional quantum teleportation protocol, where the residual qubits system collapses into the states,

$$|\varphi\rangle_{BD235} = (a_0b_0|00000\rangle + a_0b_1|01011\rangle + a_1b_0|10111\rangle + a_1b_1|11100\rangle)_{BD235}. \tag{9}$$

Alice takes X-basis measurement on qubit B and conveys her result to Bob, where X-basis is orthogonal basis including vectors,

$$\begin{Bmatrix} |+\rangle = |0\rangle + |1\rangle \\ |-\rangle = |0\rangle - |1\rangle \end{Bmatrix} \tag{10}$$

If Alice's measurement result is $|+\rangle_B$, then the state of the remaining qubits D, 2, 3 and 5 collapse into the state as

$$|\varphi^1\rangle_{D235} = (a_0b_0|0000\rangle + a_0b_1|1011\rangle + a_1b_0|0111\rangle + a_1b_1|1100\rangle)_{D235}. \tag{11}$$

On the other hand, Bob takes X-basis measurement on qubit D and conveys his result to Alice. If Bob's measurement result is $|+\rangle_D$, then the state of the remaining qubits 2, 3 and 5 collapse into the state as

$$|\varphi^2\rangle_{235} = (a_0b_0|000\rangle + a_0b_1|011\rangle + a_1b_0|111\rangle + a_1b_1|100\rangle)_{235}. \tag{12}$$

Also, Alice introduces an auxiliary particle 6 with an initial state$|0\rangle_6$, then the state $|\varphi^2\rangle_{235}$ becomes the following state,

$$|\varphi^3\rangle_{2356} = (a_0b_0|0000\rangle + a_0b_1|0110\rangle + a_1b_0|1110\rangle + a_1b_1|1000\rangle)_{2356}. \tag{13}$$

Then Alice and Bob perform joint unitary transformation on particle 2, 3, 5, 6. In order to reincarnate the original state under the basis $\{|0000\rangle_{2356}, |0001\rangle_{2356}, |0010\rangle_{2356}, |0011\rangle_{2356}, |0100\rangle_{2356}, |0101\rangle_{2356}, |0110\rangle_{2356}, |0111\rangle_{2356}, |1000\rangle_{2356}, |1001\rangle_{2356}, |1010\rangle_{2356}, |1011\rangle_{2356}, |1100\rangle_{2356}, |1101\rangle_{2356}, |1110\rangle_{2356}, |1111\rangle_{2356}\}$, the unitary transformation (a 16×16 matrix) may take the form

$$U_{1.1}=\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0
\end{pmatrix},$$

Having performed the joint unitary transformation, the state $|\varphi^3\rangle_{2356}$ becomes the following state,

$$\begin{aligned}|\phi\rangle_{2356} &= (a_0b_0|0000\rangle + a_0b_1|0011\rangle + a_1b_0|1100\rangle + a_1b_1|1111\rangle)\\ &= (a_0|00\rangle + a_1|11\rangle)_{23}\otimes(b_0|00\rangle + b_1|11\rangle)_{56}\end{aligned} \tag{14}$$

.

After doing those operations, the symmetric bidirectional quantum teleportation is successfully realized. And other cases are given in Table 1 and 2, where Unitary transformation is given in Table 3.

So the total successful probability is equal to one.

3. Conclusion and discussion

In conclusion, we have proposed a theoretical scheme for symmetric bidirectional quantum teleportation. In our scheme, five-qubit cluster state is considered as the quantum channel, while Alice and Bob are not noly senders but also receivers. In this paper, Alice has two particles A and B in an unknown state, she wants to transmit the state of particles A and B to Bob; at the same time, Bob has two particles C and D in an unknown state, he wants to transmit the state of particles C and D to Alice. We hope that such a quantum teleportation scheme can be realized experimentally in the future.

Conflict of Interests

The authors declare that there is no conflict of interests.

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REFERENCES

[1] Zha X W, Zou Z C, Qi J X, et al. Bidirectional Quantum Controlled Teleportation via Five-Qubit Cluster State. Int. J. Theor. Phys. 52 (6) (2013):1740-1744.

[2] Li Y H, Li X L, Sang M H, et al. Bidirectional controlled quantum teleportation and secure direct communication using five-qubit entangled state. Quantum Inf. Process. 12 (12) (2013): 3835-3844.

