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DYNAMICAL RESPONSE BY THE INSTANT BUYER AND THINKER BUYER IN AN INNOVATION DIFFUSION MARKETING MODEL WITH MEDIA COVERAGE

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Abstract. In this exposition, a three compartment model has established to analyze the purchasing behavior of buyer. Population for a specific product marketing is classified into non-adopter group N(t), thinker group I(t) and adopter group A(t). The relation within adopter and the non-adopter group is word of mouth. For enhance the impact of word of mouth, media is the another dominating factor for impressing the non-adopter population, who has to become an adopter. In this model, there are two types of buyers instant buyers and thinker buyers. Instant purchasing is an unplanned decision to purchase a brand-new product, done just before purchase. These types of non-adopters become instant adopters. Thinker buyer is not immediate-adopter, and it will take the time for the buying of a brand-new product. Boundedness, Positivity and Basic influence numbers(BINs) of the model is studied. Stability analysis is carried out for all the possible equilibrium points. Hopf bifurcation analysis has also been determined at the adopter-free state and interior equilibrium. Sensitivity analysis for the basic reproduction number and variables about the parameters of the model has been examined for the internal steady state. Finally, numerically experimentations have been carried out to support our analytical findings with a different set of parameters.

Keywords: boundedness; positivity; delay; Hopf bifurcation; sensitivity analysis.

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1. Introduction

As the issues concerning the Bass model in [1] study about the modeling regarding the diffusion of innovations has received into a form of research consisting of several handful studies, publications, also different other papers. Analysis has been established to examine the basic and logical opinions and evaluation issues underlying the diffusion models of current product recognition. The critics estimate that development during the earlier two decades. Others find with an investigation plan on offer of diffusion models assumption-based more quality and realistically higher efficient including real to be. Diffusion is the plan of action by that innovation is transmitted within specific channels across time between the part of a social arrangement. Since a hypothesis of developments, diffusion system has the major objective on transmission channels, that are the means through which knowledge regarding reform is imparted into the social arrangement. Researchers in consumer behavior has been associated with assessing the applicability of hypothesis extended in the global diffusion field to buyer analysis [2]. Throughout its introduction into marketing during the 1960s, [1, 3, 4, 5], innovation diffusion system possesses flashed considerable investigation among customer response, marketing administration including supervision and marketing science critics. Specific marketing management articles become targeted toward the involvement of those theories which target new product possibilities also as discovering marketing strategies directed to possible adopters[6, 7]. Researchers within management plus retailing science should be devoted to these extensions from diffusion system by suggesting models analytically for explaining and projecting the diffusion about innovation within any social order. Currently, research, as mentioned earlier more, has been concerned growing standardizing rules as for how a modification should be diffused into the social arrangement. As a special edition regarding particular Bass model, an investigation concerning the modeling of the diffusion about reform within marketing has the outcome of extensive research. The contribution regarding the analysis during the 1970s was observed by[8]. However, during the ensuing decade, a plethora regarding investigations has provided to ours knowledge regarding particular fundamental, evaluation and conceptual opinions holding diffusion models. Modeling concerning innovation diffusion has been significantly performed to the obstacles from a wide field of disciplines [9]. Models recommended dealing with the aspect through the aggregate and non-aggregate levels must evolve over a significant period. Status of uncertainty into the models like innovation diffusion can be incorporated within these model parameters, its arrangement, or both. The resulting models should describe the business to the aggregate or disaggregate level also be employed as predictive or standardizing purposes [10]. The author introduced an epidemic model as the infection spread from an interaction of infected individual to premature individual. Similarly, in the diffusion process non-adopter population can become the user of innovation by the direct interaction with the adopter population[11]. Further, The author proposed an epidemic model; the infection can be control by vaccination and media information as a key factor to aware the people. Several researchers analyzed the epidemics models without consolidating the media awareness in controlling the diseases [12, 13, 14, 15]. Many researchers studied the influence of media coverage in controlling the infectious diseases[16, 17, 18]. At present scenario, media play a significant role in product expansion [19]. In the present study, we make some modifications in the model presented by [20].

This paper is planned as follows: In section 1, describe the introduction of the research study. In section 2, representation of the proposed model. In section 3, positivity along with boundedness of the proposed model has been studied. In section 4, basic influence number of the model, asymptotic stability for every possible steady state has been analyzed the stipulation for the occurrence of Hopf bifurcation about the adopter-free equilibrium and the interior steady state. In section 5, the sensitivity analysis of the basic reproduction number and essential variables at interior steady state concerning model parameters has been achieved. At last, in section 6, numerically experimentations have been carried out to support our analytical findings with a different set of parameters.

2. Representation of the Proposed Model

The supposition of the Proposed model is discussed below:

(i) Assuming that $N(t)$, $I(t)$, $A(t)$ the population densities of the non-adopter group, thinker group and adopter group.

(ii) The relationship among non-adopter and adopter group is through and word of mouth. For enhance the impact of word of mouth, media plays a significant role in influencing the non-adopter population, who has to become an adopter of a new product.

(iii)Due to media coverage and active command by word of mouth, some non-adopters will purchase the new product and meet the adopter group instantaneous. That type of the non-adopter is known as the instant buyer. On the other side, the non-adopter who are not immediately affected through media and word of mouth, but they can buy the product in eventuality. This type of non-adopter will join into the thinker group and known as thinker buyer.

(iv) Here, β is the instantaneous transformation rate non-adopter N(t) to adopter A(t).

(v) Here α fraction of non-adopter N(t) people will join to thinker group I(t), we have recognized τ is the lag period in thinker group as the effect of media and word of mouth, in this group is not instantaneous. There is some time gap between non-adopter to the adopter. The part of thinker group γ I will meet the adopter group after the delay τ.

(vi) Assume K is the recruitment rate of non-adopter N(t). ξ is the frustration rate of adopter A(t) and join the non-adopter group N(t) and m is coefficient of media coverage. Finally, μ is the death rate of non-adopter N(t) and adopter A(t).

The proposed system is of the form:

$$\frac{dN}{dt} = K - \alpha NA - \beta e^{-mA} NA + \xi A - \mu N, \tag{1}$$

$$\frac{dI}{dt} = \alpha NA - \gamma I(t-\tau), \tag{2}$$

$$\frac{dA}{dt} = \gamma I + \beta e^{-mA} NA - \xi A - \mu A, \tag{3}$$

with initial values: $N(0) > 0, I(0) > 0, A(0) > 0$ for all $t \geq 0$.

