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EXISTENCE OF EQUILIBRIUM POINTS IN THE MAGNETIC BINARY PROBLEM WITH VARIABLE MASS

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Abstract: The present paper deals with the existence of equilibrium points in the magnetic binary problem when the infinitesimal body is of variable mass. We have observed that there exists nine collinear and two non-collinear equilibrium points we have also observed that the mass reduction factor has a significant role on the existence of the equilibrium points.

Keywords: equilibrium points; magnetic binary problem; Jean's law; variable mass.

2010 AMS Subject Classification: 70P05.

1. Introduction

In 1928 Jeans [7] has studied the two-body problem with variable mass. Omarov [13] has also discussed the restricted problem of perturbed motion of two bodies with variable mass. Shrivastava and Ishwar [14] have studied the circular restricted three body problem with variable mass with the assumption that the mass of the infinitesimal body varies with respect to time. Singh

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and Ishwar [15] showed the effect of perturbation on the location and stability of the triangular equilibrium points in the restricted three-body problem. Lukyanov [8] discussed the stability of equilibrium points in the restricted three problem with variable mass. He found that for any set of parameters, all the equilibriums points in the problem (Collinear, Triangular and Coplanar) are stable with respect to the conditions considered in the Meshcherskii space-time transformation. Singh et al. [6] has discussed the non-linear stability of equilibrium points in the restricted three body with variable mass. They have also found that in non-linear sense, collinear points are unstable for all mass ratios and the triangular points are stable in the range of linear stability except for three mass ratios which depend upon, the constant due to the variation in mass governed by Jean's law. Jagdish Singh [5] discussed the photogravitational restricted three body problem with variable mass. M. R. Hassan et al. [4] has studied the existence of equilibrium points in the restricted three body problem with variable mass when the smaller primary is an oblate spheroid.

A. Mavragnais [9-12] and Mohd Arif [1-3] have studied the motion of a charge particle which is moving in the field of two rotating magnetic dipoles which are moving in the circular motion around their centre of mass in a uniform motion. In this article we have discussed the motion of a charged particle of variable mass which is moving in the field of two rotating magnetic dipoles.

2. Equation of motion

Two dipoles (the primaries), with magnetic fields move under the influence of gravitational forces and a charged particle P of charge q_1 and variable mass m moves in the vicinity of these dipoles. The question of the magnetic-binaries problem is to describe the motion of this particle. The equation of motion in the rotating coordinate system including the effect of the gravitational forces of the primaries on the charged particle P written as:

$$\ddot{x} + \frac{\dot{m}}{m}\ (\dot{x} - y) - 2\dot{y}\ f = -\frac{1}{m}\ U_x \tag{1}$$

$$\ddot{y} + \frac{\dot{m}}{m}\ (\dot{y} + x) + 2\dot{x}\ f = -\frac{1}{m}\ U_y \tag{2}$$

Where

$$f = 1 - \frac{1}{m}\left(\frac{1}{\rho_1^3} + \frac{\lambda}{\rho_2^3}\right), \quad U_x = \frac{\partial U}{\partial x} \quad \text{and} \quad U_y = \frac{\partial U}{\partial y} \tag{3}$$

$$U = -\frac{m}{2}(x^2+y^2) - (x^2+y^2)\left\{\frac{1}{\rho_1^3} + \frac{\lambda}{\rho_2^3}\right\} - x\left\{\frac{\mu}{\rho_1^3} - \frac{\lambda(1-\mu)}{\rho_2^3}\right\} - \frac{m\ (1-\mu)}{\rho_1} - \frac{m\ \mu}{\rho_2} \tag{4}$$

Here we assumed

1. Primaries participate in the circular motion around their centre of mass

2. Position vector of P at any time t be $\overline{\rho} = (xi + yj + zk)$ referred to a rotating frame of reference $\mathrm{O}(x, y, z)$ which is rotating with the same angular velocity $\overline{\omega}$ = (0, 0, 1) as those the primaries.

3. Initially the primaries lie on the x-axis.

4. The distance between the primaries as the unit of distance and the coordinate of one primary is (μ, 0, 0) then the other is ($\mu-1$, 0, 0).

5. The sum of their masses as the unit of mass. If mass of the one primaries μ then the mass of the other is ($1-\mu$).

6. The unit of time in such a way that the gravitational constant G has the value unity and

$q_1 = c$ where c is the velocity of light.