[3] Yan A. Bidirectional Controlled Teleportation via Six-Qubit Cluster State. Int. J. Theor. Phys. 52(11) (2013): 3870-3873

[4] Li Y H, Nie L P. Bidirectional Controlled Teleportation by Using a Five-Qubit Composite GHZ-Bell State. Int. J. Theor. Phys. 52(5)(2013): 1630-1634.

[5] Shukla C, Banerjee A, Pathak A. Bidirectional Controlled Teleportation by Using 5-Qubit States: A Generalized View. Int. J. Theor. Phys. 52(10)(2013): 3790-3796.

[6] Zhang D, Zha X W, Duan Y J. Bidirectional and Asymmetric Quantum Controlled Teleportation. Int. J. Theor. Phys. 54(5)(2015): 1-9.

[7] Duan Y J, Zha X W, Sun X M, et al. Bidirectional Quantum Controlled Teleportation via a Maximally Seven-qubit Entangled State. Int. J. Theor. Phys. 53(8)(2014): 2697-2707.

[8] Chen Y. Bidirectional Quantum Controlled Teleportation by Using a Genuine Six-qubit Entangled State. Int. J. Theor. Phys. , 54(1)(2015): 269-272.

[9] Sang M H. Bidirectional Quantum Teleportation by Using Five-qubit Cluster State[J]. Int. J. Theor. Phys. 55(3)(2016): 1333-1335.

[10] Li Y H, Jin X M. Bidirectional controlled teleportation by using nine-qubit entangled state in noisy environments. Quantum Inf. Process. 15(2)(2015): 929-945.

[11] Karlsson A, Bourennane M. Quantum teleportation using three-particle entanglement. Phys. Rev. A, 58(6)(1998): 4394-4400.

[12] Yang C P, Chu S I, Han S. Efficient many-party controlled teleportation of multiqubit quantum information via entanglement. Phys. Rev. A, 70(2)(2004): 022329.

[13] Deng F G, Li C Y, Li Y S, et al. Symmetric multiparty-controlled teleportation of an arbitrary two-particle entanglement, Phys. Rev. A, 72(2)(2005): 656-665.

[14] Man Z X, Xia Y J, An N B. Genuine multiqubit entanglement and controlled teleportation. Phys. Rev. A, 75(5)(2007): 497-500.

[15] Saitoh A, Rahimi R, Nakahara M. Economical, (k, m) -threshold controlled quantum teleportation. Phys. Rev. A, 79(6)(2009): 1989-1990.

[16] Chen Y. Bidirectional Quantum Controlled Teleportation by Using a Genuine Six-qubit Entangled State. Int. J. Theor. Phys. 54(1)(2015): 269-272.

[17] Nie Y Y, Hong Z H, Huang Y B. Non-Maximally Entangled Controlled Teleportation Using Four Particles Cluster States. Int. J. Theor. Phys. 48(5)(2009): 1485-1490.

[18] Ma Y H, Wu E. Macroscopic Entanglement of Remote Optomechanical Systems Assisted by Parametric Interactions. Int. J. Theor. Phys. 54(4)(2015): 1334-1341.