3. Positivity and boundedness

For the positivity along with the boundedness of the proposed system $(1)-(3)$, we state and evaluate the below mentioned lemmas:

Lemma 3.1. *The solution for the proposed system* $(1)-(3)$*, with initial values are non-negative, for all* $t \geq 0$*.*

Proof. Let N, I, A be the solution for the proposed system $(1)-(3)$, with positive initial populations. We suppose N for $t \in [0,\tau]$. On solving the expression (1), may be written as

$$\frac{dN}{dt} \geq -\alpha NA - \beta e^{-mA} NA - \mu N,$$

which exhibit that

$$N(t) \geq \ N(0)\exp\left\{-\int_0^t \left(\alpha A + \beta e^{-mA} A + \mu\right) dv\right\} > 0.$$

For the expression (2), $t \in [0,\tau]$ may be recast as

$$\frac{dI}{dt} \geq -\gamma I,$$

which exhibit that

$$I(t) \geq \ I(0)\exp\left\{-\int_0^t (\gamma)\, dv\right\} > 0.$$

For the expression (3), $t \in [0,\tau]$ may be recast as

$$\frac{dA}{dt} \geq -\xi A - \mu A,$$

which exhibit that

$$A(t) \geq A(0)\exp\left\{-\int_0^t (\xi + \mu)\, dv\right\} > 0.$$

With the above discussion, we prove that $N(t) > 0, I(t) > 0$ and $A(t) > 0$ for all $t \geq 0$.

Lemma 3.2. *The solution for the proposed system* $(1)-(3)$*, with initial conditions is bounded uniformly in* ζ*, where*

$$\zeta = \{(N,I,A) : 0 \leq N(t) + I(t) + A(t) \leq \frac{K}{\mu}\}.$$

.

Proof. Let $W(t) = N(t) + A(t) + I(t)$. Differentiate $W(t)$ with respect to t, we have

$$\frac{dW}{dt} = K - \mu(N + A).$$

On solving, we notice $0 \leq W(t) \leq \frac{K}{\mu}$ as $t \to \infty$. Hence, $W(t)$ is bounded. Therefore, the proposed system is bounded.

4. Reproduction Number and Dynamical Nature of the system

Now, we will explain one valuable, significant parameter which explains mathematical problems regarding new product adoption is the basic influence rate, basic influence number or basic influence ratio. The basic influence number is stated as the required fraction of subsequent adoption created by an adopted person that is related to basic influence number in disease spreading models [15]. The advantage of that matrix is to define either or no the products will diffuse into people. For the projected model equation $(1)-(3)$ has an adopter-free equilibrium position, i.e. $E_0(\frac{K}{\mu},0,0)$. Using the notation in [15, 21], where F and V are two vectors to express the new adopters with the personal touch of adopter community with the non-adopter community and outlasting transfer terms, sequentially concerning the model expression of adopter sections.

$$F = \begin{pmatrix} \alpha NA \\ \beta e^{-mA} NA \end{pmatrix}, V = \begin{pmatrix} \gamma I \\ -\gamma I + \xi A + \mu A \end{pmatrix}.$$

Now, the jacobian of F and V by using the adopter-free equilibrium E_0.

$$F = J(F) = \begin{pmatrix} 0 & \frac{\alpha K}{\mu} \\ 0 & \frac{\beta K}{\mu} \end{pmatrix},$$

$$V = J(V) = \begin{pmatrix} \gamma & 0 \\ -\gamma & \xi + \mu \end{pmatrix}, V^{-1} = \begin{pmatrix} \frac{1}{\gamma} & 0 \\ \frac{1}{\xi+\mu} & \frac{1}{\xi+\mu} \end{pmatrix}.$$

Where F is positive and V is a non-singular Matrix. Consequently, FV^{-1} is positive.

$$FV^{-1} = \begin{pmatrix} \frac{\alpha K}{\mu(\xi+\mu)} & \frac{\alpha K}{\mu(\xi+\mu)} \\ \frac{\beta K}{\mu(\xi+\mu)} & \frac{\beta K}{\mu(\xi+\mu)} \end{pmatrix}. \tag{4}$$

Basic influence number is represented by R_A, which is the largest eigenvalue. Where, $R_A = \frac{(\alpha+\beta)K}{(\mu+\xi)\mu}$.

The nature of stability for possible equilibrium points of the system $(1)-(3)$, is as follows:

(i)The adopter-free equilibrium $E_0(\frac{K}{\mu},0,0)$ always exists.

(ii) For $m>0$ the interior equilibrium $E^*(N^*,I^*,A^*)$ exists, where N^*,I^*,A^* is given by

$$K-\alpha N^*A^*-\beta e^{-mA^*}NA^*+\xi A^*-\mu N^*=0,$$

$$\alpha N^*A^*-\gamma I^*(t-\tau)=0,$$

$$\gamma I^*+\beta e^{-mA^*}N^*A^*-\xi A^*-\mu A^*=0. \tag{5}$$

when $m=0$, the interior equilibrium $E^*(N^*,I^*,A^*)$ exists if $R_A>1$, where N^*,I^*,A^* is given by

$$N^*=\frac{\mu+\xi}{\alpha+\beta},I^*=\frac{\alpha(\mu+\xi)^2}{\gamma(\alpha+\beta)^2}(R_A-1),A^*=\frac{(\mu+\xi)}{(\alpha+\beta)}(R_A-1). \tag{6}$$

As the transcendental polynomials equation concerning the 2nd degree of the pattern

$$\lambda^2+p\lambda+r+(s\lambda+q)e^{-\lambda\tau}=0. \tag{7}$$

was studied by [22, 23] and examined the mentioned below results:

$(B1)$ $p+s>0$;

$(B2)$ $q+r>0$;

$(B3)$ either $s^2-p^2+2r<0$ and $r^2-q^2>0$ or $(s^2-p^2+2r)^2<4(r^2-q^2)$;

$(B4)$ either $r^2-q^2<0$ or $s^2-p^2+2r>0$ and $(s^2-p^2+2r)^2=4(r^2-q^2)$;

$(B5)$ either $r^2-q^2>0$, $s^2-p^2+2r>0$ and $(s^2-p^2+2r)^2>4(r^2-q^2)$.

Lemma 4.1.

(i) *If* $(B1)-(B3)$ *exists, entire roots of* (7) *possess negative real parts for all* $\tau\geq 0$.

(ii) If $(B1)$, $(B2)$ and $(B4)$ exists, also $\tau=\tau_j^+$, then equation (7) possess a pair of purely imaginary roots $\pm iw_+$. When $\tau=\tau_j^+$ then entire roots of (7) except $\pm iw_+$ have negative real parts.

(iii) If $(B1)$, $(B2)$ and $(B5)$ exists, also $\tau = \tau_j^+ (\tau = \tau_j^-$ respectively) then equation (7) possess a pair of purely imaginary

Theorem 4.2. *When $R_A < 1$, for the adopter-free equilibrium of the system $(1)-(3)$, we have:*

(i) The adopter-free equilibrium $E_0$is always locally asymptotically stable for $\tau = 0$.

(ii) If $2\beta K < \mu(R_A+1)(\mu+\xi)$ holds, then adopter-free equilibrium E_0 is locally asymptotically stable for all $\tau \in (0, \tau_0^+]$, further unstable at $\tau \geq \tau_0^+$ for $\tau > 0$.