$\rho_1^2 = (x-\mu)^2 + y^2$, $\rho_2^2 = (x+1-\mu)^2 + y^2$, $\lambda = \frac{M_2}{M_1}$, ($M_1$, M_2 are the magnetic moments of the primaries which lies perpendicular to the plane of the motion).

The variation of mass of the charged particle P is given by (Jeans law)

$$\frac{dm}{dt} = -\alpha\ m^n \quad i.e \quad \frac{\dot{m}}{m} = -\alpha\ m^{n-1} \tag{5}$$

Where α is a constant coefficient and $n \ \epsilon\ [0.4,\ 4.4]$

Now introduce the space-time transformation as:

$$x = \xi\ \gamma^{-q}, \qquad y = \eta\ \gamma^{-q}, \qquad dt = \gamma^{-k} d\tau$$

$$\rho_1 = r_1 \gamma^{-q}, \qquad \rho_2 = r_2 \gamma^{-q}\ , \qquad \gamma = \frac{m}{m_0} < 1$$

Where m_0 is the mass of the charge particle at time $t = 0$.

Differentiating x and y with respect to t twice, we get

$$\dot{x} = \xi'\ \gamma^{k-q} + \beta\ q\ \xi\ \gamma^{n-q-1}, \qquad \dot{y} = \eta'\ \gamma^{k-q} + \beta\ q\ \eta\ \gamma^{n-q-1}$$

$$\ddot{x} = \xi''\ \gamma^{2k-q} + \beta\ \xi'\ (2q-k)\ \gamma^{n+k-q-1} - \beta^2 q\ \xi\ (n-q-1)\ \gamma^{2n-q-2},$$

$$\ddot{y} = \eta'' \ \gamma^{2k-q} + \ \beta \ \ \eta' \ \ (2q-k) \ \gamma^{n+k-q-1} - \beta^2 q \ \ \eta \ \ (n-q-1) \ \gamma^{2n-q-2}.$$

Where

$$\dot{\gamma} = \ \frac{\dot{m}}{m} = - \ \beta \ \gamma^{n-1}, \quad \beta = \ \alpha \ \ {m_0}^{n-1} = \text{ constant},$$

$$- \ \frac{1}{m} \ \ U_x = - \ \frac{1}{m} \ \frac{\partial U}{\partial \xi} \ \frac{\partial \xi}{\partial x} = - \ \frac{\gamma^{q-1}}{m_0} \ \frac{\partial U}{\partial \xi} \ , \quad - \ \frac{1}{m} \ \ U_y = - \ \frac{1}{m} \ \frac{\partial U}{\partial \eta} \ \frac{\partial \eta}{\partial y} = - \ \frac{\gamma^{q-1}}{m_0} \ \frac{\partial U}{\partial \eta}.$$

Putting the values of $\dot{x}, \ \dot{y}, \ \ddot{x}, \ \ddot{y}, \ U_x, \ U_y$ and $\frac{\dot{m}}{m}$ in equations (1) and (2) and after some simplification we get,

$$\xi'' + \beta \ \ \ \xi' \ \ \ (2q-k-1) \ \gamma^{n-k-1} - \beta^2 q \ \ \xi \ \ (n-q) \ \gamma^{2(n-k-1)}$$
$$- 2 \ \eta' \ \gamma^{-k} \left[1 - \frac{\gamma^{3q}}{\gamma \ \ m_0} \left\{\frac{1}{\mathrm{r}_1^3} \ + \ \ \frac{\lambda}{r_2^3}\right\}\right]$$
$$-\beta \ \ \eta \ \ \gamma^{\frac{n-q-1}{2k-q}} \left[1 - 2 \ \ q \left\{1 - \frac{\gamma^{3q}}{\gamma \ \ m_0} \left(\frac{1}{\mathrm{r}_1^3} \ + \ \ \frac{\lambda}{r_2^3}\right)\right\}\right] = - \ \frac{\gamma^{2q-2k-1}}{m_0} \ \frac{\partial U}{\partial \xi} \tag{6}$$