TABLE 1

Alice and Bob's results	State of particles 2, 3, 5 and 6	Unitary transformation
$\|\Phi^+\rangle_{A1}\|\Phi^+\rangle_{C4}\|+\rangle_B\|+\rangle_D$	$a_0b_0\|0000\rangle+a_0b_1\|0110\rangle+a_1b_0\|1110\rangle+a_1b_1\|1000\rangle$	$U_{1.1}$
$\|\Phi^+\rangle_{A1}\|\Phi^+\rangle_{C4}\|+\rangle_B\|-\rangle_D$	$a_0b_0\|0000\rangle-a_0b_1\|0110\rangle+a_1b_0\|1110\rangle-a_1b_1\|1000\rangle$	$U_{1.2}$
$\|\Phi^+\rangle_{A1}\|\Phi^+\rangle_{C4}\|-\rangle_B\|+\rangle_D$	$a_0b_0\|0000\rangle+a_0b_1\|0110\rangle-a_1b_0\|1110\rangle-a_1b_1\|1000\rangle$	$U_{1.3}$
$\|\Phi^+\rangle_{A1}\|\Phi^+\rangle_{C4}\|-\rangle_B\|-\rangle_D$	$a_0b_0\|0000\rangle-a_0b_1\|0110\rangle-a_1b_0\|1110\rangle+a_1b_1\|1000\rangle$	$U_{1.4}$
$\|\Phi^+\rangle_{A1}\|\Phi^-\rangle_{C4}\|+\rangle_B\|+\rangle_D$	$a_0b_0\|0000\rangle-a_0b_1\|0110\rangle+a_1b_0\|1110\rangle-a_1b_1\|1000\rangle$	$U_{1.2}$
$\|\Phi^+\rangle_{A1}\|\Phi^-\rangle_{C4}\|+\rangle_B\|-\rangle_D$	$a_0b_0\|0000\rangle+a_0b_1\|0110\rangle+a_1b_0\|1110\rangle+a_1b_1\|1000\rangle$	$U_{1.1}$
$\|\Phi^+\rangle_{A1}\|\Phi^-\rangle_{C4}\|-\rangle_B\|+\rangle_D$	$a_0b_0\|0000\rangle-a_0b_1\|0110\rangle-a_1b_0\|1110\rangle+a_1b_1\|1000\rangle$	$U_{1.4}$
$\|\Phi^+\rangle_{A1}\|\Phi^-\rangle_{C4}\|-\rangle_B\|-\rangle_D$	$a_0b_0\|0000\rangle+a_0b_1\|0110\rangle-a_1b_0\|1110\rangle-a_1b_1\|1000\rangle$	$U_{1.3}$
$\|\Phi^-\rangle_{A1}\|\Phi^+\rangle_{C4}\|+\rangle_B\|+\rangle_D$	$a_0b_0\|0000\rangle+a_0b_1\|0110\rangle-a_1b_0\|1110\rangle-a_1b_1\|1000\rangle$	$U_{1.3}$
$\|\Phi^-\rangle_{A1}\|\Phi^+\rangle_{C4}\|+\rangle_B\|-\rangle_D$	$a_0b_0\|0000\rangle-a_0b_1\|0110\rangle-a_1b_0\|1110\rangle+a_1b_1\|1000\rangle$	$U_{1.4}$
$\|\Phi^-\rangle_{A1}\|\Phi^+\rangle_{C4}\|-\rangle_B\|+\rangle_D$	$a_0b_0\|0000\rangle+a_0b_1\|0110\rangle+a_1b_0\|1110\rangle+a_1b_1\|1000\rangle$	$U_{1.1}$
$\|\Phi^-\rangle_{A1}\|\Phi^+\rangle_{C4}\|-\rangle_B\|-\rangle_D$	$a_0b_0\|0000\rangle-a_0b_1\|0110\rangle+a_1b_0\|1110\rangle-a_1b_1\|1000\rangle$	$U_{1.2}$
$\|\Phi^-\rangle_{A1}\|\Phi^-\rangle_{C4}\|+\rangle_B\|+\rangle_D$	$a_0b_0\|0000\rangle-a_0b_1\|0110\rangle-a_1b_0\|1110\rangle+a_1b_1\|1000\rangle$	$U_{1.4}$
$\|\Phi^-\rangle_{A1}\|\Phi^-\rangle_{C4}\|+\rangle_B\|-\rangle_D$	$a_0b_0\|0000\rangle+a_0b_1\|0110\rangle-a_1b_0\|1110\rangle-a_1b_1\|1000\rangle$	$U_{1.3}$
$\|\Phi^-\rangle_{A1}\|\Phi^-\rangle_{C4}\|-\rangle_B\|+\rangle_D$	$a_0b_0\|0000\rangle+a_0b_1\|0110\rangle+a_1b_0\|1110\rangle+a_1b_1\|1000\rangle$	$U_{1.1}$
$\|\Phi^-\rangle_{A1}\|\Phi^-\rangle_{C4}\|-\rangle_B\|-\rangle_D$	$a_0b_0\|0000\rangle-a_0b_1\|0110\rangle+a_1b_0\|1110\rangle-a_1b_1\|1000\rangle$	$U_{1.2}$