Proof. The variational matrix corresponding to model equation $(1)-(3)$, is given by

$$J = \begin{pmatrix} -A\alpha - A\beta e^{-mA} - \mu & 0 & -\alpha N - \beta N e^{-mA} + \beta m N A e^{-mA} + \xi \\ A\alpha & -\gamma e^{-\lambda\tau} & \alpha N \\ A\beta e^{-mA} & \gamma & -\mu + \beta e^{-mA} N - \beta m N A e^{-mA} - \xi \end{pmatrix}.$$

The characteristic equation about the equilibrium point $E_0(\frac{K}{\mu}, 0, 0)$ is given by: $(\lambda+\mu)F(\lambda) = 0$. Then

$$\lambda = -\mu \;\text{ and }\; F(\lambda) = \lambda^2 + (\mu - \beta N + \xi)\lambda - \alpha\gamma N + (\gamma\lambda + \mu\gamma + \xi\gamma - \beta N\gamma)e^{-\lambda\tau}. \tag{8}$$

From equation (8), $F(\lambda)$ may be written as:

$$\lambda^2 + p\lambda + r + (s\lambda + q)e^{-\lambda\tau} = 0. \tag{9}$$

where

$p = \mu + \xi - \beta N$, $r = -\alpha\gamma N$, $s = \gamma$, $q = \mu\gamma + \xi\gamma - \beta\gamma N$.

Case I: When $\tau = 0$, the transcendental equation (8), represented as

$$\lambda^2 + (p+s)\lambda + (q+r) = 0. \tag{10}$$

Let $C_1 = p + s = \gamma + \alpha N + (\mu+\xi)(1-R_A)$,

$C_2 = q + r = (\mu+\xi)(1-R_A)$.

With Routh-Hurwitz method, entire roots of equation (10), have negative real parts, If $R_A < 1$ then $C_1 > 0, C_2 > 0$.

Case II: When $\tau > 0$, we have

By Lemma (4.1), the conditions $[B_1], [B_2]$ and $[B_4]$ holds good, if $2\beta K < \mu(R_A+1)(\mu+\xi)$

exists, then the proposed system (1)-(3), possess a pair of purely imaginary roots.

Substitute $\lambda = iw$ in (10), we make

$$(iw)^2 + p(iw) + r + (iws + q)e^{-iw\tau} = 0. \tag{11}$$

Compare real and imaginary part from (11), we make

$$-w^2 + r + sw\sin w\tau + q\cos w\tau = 0, \tag{12}$$

$$pw + sw\cos w\tau_1 - q\sin w\tau = 0. \tag{13}$$

On simplifying (12) – (13), we make

$$\sin w\tau = \frac{sw^3 + (pq - rs)w}{s^2w^2 + q^2}, \tag{14}$$

$$\cos w\tau = \frac{(q - ps)w^2 - qr}{s^2w^2 + q^2}. \tag{15}$$

also

$$w^4 + (p^2 - 2r - s^2)w^2 + (r^2 - q^2) = 0. \tag{16}$$

Let us take

$$F(w) = w^4 + (p^2 - 2r - s^2)w^2 + (r^2 - q^2) = 0.$$

With Descarte's rule concerning the sign, there is at least one positive root of $F(w) = 0$. Suppose w_0 is the non-negative root of $F(w) = 0$. By (15), we make

$$\tau_k^+ = \frac{1}{w_0}\left[\cos^{-1}\left(\frac{(q - ps){w_0}^2 - qr}{s^2{w_0}^2 + q^2}\right) + 2k\pi\right],$$

where $k = 0, 1, 2, \ldots$. As for the occurrence concerning Hopf bifurcation at τ_0^+, it is mandatory that the transversality situation $Re\left[\left(\frac{d\lambda}{d\tau}\right)^{-1}\right]_{\tau=\tau_0^+} \neq 0$ should exists, by taking the derivative of λ concerning τ in (9), we make

$$\frac{d\lambda}{d\tau} = \frac{\lambda(s\lambda + q)e^{-\lambda\tau}}{2\lambda + p + se^{-\lambda\tau} - (s\lambda + q)\tau e^{-\lambda\tau}}.$$

Put $\lambda = iw_0$ and $\tau = \tau_0^+$, we obtain

$$Re\left(\frac{d\lambda}{d\tau}\right)^{-1} = \frac{qG - sw_0H}{w_0(q^2 + s^2w_0^2)}. \tag{17}$$

here $G = psinw_0\tau_0 + 2w_0cosw_0\tau_0$ and $H = s + pcosw_0\tau_0 - 2w_0sinw_0\tau_0$

On analyzing (17), we make

$$Re\left[\left(\frac{d\lambda}{d\tau}\right)^{-1}\right]_{\tau=\tau_0^+} \neq 0,$$

if $qG \neq sw_0H$.

Theorem 4.3. *The interior equilibrium is locally unstable for $R_A > 1$, but close to 1.*

Proof. Now, we apply the specific method depends upon the central manifold system to investigate the nature of local stability of interior steady state using ξ as the bifurcation parameter[24]. We consider the adopter-free equilibrium $E_0(\frac{K}{\mu},0,0)$ and recognize the significant condition of the bifurcation parameter about $R_A = 1$ is $\xi^* = (\alpha+\beta)\frac{K}{\mu} - \mu$. The eigenvalue of the matrix $(\tau = 0)$ is

$$J_0 = \begin{pmatrix} -\mu & 0 & -\mu \\ 0 & -\gamma & \frac{\alpha K}{\mu} \\ 0 & \gamma & -\frac{\alpha K}{\mu} \end{pmatrix}.$$

On solving $\lambda = -\mu, 0, -(\gamma + \frac{\alpha K}{\mu})$. Thus $\lambda = 0$ is the simple eigenvalue and remaining two eigenvalues have negative real parts. Therefor, we can use the central manifold theory. It can be simply determined that the Jacobian J_0 about $\xi = \xi^*$ posses right eigenvector provided with $W = (w_1, w_2, w_3)^T$ so that $J_0W = 0$, where

$$w_1 = -\sigma, w_2 = \frac{\alpha K}{\gamma\mu}, w_3 = \sigma. \tag{18}$$

The left eigenvector is presented with $V = (v_1, v_2, v_3)$ that meet $VJ_0 = 0$ and $V.W = 1$, where

$$v_1 = 0, v_2 = 0, v_3 = \frac{1}{\sigma}. \tag{19}$$

Here we take the notation $x_1 = N, x_2 = I, x_3 = A$.

Evaluating the 2nd order partial derivatives about the adopter-free equilibrium, we get $\frac{\partial^2 f_1}{\partial x_1 \partial x_3} =$

$-(\alpha+\beta)$, $\frac{\partial^2 f_1}{\partial x_3^2}=\frac{2\beta mK}{\mu}$, $\frac{\partial^2 f_2}{\partial x_1\partial x_3}=\alpha$, $\frac{\partial^2 f_3}{\partial x_1\partial x_3}=\beta$, $\frac{\partial^2 f_3}{\partial x_3^2}=-\frac{2\beta mK}{\mu}$ and rest 2nd order partial derivatives are equal to zero. Consequently, we compute the coefficient of a and b by applying the resulting formula mentioned below.

$$a=\sum_{k,i,j=1}^{n} v_k w_i w_j \frac{\partial^2 f_k}{\partial x_i \partial x_j}$$

$$b=\sum_{k,i,j=1}^{n} v_k w_i \frac{\partial^2 f_k}{\partial x_i \partial \phi}$$

On simplifying, $a=-\sigma\beta(1+2m\frac{K}{\mu})$ and $b=-1$.