$$\eta'' + \beta \ \ \ \eta' \ \ \ (2q-k-1) \ \gamma^{n-k-1} - \beta^2 q \ \ \eta \ \ (n-q) \ \gamma^{2(n-k-1)} \qquad +2 \ \ \xi' \ \ \gamma^{-k} \left[1 - \frac{\gamma^{3q}}{\gamma \ \ m_0} \left\{\frac{1}{\mathrm{r}_1^3} \ + \ \ \frac{\lambda}{r_2^3}\right\}\right]$$
$$+\beta \ \ \xi \ \ \gamma^{\frac{n-q-1}{2k-q}} \left[1 - 2 \ \ q \left\{1 - \frac{\gamma^{3q}}{\gamma \ \ m_0} \left(\frac{1}{\mathrm{r}_1^3} \ + \ \ \frac{\lambda}{r_2^3}\right)\right\}\right] = - \ \frac{\gamma^{2q-2k-1}}{m_0} \ \frac{\partial U}{\partial \eta} \tag{7}$$

To eliminate the non-variational factor from equations (6) and (7) we assume

$$2q-k-1 = 0, \quad n-k-1 = 0, \quad n = 1, \ k = 0, \quad q = \ \frac{1}{2}, \quad \beta = \ \alpha.$$

Thus we have

$$\xi'' - 2 \ \ \eta' \ \ \left[1 - \frac{\sqrt{\gamma}}{m_0} \left\{\frac{1}{\mathrm{r}_1^3} \ + \ \ \frac{\lambda}{r_2^3}\right\}\right] = \ \frac{\beta^2 \ \ \xi}{4} - \frac{\beta \ \ \eta \ \ \gamma^{\frac{3}{2}}}{m_0} \ \left(\frac{1}{\mathrm{r}_1^3} \ + \ \ \frac{\lambda}{r_2^3}\right) \ - \ \frac{1}{m_0} \ \frac{\partial U}{\partial \xi} \tag{8}$$

$$\eta'' + 2 \ \ \xi' \ \ \left[1 - \frac{\sqrt{\gamma}}{m_0} \left\{\frac{1}{\mathrm{r}_1^3} \ + \ \ \frac{\lambda}{r_2^3}\right\}\right] = \ \frac{\beta^2 \eta}{4} + \ \frac{\beta \ \ \xi \ \ \gamma^{\frac{3}{2}}}{m_0} \ \left(\frac{1}{\mathrm{r}_1^3} \ + \ \ \frac{\lambda}{r_2^3}\right) - \ \frac{1}{m_0} \ \frac{\partial U}{\partial \eta} \tag{9}$$

Where

$$U \ \ = - \frac{m_0}{2} \ \ (\xi^2 \ + \ \eta^2) - (\xi^2 + \eta^2) \ \left\{\frac{1}{\mathrm{r}_1^3} \ + \ \ \frac{\lambda}{r_2^3}\right\} \gamma^{\frac{1}{2}} - \gamma \ \ \xi \ \ \left\{\frac{\mu}{\mathrm{r}_1^3} - \ \ \frac{\lambda(1-\mu)}{\mathrm{r}_2^3}\right\} - \gamma^{\frac{3}{2}} \left(\frac{m_0 \ \ (1-\mu)}{\mathrm{r}_1} + \frac{m_0 \ \ \mu}{\mathrm{r}_2}\right) \tag{10}$$

3. Existence of Equilibrium Points

The Equilibrium Points are the solution of

$$\frac{\beta^2\ \xi}{4} - \frac{\beta\ \eta\ \gamma^{\frac{3}{2}}}{m_0}\ \left(\frac{1}{\mathrm{r}_1^3}\ +\ \ \frac{\lambda}{r_2^3}\right)\ -\ \frac{1}{m_0}\ \frac{\partial U}{\partial \xi} = 0 \tag{11}$$

$$\frac{\beta^2 \eta}{4} + \ \frac{\beta\ \xi\ \gamma^{\frac{3}{2}}}{m_0}\ \left(\frac{1}{\mathrm{r}_1^3}\ +\ \ \frac{\lambda}{r_2^3}\right) - \ \frac{1}{m_0}\ \frac{\partial U}{\partial \eta} = 0 \tag{12}$$

The solution of equations (11) and (12) results the equilibrium points, this solution divided in two group those with $y = 0$, called the collinear equilibrium points and other are on xy-plane $(y \neq 0)$ called the non-collinear equilibrium points (ncep). For $\lambda > 0$ we found that there exist three collinear equilibrium points within the interval $\{-\infty, -(1-\mu)\}$, $\{-(1-\mu), \mu\}, (\mu, +\infty)$ which we denote by L_i, $(i = 1,2,3)$ respectively and two non-collinear equilibrium points.