$\|\Phi^+\rangle_{A1}\|\Psi^+\rangle_{C4}\|+\rangle_B\|+\rangle_D$	$a_0b_0\|0110\rangle+a_0b_1\|0000\rangle+a_1b_0\|1000\rangle+a_1b_1\|1110\rangle$	$U_{2.1}$
$\|\Phi^+\rangle_{A1}\|\Psi^+\rangle_{C4}\|+\rangle_B\|-\rangle_D$	$a_0b_0\|0110\rangle-a_0b_1\|0000\rangle+a_1b_0\|1000\rangle-a_1b_1\|1110\rangle$	$U_{2.2}$
$\|\Phi^+\rangle_{A1}\|\Psi^+\rangle_{C4}\|-\rangle_B\|+\rangle_D$	$a_0b_0\|0110\rangle+a_0b_1\|0000\rangle-a_1b_0\|1000\rangle-a_1b_1\|1110\rangle$	$U_{2.3}$
$\|\Phi^+\rangle_{A1}\|\Psi^+\rangle_{C4}\|-\rangle_B\|-\rangle_D$	$a_0b_0\|0110\rangle-a_0b_1\|0000\rangle-a_1b_0\|1000\rangle+a_1b_1\|1110\rangle$	$U_{2.4}$
$\|\Phi^+\rangle_{A1}\|\Psi^-\rangle_{C4}\|+\rangle_B\|+\rangle_D$	$a_0b_0\|0110\rangle-a_0b_1\|0000\rangle+a_1b_0\|1000\rangle-a_1b_1\|1110\rangle$	$U_{2.2}$
$\|\Phi^+\rangle_{A1}\|\Psi^-\rangle_{C4}\|+\rangle_B\|-\rangle_D$	$a_0b_0\|0110\rangle+a_0b_1\|0000\rangle+a_1b_0\|1000\rangle+a_1b_1\|1110\rangle$	$U_{2.1}$
$\|\Phi^+\rangle_{A1}\|\Psi^-\rangle_{C4}\|-\rangle_B\|+\rangle_D$	$a_0b_0\|0110\rangle-a_0b_1\|0000\rangle-a_1b_0\|1000\rangle+a_1b_1\|1110\rangle$	$U_{2.4}$
$\|\Phi^+\rangle_{A1}\|\Psi^-\rangle_{C4}\|-\rangle_B\|-\rangle_D$	$a_0b_0\|0110\rangle+a_0b_1\|0000\rangle-a_1b_0\|1000\rangle-a_1b_1\|1110\rangle$	$U_{2.3}$
$\|\Phi^-\rangle_{A1}\|\Psi^+\rangle_{C4}\|+\rangle_B\|+\rangle_D$	$a_0b_0\|0110\rangle+a_0b_1\|0000\rangle-a_1b_0\|1000\rangle-a_1b_1\|1110\rangle$	$U_{2.3}$
$\|\Phi^-\rangle_{A1}\|\Psi^+\rangle_{C4}\|+\rangle_B\|-\rangle_D$	$a_0b_0\|0110\rangle-a_0b_1\|0000\rangle-a_1b_0\|1000\rangle+a_1b_1\|1110\rangle$	$U_{2.4}$
$\|\Phi^-\rangle_{A1}\|\Psi^+\rangle_{C4}\|-\rangle_B\|+\rangle_D$	$a_0b_0\|0110\rangle+a_0b_1\|0000\rangle+a_1b_0\|1000\rangle+a_1b_1\|1110\rangle$	$U_{2.1}$
$\|\Phi^-\rangle_{A1}\|\Psi^+\rangle_{C4}\|-\rangle_B\|-\rangle_D$	$a_0b_0\|0110\rangle-a_0b_1\|0000\rangle+a_1b_0\|1000\rangle-a_1b_1\|1110\rangle$	$U_{2.2}$
$\|\Phi^-\rangle_{A1}\|\Psi^-\rangle_{C4}\|+\rangle_B\|+\rangle_D$	$a_0b_0\|0110\rangle-a_0b_1\|0000\rangle-a_1b_0\|1000\rangle+a_1b_1\|1110\rangle$	$U_{2.4}$
$\|\Phi^-\rangle_{A1}\|\Psi^-\rangle_{C4}\|+\rangle_B\|-\rangle_D$	$a_0b_0\|0110\rangle+a_0b_1\|0000\rangle-a_1b_0\|1000\rangle-a_1b_1\|1110\rangle$	$U_{2.3}$
$\|\Phi^-\rangle_{A1}\|\Psi^-\rangle_{C4}\|-\rangle_B\|+\rangle_D$	$a_0b_0\|0110\rangle-a_0b_1\|0000\rangle+a_1b_0\|1000\rangle-a_1b_1\|1110\rangle$	$U_{2.2}$
$\|\Phi^-\rangle_{A1}\|\Psi^-\rangle_{C4}\|-\rangle_B\|-\rangle_D$	$a_0b_0\|0110\rangle+a_0b_1\|0000\rangle+a_1b_0\|1000\rangle+a_1b_1\|1110\rangle$	$U_{2.1}$