Since $a<0$ and $b<0$ at $\xi=\xi^*$, a transcritical bifurcation appears about $R_A=1$ and unique interior equilibrium is locally unstable for $R_A>1$.

Presently, we state the following lemma as related as presented within Song et al. [23, 25];

Lemma 4.4. *As this polynomial equation* $z^3+pz^2+qz+r=0$,

(i)If $r<0$, then the equation possess at least one non-negative root;

(ii) If $r\geq 0$ and $\nabla=p^2-3q\leq 0$, then equation possess no positive root;

(iii)If $r\geq 0$ and $\nabla=p^2-3q>0$, then equation possess non-negative roots if $z_1^*=\frac{-p+\sqrt{\nabla}}{3}$ and $h(z_1^*)\leq 0$, where $h(z)=z^3+pz^2+qz+r$.

Theorem 4.5. *Let* $R_A>1$ *exists, for the interior equilibrium system* $(1)-(3)$,

(i) If $m>0$ or $m=0$, then interior equilibrium E^* is always locally asymptotically stable for $\tau=0$.

(ii) If $m>0$ or $m=0$, then interior equilibrium E^* is locally asymptotically stable for all $\tau\in(0,\tau_0^+]$ for $\tau>0$.

(iii) If $\tau\geq\tau_0^+$, then the interior equilibrium E^* is unstable and the Hopf bifurcation occurs in the system, for $m>0$ or $m=0$.

Proof. This corresponding characteristic expression for the Jacobian matrix about the interior equilibrium point $E^*(N^*,I^*,A^*)$, is given as:

$$\lambda^3+A^{**}\lambda^2+B\lambda+C+(F\lambda^2+E\lambda+D)e^{-\lambda\tau}. \tag{20}$$

where

$$A^{**} = A^*\alpha + A^*\beta e^{-mA^*}(1+mN^*) + \frac{I^*\gamma}{A^*} + \mu,$$

$$B = I^*\gamma(\alpha + \frac{\mu}{A^*}) - \alpha\gamma N^* + A^*\beta e^{-mA^*}(\mu + \alpha N^* + N^*A^*m\alpha + m\mu N^*),$$

$$C = -I^*\alpha\gamma^2 + \alpha\gamma\mu(A^* - N^*) - A^*N^*\alpha\beta\gamma e^{-mA^*}(1+mA^*),$$

$$D = I^*\gamma^2(\alpha + \frac{\mu}{A^*}) + A^*\beta\gamma e^{-mA^*}(\mu + \alpha N^* + N^*A^*m\alpha + m\mu N^*),$$

$$E = A^*\alpha\gamma + A^*\beta\gamma e^{-mA}(1+mN^*) + \frac{I^*\gamma^2}{A^*} + \gamma\mu,$$

$$F = \gamma. \tag{21}$$

Case I: When $\tau = 0$ and $m > 0$, the transcendental equation (20), reduces so

$$\lambda^3 + (A^{**} + F)\lambda^2 + (B+E)\lambda + (C+D) = 0. \tag{22}$$

Let $X = A^{**} + F, Y = B + E, Z = C + D$. On solving

$$X = A^*\alpha + A^*\beta e^{-mA^*}(1+mN^*) + \frac{I^*\gamma}{A^*} + \mu + \gamma,$$

$$Y = I^*\gamma(\alpha + \frac{\mu}{A^*}) + A^*\beta e^{-mA^*}(\mu + \alpha N^* + N^*A^*m\alpha + m\mu N^* + \gamma + m\gamma N^*) + A^*\alpha\gamma + \gamma\mu,$$

$$Z = A^*\alpha\gamma\mu + A^*\beta\gamma\mu e^{-mA}(1+N^*m)$$

By using Routh-Hurwitz method, entire roots of equation (22), have negative real parts and the interior equilibrium E^* with media effect is locally asymptotically stable because $X > 0, Y > 0, Z > 0, XY - Z > 0$.

Case II: When $\tau = 0$ and $m = 0$, the transcendental equation (20), reduces so

$$\lambda^3 + (A^{**} + F)\lambda^2 + (B+E)\lambda + (C+D) = 0. \tag{23}$$

Let $X_1 = A^{**} + F, Y_1 = B + E, Z_1 = C + D$. On solving

$$X_1 = (\mu + \xi)(R_A - 1) + \gamma + \mu + \alpha N^*,$$

$$Y_1 = I^*\gamma(\alpha + \frac{\mu}{A^*}) + A^*\alpha\gamma + \gamma\mu + A^*\beta(\mu + \gamma + \alpha N^*),$$

$$Z_1 = \gamma\mu(\mu + \xi)(R_A - 1),$$

By using Routh-Hurwitz method, entire roots of equation (23), have negative real parts and the interior equilibrium E^* without media effect is locally asymptotically stable because $X_1 > 0, Y_1 > 0, Z_1 > 0, X_1Y_1 - Z_1 > 0$.

Case III: When $\tau > 0$ and $m > 0$.

Let $\lambda = iw$ is root of (20), as we make

$$(iw)^3 + A^{**}(iw)^2 + B(iw) + C + (F(iw)^2 + E(iw) + D)e^{-iw\tau} = 0. \quad (24)$$

compare real and imaginary parts from (22), we have

$$Ew\sin w\tau + (D - Fw^2)\cos w\tau = A^{**}w^2 - C, \quad (25)$$

$$Ew\cos w\tau - (D - Fw^2)\sin w\tau = w^3 - Bw. \quad (26)$$

Simplifying (25) − (26), we make

$$w^6 + pw^4 + qw^2 + r = 0, \quad (27)$$

where $p = A^{**2} - 2B - F^2$, $q = B^2 - 2A^{**}C + 2DF - E^2$, $r = C^2 - D^2$.