Case I when $L_1 \in \{-\infty, -(1-\mu)\}$

The substitution $r_1 = \ \mu - \xi = \tau + 1,\ r_2 = -((1-\mu) + \ \xi) = \tau$ in equations (11) and (12), we have

$$\left(\frac{\alpha^2\ \xi}{4} - m_0\right)(\mu - \tau - 1)\ (\tau+1)^5\tau^5 + 2\ (\mu - \tau - 1)\ \ {}^{\frac{1}{2}}\{(\tau+1)^2\tau^5 + \lambda\ (\tau+1)^5\tau^2\} -$$
$$-\ 3\ (\mu - \tau - 1)^2\ \gamma^{\frac{1}{2}}\ \left[\left\{(\mu - \tau - 1) - \ \mu\ \ \gamma^{\frac{1}{2}}\right\}\ \tau^5\right.$$
$$\left. + \left\{(\mu - \tau - 1) + (1 - \ \mu\ )\ \gamma^{\frac{1}{2}}\right\}\ (\tau+1)^5\right] -$$
$$-3\gamma\ \ \left[\mu\left\{(\mu - \tau - 1) - \ \mu\ \ \gamma^{\frac{1}{2}}\right\}\ \tau^5 - \lambda\ (1 - \ \mu\ )\ \left\{(\mu - \tau - 1) + \ (1 - \ \mu\ )\ \gamma^{\frac{1}{2}}\right\}\ (\tau + 1)^5\right](\mu - \tau - 1) - \ m_0\ \gamma^{\frac{3}{2}}\ \left[(1-\mu)\left\{(\mu - \tau - 1) - \ \mu\ \ \gamma^{\frac{1}{2}}\right\}\ \tau^5(\tau+1)^2 + \mu\ \left\{(\mu - \tau - 1) + (1 - \ \mu\ )\ \gamma^{\frac{1}{2}}\right\}\ (\tau+1)^5\tau^2\right] + \ \{\mu\ (\tau+1)^2\tau^5 - \lambda\ (1 - \ \mu\ )(\tau+1)^5\tau^2\} = 0 \tag{13}$$

And

$$\tau^3 + \lambda\ (\tau+1)^3 \tag{14}$$

Case II when $L_2 \in \{-(1-\mu), \mu\}$

The substitution $r_1 = \ \mu - \xi = 1 - \tau,\ r_2 = -(1-\mu) + \ \xi = \tau$ in equations (11) and (12), we have

$$\left(\frac{\alpha^2\ \xi}{4}-m_0\right)(\mu+\tau-1)\ (1-\tau)^5\tau^5+2\ (\mu+\tau-1)\ \ \gamma^{\frac{1}{2}}\{(1-\tau)^2\tau^5+\lambda\ (1-\tau)^5\tau^2\}-$$

$$-\ 3\ (\mu+\tau-1)^2\ \gamma^{\frac{1}{2}}\ \left[\left\{(\mu+\tau-1)-\ \mu\ \ \gamma^{\frac{1}{2}}\right\}\ \tau^5\right.$$

$$\left.+\left\{(\mu+\tau-1)+(1-\ \mu\ )\ \gamma^{\frac{1}{2}}\right\}\ (1-\tau)^5\right]-$$

$$-3\gamma\ \ \left[\mu\left\{(\mu+\tau-1)-\ \mu\ \ \gamma^{\frac{1}{2}}\right\}\ \tau^5-\lambda\ (1-\ \mu\ )\ \left\{(\mu+\tau-1)+\ (1-\ \mu\ )\ \gamma^{\frac{1}{2}}\right\}\ (1-\tau)^5\right](\mu+\tau-1)-\ m_0\ \gamma^{\frac{3}{2}}\ \left[(1-\mu)\left\{(\mu+\tau-1)-\ \mu\ \ \gamma^{\frac{1}{2}}\right\}\ \tau^5(1-\tau)^2+\mu\ \left\{(\mu+\tau-1)+(1-\ \mu\ )\ \gamma^{\frac{1}{2}}\right\}\ (1-\tau)^5\tau^2\right]+\ \{\mu\ (1-\tau)^2\tau^5-\lambda\ (1-\ \mu\ )(1-\tau)^5\tau^2\}=0 \quad (15)$$