TABLE 2

Alice and Bob's results	State of particles 2, 3, 5 and 6	Unitary transformation
$\|\Psi^+\rangle_{A1}\|\Phi^+\rangle_{C4}\|+\rangle_B\|+\rangle_D$	$a_0b_0\|1110\rangle+a_0b_1\|1000\rangle+a_1b_0\|0000\rangle+a_1b_1\|0110\rangle$	$U_{3.1}$
$\|\Psi^+\rangle_{A1}\|\Phi^+\rangle_{C4}\|+\rangle_B\|-\rangle_D$	$a_0b_0\|1110\rangle-a_0b_1\|1000\rangle+a_1b_0\|0000\rangle-a_1b_1\|0110\rangle$	$U_{3.2}$
$\|\Psi^+\rangle_{A1}\|\Phi^+\rangle_{C4}\|-\rangle_B\|+\rangle_D$	$a_0b_0\|1110\rangle+a_0b_1\|1000\rangle-a_1b_0\|0000\rangle-a_1b_1\|0110\rangle$	$U_{3.3}$
$\|\Psi^+\rangle_{A1}\|\Phi^+\rangle_{C4}\|-\rangle_B\|-\rangle_D$	$a_0b_0\|1110\rangle-a_0b_1\|1000\rangle-a_1b_0\|0000\rangle+a_1b_1\|0110\rangle$	$U_{3.4}$
$\|\Psi^+\rangle_{A1}\|\Phi^-\rangle_{C4}\|+\rangle_B\|+\rangle_D$	$a_0b_0\|1110\rangle-a_0b_1\|1000\rangle+a_1b_0\|0000\rangle-a_1b_1\|0110\rangle$	$U_{3.2}$
$\|\Psi^+\rangle_{A1}\|\Phi^-\rangle_{C4}\|+\rangle_B\|-\rangle_D$	$a_0b_0\|1110\rangle+a_0b_1\|1000\rangle+a_1b_0\|0000\rangle+a_1b_1\|0110\rangle$	$U_{3.1}$
$\|\Psi^+\rangle_{A1}\|\Phi^-\rangle_{C4}\|-\rangle_B\|+\rangle_D$	$a_0b_0\|1110\rangle-a_0b_1\|1000\rangle-a_1b_0\|0000\rangle+a_1b_1\|0110\rangle$	$U_{3.4}$
$\|\Psi^+\rangle_{A1}\|\Phi^-\rangle_{C4}\|-\rangle_B\|-\rangle_D$	$a_0b_0\|1110\rangle+a_0b_1\|1000\rangle-a_1b_0\|0000\rangle-a_1b_1\|0110\rangle$	$U_{3.3}$
$\|\Psi^-\rangle_{A1}\|\Phi^+\rangle_{C4}\|+\rangle_B\|+\rangle_D$	$a_0b_0\|1110\rangle+a_0b_1\|1000\rangle-a_1b_0\|0000\rangle-a_1b_1\|0110\rangle$	$U_{3.3}$
$\|\Psi^-\rangle_{A1}\|\Phi^+\rangle_{C4}\|+\rangle_B\|-\rangle_D$	$a_0b_0\|1110\rangle-a_0b_1\|1000\rangle-a_1b_0\|0000\rangle+a_1b_1\|0110\rangle$	$U_{3.4}$
$\|\Psi^-\rangle_{A1}\|\Phi^+\rangle_{C4}\|-\rangle_B\|+\rangle_D$	$a_0b_0\|1110\rangle+a_0b_1\|1000\rangle+a_1b_0\|0000\rangle+a_1b_1\|0110\rangle$	$U_{3.1}$
$\|\Psi^-\rangle_{A1}\|\Phi^+\rangle_{C4}\|-\rangle_B\|-\rangle_D$	$a_0b_0\|1110\rangle-a_0b_1\|1000\rangle+a_1b_0\|0000\rangle-a_1b_1\|0110\rangle$	$U_{3.2}$
$\|\Psi^-\rangle_{A1}\|\Phi^-\rangle_{C4}\|+\rangle_B\|+\rangle_D$	$a_0b_0\|1110\rangle-a_0b_1\|1000\rangle-a_1b_0\|0000\rangle+a_1b_1\|0110\rangle$	$U_{3.4}$
$\|\Psi^-\rangle_{A1}\|\Phi^-\rangle_{C4}\|+\rangle_B\|-\rangle_D$	$a_0b_0\|1110\rangle+a_0b_1\|1000\rangle-a_1b_0\|0000\rangle-a_1b_1\|0110\rangle$	$U_{3.3}$
$\|\Psi^-\rangle_{A1}\|\Phi^-\rangle_{C4}\|-\rangle_B\|+\rangle_D$	$a_0b_0\|1110\rangle-a_0b_1\|1000\rangle+a_1b_0\|0000\rangle-a_1b_1\|0110\rangle$	$U_{3.2}$
$\|\Psi^-\rangle_{A1}\|\Phi^-\rangle_{C4}\|-\rangle_B\|-\rangle_D$	$a_0b_0\|1110\rangle+a_0b_1\|1000\rangle+a_1b_0\|0000\rangle+a_1b_1\|0110\rangle$	$U_{3.1}$