On substituting $w^2 = z$ in equation (27), we state

$$F(z) = z^3 + pz^2 + qz + r.$$

With Lemma (4.3), there holds at least one non-negative root $w = w_0$ of equation (27) satisfying (25) and (26), that means the equation (20) possess a pair of purely imaginary roots $\pm iw_0$. Simplifying (25) and (26) for τ and put the value of $w = w_0$, the corresponding $\tau_k > 0$ is given by

$$\tau_k^+ = \frac{1}{w_0}\left[\cos^{-1}\left(\frac{(E - A^{**}F)w_0^4 + (A^{**}D + CF - BE)w_0^2 + CD}{E^2w_0^2 + (D - Fw_0^2)^2}\right) + 2k\pi\right],$$

where $k = 0, 1, 2, ...$

As for the occurrence concerning Hopf bifurcation at τ_0^+, it is mandatory that the transversally condition $Re\left[\left(\frac{d\lambda}{d\tau}\right)^{-1}\right]_{\tau=\tau_0^+} \neq 0$ should hold, therefore taking the derivative of λ concerning τ in (20), we get

$$\left(\frac{d\lambda}{d\tau_2}\right)^{-1} = \frac{(3\lambda^2 + 2A^{**}\lambda + B)e^{\lambda\tau_2} + (2F\lambda + E)}{\lambda(F\lambda^2 + E\lambda + D)} - \frac{\tau}{\lambda}.$$

At $\lambda = iw_0$ and $\tau = \tau_0^+$, we have

$$Re\left[\left(\frac{d\lambda}{d\tau}\right)^{-1}\right] = \frac{MQ - NR}{w_0(L^2 + M^2)},$$

On solving $K = -3w_0^2 + B$, $L = 2A^{**}w_0$, $M = D - Fw_0^2$, $N = Ew_0$, $Q = Ksinw_0\tau_0 + Lcosw_0\tau_0 + 2Fw_0$ and $R = Kcosw_0\tau_0 - Lsinw_0\tau_0 + E$.

Now, we have,

$$Re\left[\left(\frac{d\lambda}{d\tau_2}\right)^{-1}\right]_{\tau_2=\tau_{20}^+} \neq 0,$$

if $MQ \neq NR$.

Case iv: When $\tau > 0$ and $m = 0$.

The equation (21), reduces to

$$A^{**} = A^*\alpha + A^*\beta + \frac{I^*\gamma}{A^*} + \mu,$$

$$B = I^*\gamma(\alpha + \frac{\mu}{A^*}) - \alpha\gamma N^* + A^*\beta(\mu + \alpha N^*),$$

$$C = -I^*\alpha\gamma^2 + \alpha\gamma\mu(A^* - N^*) - A^*N^*\alpha\beta\gamma,$$

$$D = I^*\gamma^2(\alpha + \frac{\mu}{A^*}) + A^*\beta\gamma(\mu + \alpha N^*),$$

$$E = A^*\alpha\gamma + A^*\beta\gamma + \frac{I^*\gamma^2}{A^*} + \gamma\mu,$$

$$F = \gamma.$$

where $p = A^{**2} - 2B - F^2$, $q = B^2 - 2A^{**}C + 2DF - E^2$, $r = C^2 - D^2$.

On solving, by Lemma (4.4) the condition *[iii]* holds good. We can get the value of critical τ_k^+ by the similar method as mentioned in case(III).

5. Sensitivity Analysis

In this section, sensitivity analysis for the basic influence number R_A and state variables for the proposed system $(1) - (3)$, concerning the context of the model parameters at the interior equilibrium position, has achieved. The corresponding sensitive parameters for the basic influence number R_A and for the state variables about the interior equilibrium are presented in Table 2 and Table 3 respectively. Using parametric conditions shown in Table 1. It is established that

K, α, β are the positive influence parameters to R_A and ξ, μ are the negative influence parameters to R_A. Next, α, β are the negative influence parameters and ξ, μ, m are the positive influence parameters for N^*, So we notice large changes in N^* by a small variation in the parameter ξ. Succeeding, K is the most delicate parameter into I^* also the resting parameters are less delicate as compared to K in I^*. On end, μ is the extremely delicate parameter within A^* including all the additional parameters are less delicate to A^* as contrasted to μ.

TABLE 1. Parametric conditions applied for sensitivity analysis

Parameter	value
K	0.1
α	0.28
β	0.3
γ	0.24
ξ	0.2
μ	0.08
m	0.5

TABLE 2. The sensitivity lists $\gamma_{y_g}^{R_A} = \frac{\partial R_A}{\partial y_g} \times \frac{y_g}{R_A}$ for the basic influence number R_A to these parameters y_g as the parameter conditions presented within Table1

.

Parameter (y_g)	$\gamma_{y_g}^{R_A}$
K	1
α	0.483
β	0.518
γ	0
ξ	-0.715
μ	-1.286
m	0

TABLE 3. The sensitivity lists $\gamma_{y_g}^{x_h} = \frac{\partial x_h}{\partial y_g} \times \frac{y_g}{x_h}$ of the state variables for the proposed model equations $(1)-(3)$, to the parameters y_g as the parameter conditions presented within Table1

Parameter (y_g)	$\gamma_{y_g}^{N^*}$	$\gamma_{y_g}^{I^*}$	$\gamma_{y_g}^{A^*}$
K	0	1.831	1.831
α	-0.567	0.904	0.471
β	-0.432	-0.0730	0.359
γ	0	-1	0
ξ	0.714	0.121	-0.593
μ	0.286	-1.782	-2.068
m	0.147	0.249	- 0.122

6. Numerical Simulations

To verify the analytic conclusions of the model $(1)-(3)$, numerical simulations are executed by using MATLAB. It provides an impression of completeness to the analytic conclusions. It is examined that the adopter-free equilibrium point without delay $E_0(1.4,0,0)$ is stable for the parametric conditions: $K = 0.07; \alpha = 0.04; \beta = 0.04; \gamma = .03; \xi = 0.1; \mu = 0.05$ and $R_A = 0.75$. (see Fig.1).

The adopter-free equilibrium with delay $E_0(1.15,0,0)$ is stable for the parametric values: $K = .03; \alpha = 0.07; \beta = 0.09; \gamma = .06; \xi = 0.2; \mu = 0.026; m = 0.2; \tau = 24.58 < \tau_0^+ = 24.7$ (see Fig.2) also this equilibrium is unstable and Hopf bifurcation occurs at $\tau = 24.75 > \tau_0^+ = 24.7$. (see Fig.3). Here $R_A = 0.82$.

When $m > 0$ the interior equilibrium without delay $E^*(0.39,0.51,0.90)$ is stable for the parametric values: $K = .09; \alpha = 0.29; \beta = 0.33; \gamma = 0.2; \xi = 0.15; \mu = 0.07$; m= 0.2 and $R_A = 3.62$. (see Fig.4).