And

$$\tau^3+\lambda\ (1-\tau)^3 \quad (16)$$

Case III when $L_3\ \in(\mu,+\infty)$

The substitution $r_1=\ \xi-\ \mu=\tau,\ r_2=(1-\mu)+\ \xi=\tau+1$ in equations (11) and (12), we have

$$\left(\frac{\alpha^2\ \xi}{4}-m_0\right)(\mu+\tau)\ \ (\tau+1)^5\tau^5+2\ \ (\mu+\tau)\ \ \gamma^{\frac{1}{2}}\{(\tau+1)^5\tau^2+\lambda\ (\tau+1)^2\tau^5\}-\ 3\ (\mu+\tau)^2\ \gamma^{\frac{1}{2}}$$

$$\left[\left\{(\mu+\tau)-\ \mu\ \ \gamma^{\frac{1}{2}}\right\}\ (1+\tau)^5+\left\{(\mu+\tau)+(1-\ \mu\ )\ \gamma^{\frac{1}{2}}\right\}\ \tau^5\right]-3\gamma\ \left[\mu\left\{(\mu+\tau)-\ \mu\ \ \gamma^{\frac{1}{2}}\right\}\ (1+\tau)^5-\lambda\ (1-\ \mu\ )\ \left\{(\mu+\tau)+\ (1-\ \mu\ )\ \gamma^{\frac{1}{2}}\right\}\ \tau^5\right](\mu+\tau)-\ m_0\ \gamma^{\frac{3}{2}}\ \left[(1-\mu)\left\{(\mu+\tau)--\ \mu\ \ \gamma^{\frac{1}{2}}\right\}\ \tau^2(\tau+1)^5+\mu\ \left\{(\mu+\tau)+(1-\ \mu\ )\ \gamma^{\frac{1}{2}}\right\}\ (\tau+1)^2\tau^5\right]+$$

$$\{\mu\ (\tau+1)^5\tau^2-\lambda\ (1-\ \mu\ )(\tau+1)^2\tau^5\}=0 \quad (17)$$

And

$$(1+\tau)^3+\lambda\ \tau^3 \quad (18)$$

In figs 1, 2 and 3 we give the positions of the points L_1 L_2 and L_3 for $\lambda=1,$ respectively for various values of μ. These figure shows that the points L_1 and L_2 move towards the origin whereas the point L_3 go away from the origin as μ increases. We have also observed that the these points have the different positions for different values of mass reduction factor γ and small values of μ and for L_1 and L_2 this variation tends to zero as μ increases and γ decreases but for L_3 this variation increases as μ increases and γ decreases The combine position of L_1 L_2 and L_3 shows in fig (4).

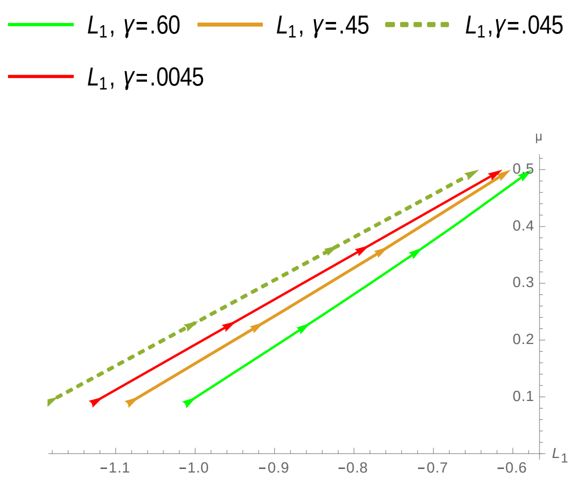
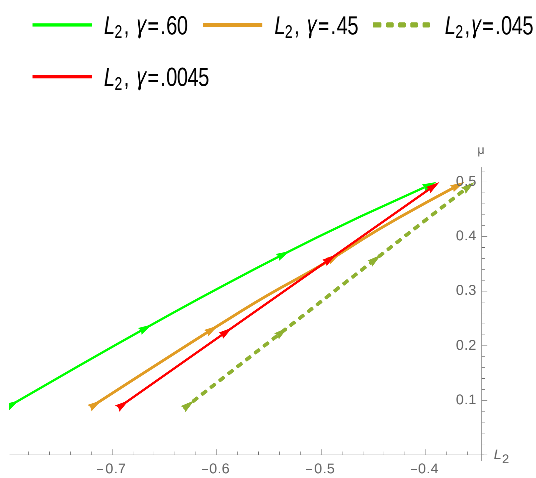