$\|\Psi^+\rangle_{A1}\|\Psi^+\rangle_{C4}\|+\rangle_B\|+\rangle_D$	$a_0b_0\|1000\rangle+a_0b_1\|1110\rangle+a_1b_0\|0110\rangle+a_1b_1\|0000\rangle$	$U_{4.1}$
$\|\Psi^+\rangle_{A1}\|\Psi^+\rangle_{C4}\|+\rangle_B\|-\rangle_D$	$a_0b_0\|1000\rangle-a_0b_1\|1110\rangle+a_1b_0\|0110\rangle-a_1b_1\|0000\rangle$	$U_{4.2}$
$\|\Psi^+\rangle_{A1}\|\Psi^+\rangle_{C4}\|-\rangle_B\|+\rangle_D$	$a_0b_0\|1000\rangle+a_0b_1\|1110\rangle-a_1b_0\|0110\rangle-a_1b_1\|0000\rangle$	$U_{4.3}$
$\|\Psi^+\rangle_{A1}\|\Psi^+\rangle_{C4}\|-\rangle_B\|-\rangle_D$	$a_0b_0\|1000\rangle-a_0b_1\|1110\rangle-a_1b_0\|0110\rangle+a_1b_1\|0000\rangle$	$U_{4.4}$
$\|\Psi^+\rangle_{A1}\|\Psi^-\rangle_{C4}\|+\rangle_B\|+\rangle_D$	$a_0b_0\|1000\rangle-a_0b_1\|1110\rangle+a_1b_0\|0110\rangle-a_1b_1\|0000\rangle$	$U_{4.2}$
$\|\Psi^+\rangle_{A1}\|\Psi^-\rangle_{C4}\|+\rangle_B\|-\rangle_D$	$a_0b_0\|1000\rangle+a_0b_1\|1110\rangle+a_1b_0\|0110\rangle+a_1b_1\|0000\rangle$	$U_{4.1}$
$\|\Psi^+\rangle_{A1}\|\Psi^-\rangle_{C4}\|-\rangle_B\|+\rangle_D$	$a_0b_0\|1000\rangle-a_0b_1\|1110\rangle-a_1b_0\|0110\rangle+a_1b_1\|0000\rangle$	$U_{4.4}$
$\|\Psi^+\rangle_{A1}\|\Psi^-\rangle_{C4}\|-\rangle_B\|-\rangle_D$	$a_0b_0\|1000\rangle+a_0b_1\|1110\rangle-a_1b_0\|0110\rangle-a_1b_1\|0000\rangle$	$U_{4.3}$
$\|\Psi^-\rangle_{A1}\|\Psi^+\rangle_{C4}\|+\rangle_B\|+\rangle_D$	$a_0b_0\|1000\rangle+a_0b_1\|1110\rangle-a_1b_0\|0110\rangle-a_1b_1\|0000\rangle$	$U_{4.3}$
$\|\Psi^-\rangle_{A1}\|\Psi^+\rangle_{C4}\|+\rangle_B\|-\rangle_D$	$a_0b_0\|1000\rangle-a_0b_1\|1110\rangle-a_1b_0\|0110\rangle+a_1b_1\|0000\rangle$	$U_{4.4}$
$\|\Psi^-\rangle_{A1}\|\Psi^+\rangle_{C4}\|-\rangle_B\|+\rangle_D$	$a_0b_0\|1000\rangle+a_0b_1\|1110\rangle+a_1b_0\|0110\rangle+a_1b_1\|0000\rangle$	$U_{4.1}$
$\|\Psi^-\rangle_{A1}\|\Psi^+\rangle_{C4}\|-\rangle_B\|-\rangle_D$	$a_0b_0\|1000\rangle-a_0b_1\|1110\rangle+a_1b_0\|0110\rangle-a_1b_1\|0000\rangle$	$U_{4.2}$
$\|\Psi^-\rangle_{A1}\|\Psi^-\rangle_{C4}\|+\rangle_B\|+\rangle_D$	$a_0b_0\|1000\rangle-a_0b_1\|1110\rangle-a_1b_0\|0110\rangle+a_1b_1\|0000\rangle$	$U_{4.4}$
$\|\Psi^-\rangle_{A1}\|\Psi^-\rangle_{C4}\|+\rangle_B\|-\rangle_D$	$a_0b_0\|1000\rangle+a_0b_1\|1110\rangle-a_1b_0\|0110\rangle-a_1b_1\|0000\rangle$	$U_{4.3}$
$\|\Psi^-\rangle_{A1}\|\Psi^-\rangle_{C4}\|-\rangle_B\|+\rangle_D$	$a_0b_0\|1000\rangle-a_0b_1\|1110\rangle+a_1b_0\|0110\rangle-a_1b_1\|0000\rangle$	$U_{4.2}$
$\|\Psi^-\rangle_{A1}\|\Psi^-\rangle_{C4}\|-\rangle_B\|-\rangle_D$	$a_0b_0\|1000\rangle+a_0b_1\|1110\rangle+a_1b_0\|0110\rangle+a_1b_1\|0000\rangle$	$U_{4.1}$