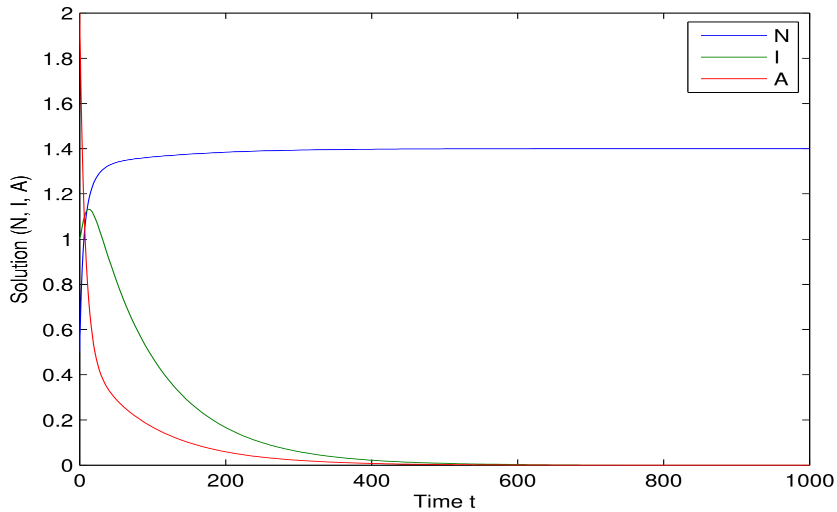

FIGURE 1. The adopter-free equilibrium without delay $E_0(1.4,0,0)$ is stable for the parametric values: $K = 0.07; \alpha = 0.04; \beta = 0.04; \gamma = .03; \xi = 0.1; \mu = 0.05$ and $R_A = 0.75$.

When $m = 0$ the interior equilibrium without delay $E^*(0.35,0.48,0.93)$ is stable for the parametric values: $K = .09; \alpha = 0.29; \beta = 0.33; \gamma = 0.2; \xi = 0.15; \mu = 0.07$; m=0 and $R_A = 3.62$. (see Fig.5).

When $m > 0$ the interior equilibrium with delay $E^*(0.57,0.45,0.68)$ is stable for the parametric values: $K = 0.1; \alpha = 0.28; \beta = 0.3; \gamma = 0.24; \xi = 0.2; \mu = 0.08; m = 0.5; \tau = 7.51 < \tau_0^+ = 7.53$ (see Fig.6) also this equilibrium is unstable and Hopf bifurcation occurs at $\tau = 7.55 > \tau_0^+ = 7.53$. (see Fig.7). Here $R_A = 2.59$.

When $m = 0$ the interior equilibrium with delay $E^*(0.48,0.43,0.76)$ is stable for the parametric values: $K = 0.1; \alpha = 0.28; \beta = 0.3; \gamma = 0.24; \xi = 0.2; \mu = 0.08; m = 0; \tau = 7.57 < \tau_0^+ = 7.59$ (see Fig.8) also this equilibrium is unstable and Hopf bifurcation occurs at $\tau = 7.61 > \tau_0^+ = 7.59$. (see Fig.9). Here $R_A = 2.59$.

The effect of media coverage m on the fraction of thinker population is shown in figure (10) and (11). We established that the level of the interior steady state is significantly affected by the media coefficient m.

7. Conclusion

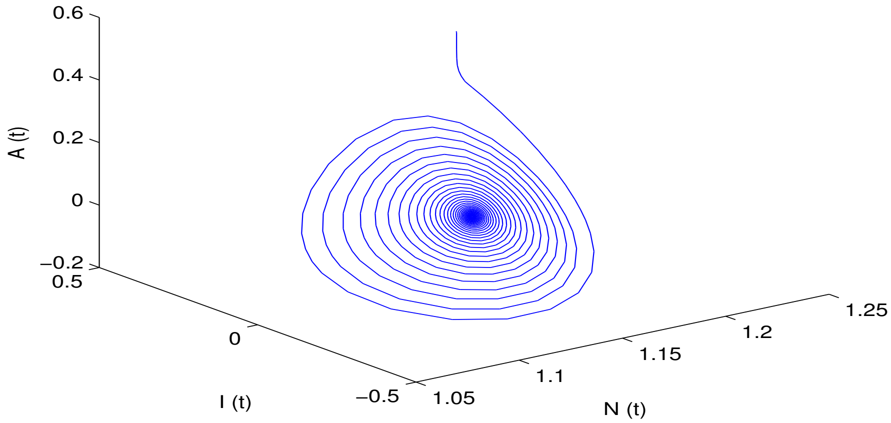

FIGURE 2. The adopter-free equilibrium with delay $E_0(1.15,0,0)$ is stable for the parametric values: $K = .03; \alpha = 0.07; \beta = 0.09; \gamma = .06; \xi = 0.2; \mu = 0.026; m = 0.2; \tau = 24.58 < \tau_0^+ = 24.7$.

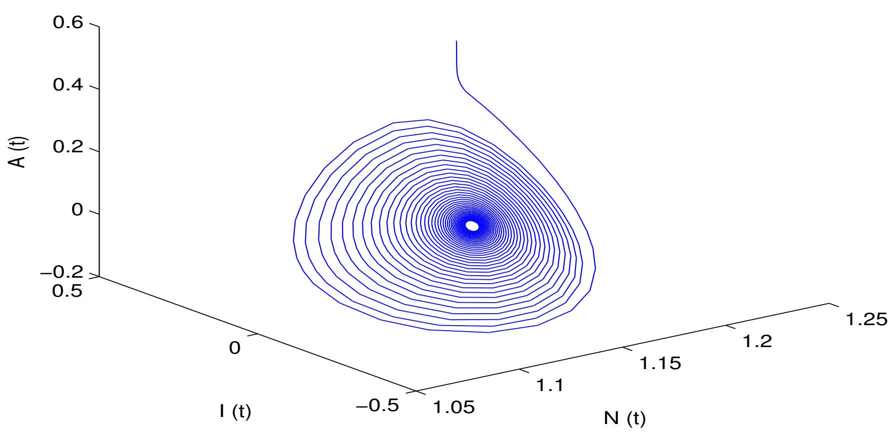

FIGURE 3. The adopter-free equilibrium with delay $E_0(1.15,0,0)$ is unstable for the parametric values: $K = .03; \alpha = 0.07; \beta = 0.09; \gamma = .06; \xi = 0.2; \mu = 0.026; m = 0.2$ and Hopf bifurcation occurs at $\tau = 24.75 > \tau_0^+ = 24.7$.

In this paper, a three compartment model has developed with the involvement of media awareness. Asymptotic stability behavior of the model is examined for adopter-free and interior steady state positions. It is established that basic influence number R_A is not affected by the

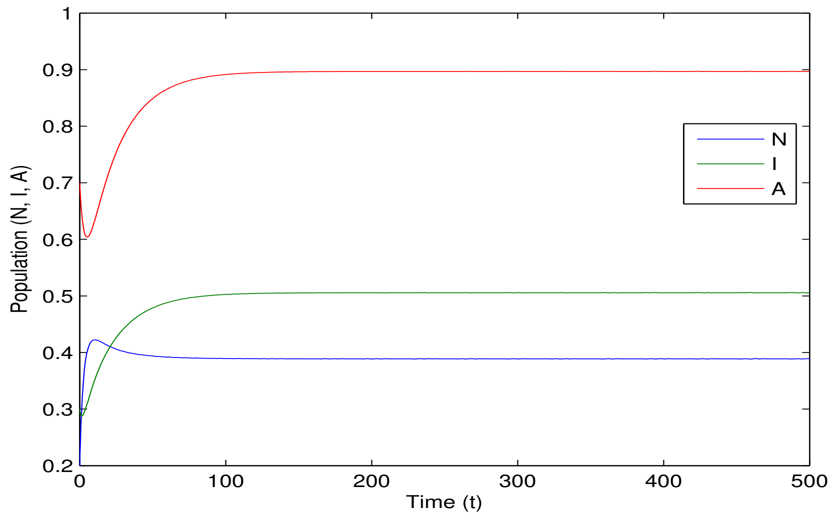

FIGURE 4. When $m > 0$ the interior equilibrium without delay $E^*(0.39, 0.51, 0.9)$ is stable for the parametric values: $K = .09; \alpha = 0.29; \beta = 0.33; \gamma = 0.2; \xi = 0.15; \mu = 0.07$; m=0.2 and $R_A = 3.62$.