Fig (1)



Fig(2)

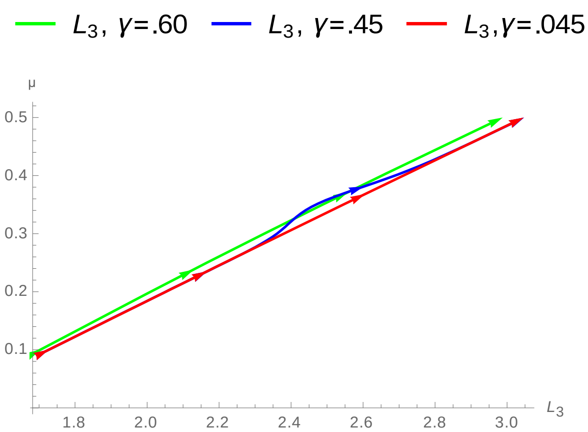


Fig (3)

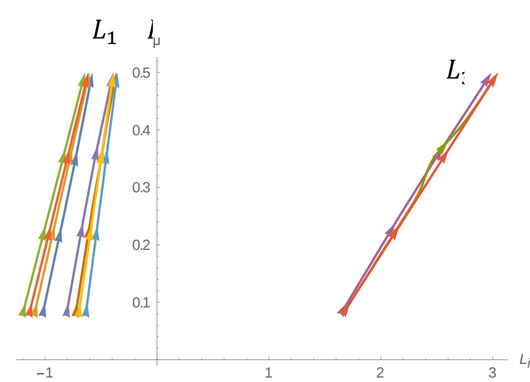


Fig (4)

Non-collinear equilibrium points $(y \neq 0)$

The non-collinear equilibrium points are the solution of the equations (11) and (12) when $y \neq 0$ and the solutions of these two equations are given in figure (5) for different value of μ and mass reduction factor γ. This figure (5) shows that there exist two non-collinear equilibrium points L_4 and L_5.We have observed that the mass reduction factor has a significant effect on the position of the non-collinear equilibrium points.

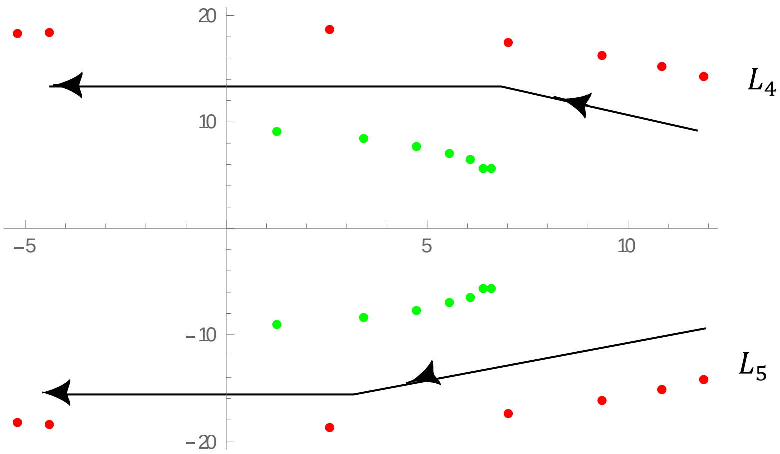


Fig (5)

In this fig (5) the green dot denotes the location of the ncep when $\gamma = .6$ and for various values of μ and red dot denotes the location of the ncep when $\gamma = .45$ and for various values of μ. We have found that these points move from the right to left as μ increases and these points also moves away from the primaries as γ decreases.

4. Conclusion

In this paper, we have studied the magnetic binary problem when the infinitesimal body is of variable mass. We have obtained the desired equations of motion and have also found the location of the collinear and non-collinear equilibrium points. We have observed that there exist three collinear and two non-collinear equilibrium points. We have found that that the points L_1 and L_2 move towards the origin whereas the point L_3 go away from the origin as μ increases. We have also observed that the these points have the different positions for different values of mass reduction factor γ and small values of μ and for L_1 and L_2 this variation tends to zero as μ increases and γ decreases but for L_3 this variation increases as μ increases and γ decreases. We have also observed that the mass reduction factor has a significant effect on the position of the non-collinear equilibrium points.

Conflict of Interests

The authors declare that there is no conflict of interests.

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