TABLE 3

Unitary transformation	
$U_{1.1} = (u_{i,j})$	$u_{1,1} = u_{2,2} = u_{3,3} = u_{4,7} = u_{5,5} = u_{6,6} = u_{7,4} = u_{8,8} = u_{9,16} = u_{10,10}$ $= u_{11,11} = u_{12,12} = u_{13,15} = u_{14,14} = u_{15,13} = u_{16,9} = 1$
$U_{1.2} = (u_{i,j})$	$u_{1,1} = u_{2,2} = u_{3,3} = u_{5,5} = u_{6,6} = u_{7,4} = u_{8,8} = u_{9,16} = u_{10,10} = u_{11,11}$ $= u_{12,12} = u_{13,15} = u_{14,14} = u_{15,13} = 1; u_{4,7} = u_{16,9} = -1$
$U_{1.3} = (u_{i,j})$	$u_{1,1} = u_{2,2} = u_{3,3} = u_{4,7} = u_{5,5} = u_{6,6} = u_{7,4} = u_{8,8} = u_{9,16} = u_{10,10}$ $= u_{11,11} = u_{12,12} = u_{14,14} = u_{15,13} = 1; u_{13,15} = u_{16,9} = -1$
$U_{1.4} = (u_{i,j})$	$u_{1,1} = u_{2,2} = u_{3,3} = u_{5,5} = u_{6,6} = u_{7,4} = u_{8,8} = u_{9,16} = u_{10,10} = u_{11,11}$ $= u_{12,12} = u_{14,14} = u_{15,13} = u_{16,9} = 1; u_{4,7} = u_{13,15} = -1$
$U_{2.1} = (u_{i,j})$	$u_{1,7} = u_{2,2} = u_{3,3} = u_{4,1} = u_{5,5} = u_{6,6} = u_{7,4} = u_{8,8} = u_{9,13} = u_{10,10}$ $= u_{11,11} = u_{12,12} = u_{13,9} = u_{14,14} = u_{15,16} = u_{16,15} = 1$
$U_{2.2} = (u_{i,j})$	$u_{1,7} = u_{2,2} = u_{3,3} = u_{5,5} = u_{6,6} = u_{7,4} = u_{8,8} = u_{9,13} = u_{10,10} = u_{11,11}$ $= u_{12,12} = u_{13,9} = u_{14,14} = u_{15,16} = 1; u_{4,1} = u_{16,15} = -1$
$U_{2.3} = (u_{i,j})$	$u_{1,7} = u_{2,2} = u_{3,3} = u_{4,1} = u_{5,5} = u_{6,6} = u_{7,4} = u_{8,8} = u_{9,13} = u_{10,10}$ $= u_{11,11} = u_{12,12} = u_{14,14} = u_{15,16} = 1; u_{13,9} = u_{16,15} = -1$
$U_{2.4} = (u_{i,j})$	$u_{1,7} = u_{2,2} = u_{3,3} = u_{5,5} = u_{6,6} = u_{7,4} = u_{8,8} = u_{9,13} = u_{10,10} = u_{11,11}$ $= u_{12,12} = u_{14,14} = u_{15,16} = u_{16,15} = 1; u_{4,1} = u_{13,9} = -1$