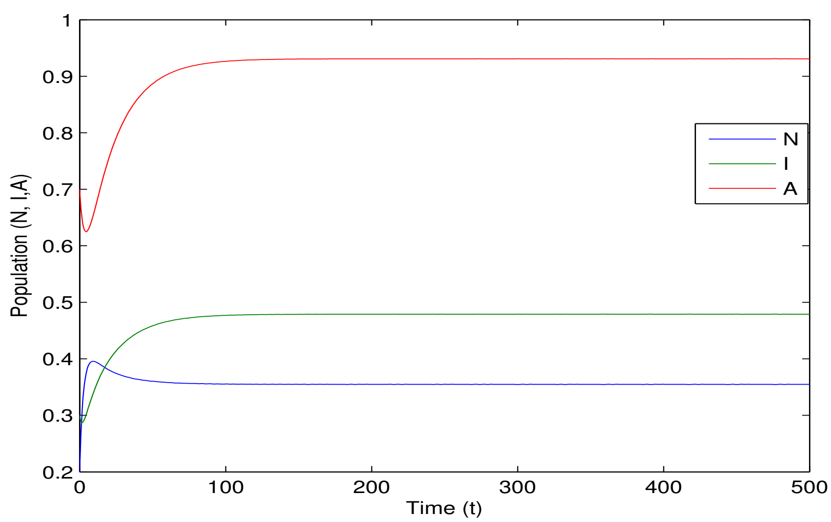

FIGURE 5. When $m = 0$ the interior equilibrium without delay $E^*(0.35, 0.48, 0.93;)$ is stable for the parametric values: $K = .09; \alpha = 0.29; \beta = 0.33; \gamma = 0.2; \xi = 0.15; \mu = 0.07$ and $R_A = 3.62$.

coefficient of media coverage (m). For the adopter-free state, basic influence number is $R_A < 1$. A transcritical bifurcation occurs at $R_A = 1$ and for the interior steady state basic influence number is $R_A > 1$. On introducing delay parameters τ Hopf-bifurcation appears in E_0 and E^*. It is

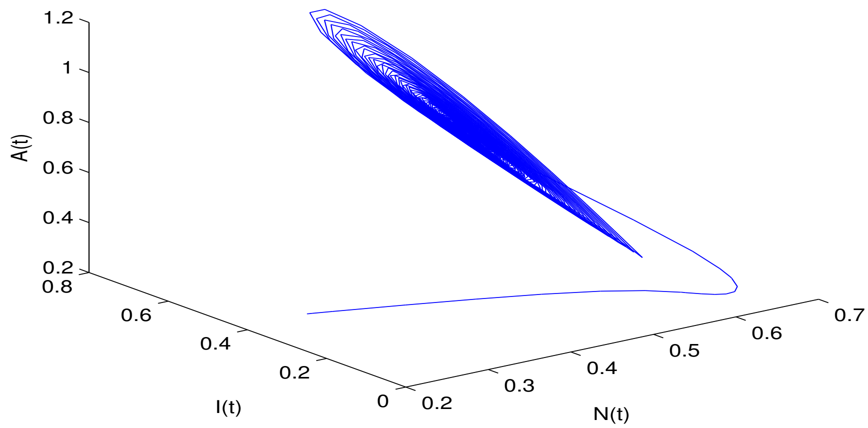

FIGURE 6. When $m > 0$ the interior equilibrium with delay $E^*(0.57, 0.45, 0.68)$ is stable for the parametric values: $K = 0.1; \alpha = 0.28; \beta = 0.3; \gamma = 0.24; \xi = 0.2; \mu = 0.08; m = 0.5; \tau = 7.51 < \tau_0^+ = 7.53$.

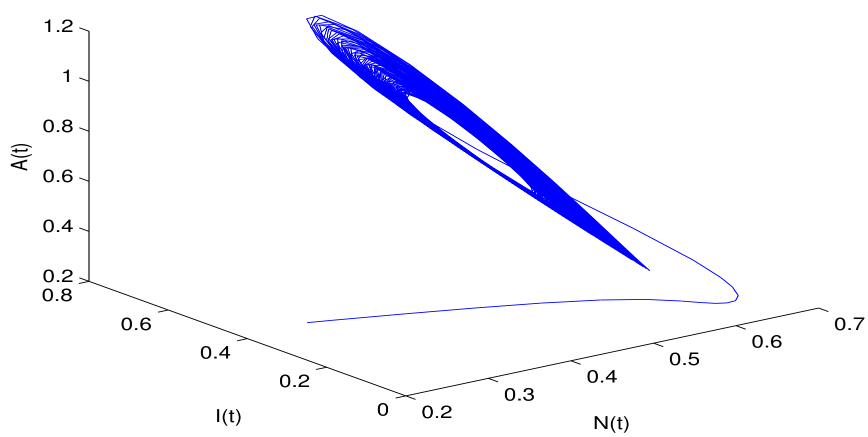

FIGURE 7. When $m > 0$ the interior equilibrium with delay $E^*(0.57, 0.45, 0.68)$ is unstable for the parametric values: $K = 0.1; \alpha = 0.28; \beta = 0.3; \gamma = 0.24; \xi = 0.2; \mu = 0.08; m = 0.5$ and Hopf bifurcation occurs at $\tau = 7.55 > \tau_0^+ = 7.53$.

examined that E_0 and E^* is stable in the range $\tau \in (0, \tau_0^+)$. After this point, the system is an unstable and oscillatory character of the system occurs. Sensitivity analysis for basic influence number R_A and for the state variables at interior steady state is also studied. At last, to defend the

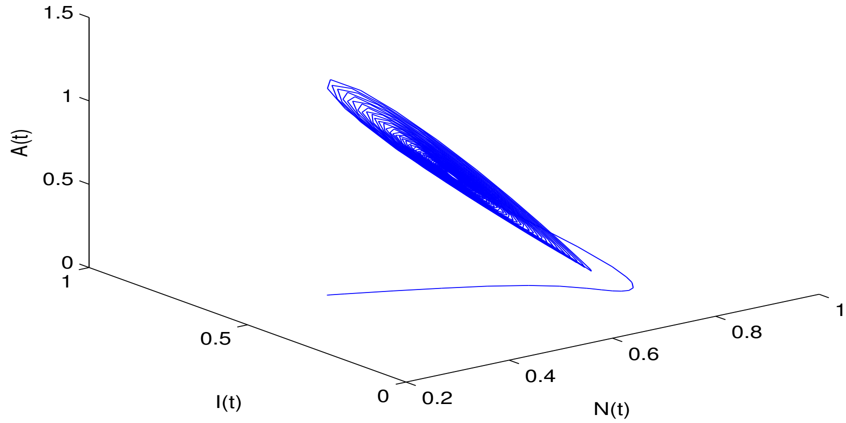

FIGURE 8. When $m=0$ the interior equilibrium with delay $E^*(0.48,0.43,0.76)$ is stable for the parametric values: $K=0.1;\alpha=0.28;\beta=0.3;\gamma=0.24;\xi=0.2;\mu=0.08;\tau=7.57<\tau_0^+=7.59$.