$U_{3.1} = (u_{i,j})$	$u_{1,15} = u_{2,2} = u_{3,3} = u_{4,9} = u_{5,5} = u_{6,6} = u_{7,16} = u_{8,8} = u_{9,4} = u_{10,10}$ $= u_{11,11} = u_{12,12} = u_{13,1} = u_{14,14} = u_{15,13} = u_{16,7} = 1$
$U_{3.2} = (u_{i,j})$	$u_{1,15} = u_{2,2} = u_{3,3} = u_{5,5} = u_{6,6} = u_{7,16} = u_{8,8} = u_{9,4} = u_{10,10} = u_{11,11}$ $= u_{12,12} = u_{13,1} = u_{14,14} = u_{15,13} = 1; u_{4,9} = u_{16,7} = -1$
$U_{3.3} = (u_{i,j})$	$u_{1,15} = u_{2,2} = u_{3,3} = u_{4,9} = u_{5,5} = u_{6,6} = u_{7,16} = u_{8,8} = u_{9,4} = u_{10,10}$ $= u_{11,11} = u_{12,12} = u_{14,14} = u_{15,13} = 1; u_{13,1} = u_{16,7} = -1$
$U_{3.4} = (u_{i,j})$	$u_{1,15} = u_{2,2} = u_{3,3} = u_{5,5} = u_{6,6} = u_{7,16} = u_{8,8} = u_{9,4} = u_{10,10} = u_{11,11}$ $= u_{12,12} = u_{14,14} = u_{15,13} = u_{16,7} = 1; u_{4,9} = u_{13,1} = -1$
$U_{4.1} = (u_{i,j})$	$u_{1,9} = u_{2,2} = u_{3,3} = u_{4,15} = u_{5,5} = u_{6,6} = u_{7,13} = u_{8,8} = u_{9,4} = u_{10,10}$ $= u_{11,11} = u_{12,12} = u_{13,7} = u_{14,14} = u_{15,1} = u_{16,16} = 1$
$U_{4.2} = (u_{i,j})$	$u_{1,9} = u_{2,2} = u_{3,3} = u_{5,5} = u_{6,6} = u_{7,13} = u_{8,8} = u_{9,4} = u_{10,10} = u_{11,11}$ $= u_{12,12} = u_{13,7} = u_{14,14} = u_{16,16} = 1; u_{4,15} = u_{15,1} = -1$
$U_{4.3} = (u_{i,j})$	$u_{1,9} = u_{2,2} = u_{3,3} = u_{4,15} = u_{5,5} = u_{6,6} = u_{7,13} = u_{8,8} = u_{9,4} = u_{10,10}$ $= u_{11,11} = u_{12,12} = u_{14,14} = u_{16,16} = 1; u_{13,7} = u_{15,1} = -1$
$U_{4.4} = (u_{i,j})$	$u_{1,9} = u_{2,2} = u_{3,3} = u_{5,5} = u_{6,6} = u_{7,13} = u_{8,8} = u_{9,4} = u_{10,10} = u_{11,11}$ $= u_{12,12} = u_{14,14} = u_{15,1} = u_{16,16} = 1; u_{4,15} = u_{13,7} = -1$