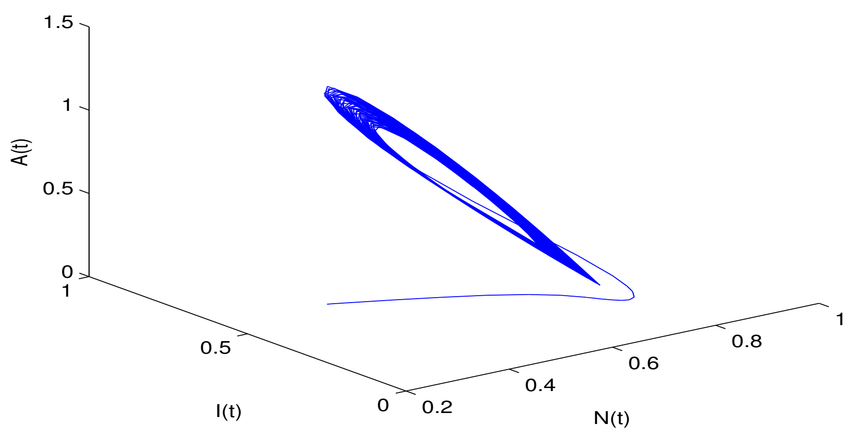

FIGURE 9. When $m=0$ the interior equilibrium with delay $E^*(0.48,0.43,0.76)$ is unstable for the parametric values: $K=0.1;\alpha=0.28;\beta=0.3;\gamma=0.24;\xi=0.2;\mu=0.08$ and Hopf bifurcation occurs at $\tau=7.61>\tau_0^+=7.59$.

analytical findings, we perform the numerical simulations with the different set of parameters. We observe that as the effect of media coverage(m) increases, the fraction of thinker population is going on increases.

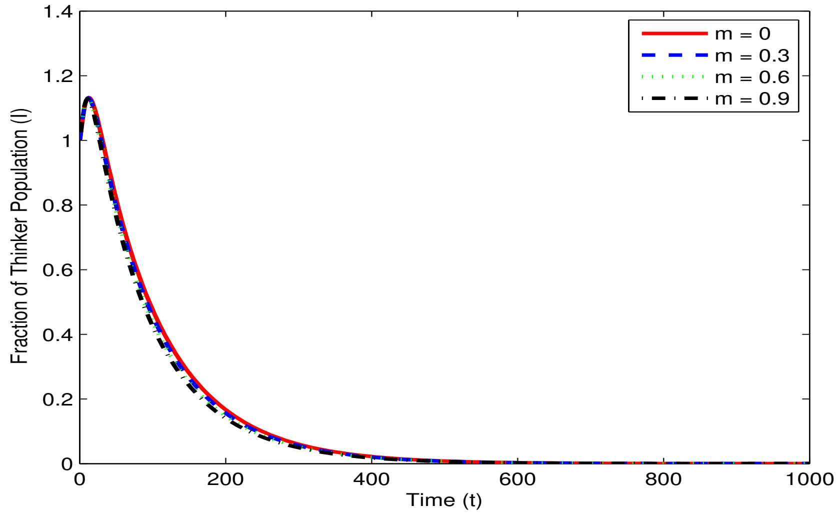

FIGURE 10. The effect of media coefficient m on I for the parametric values $K = 0.07; \alpha = 0.04; \beta = 0.04; \gamma = .03; \xi = 0.1; \mu = 0.05$ and $R_A = 0.75$.

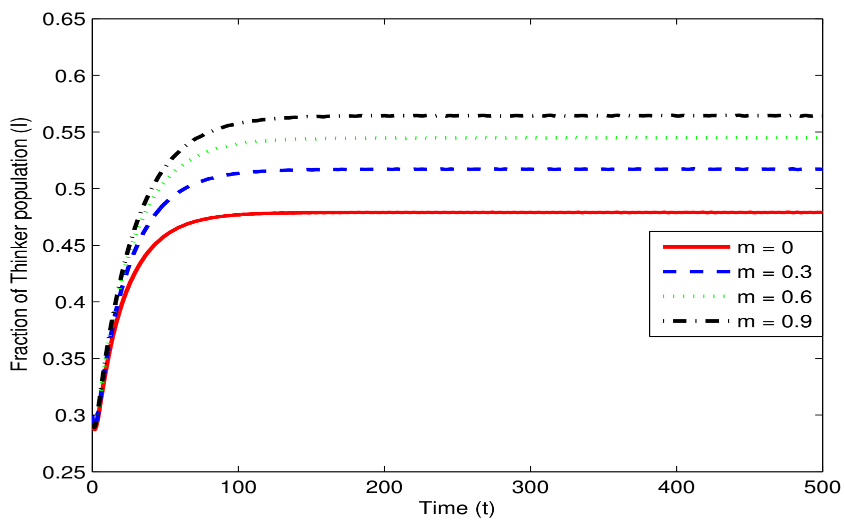

FIGURE 11. The effect of media coefficient m on I for the parametric values $K = 0.09; \alpha = 0.029; \beta = 0.33; \gamma = .02; \xi = 0.15; \mu = 0.75$ and $R_A = 3.62$.

Conflict of Interests

The author declare that there is no conflict of interests.

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REFERENCES

[1] F. M. Bass, A New Product Growth Model For Consumer Durable, Manage. Sci.15(5) (1969): 215-227.

[2] H. Gatignon, T. Robrtson, A Propositional Inventory for New Diffusion Research, J. Consumer Res. 11(4) (1985):849-867.

[3] J. Arndt, Role of Product related Conversion in the Diffussion of a New Products, J. Market. 4 (1967): 291-295.

[4] J. A. Silk, Overlap Among Self-Degignated Opinion Leader : A Study of Selected Dendal Product and Service, J. Market. Res. 3 (1966): 255-259.

[5] A. Robertson, The Nature of Quantitative Genetic Variation in Heritage from mendel, Brink University of Wisconsin Press Madison 265-280 (1967).

[6] E. James Engel, D. Roger Blackwell, W. Paul Miniard, Consumer Behavior 6th ed., The Dryden Press, Hinsdale, IL. (1990).

[7] P. Kotler, G. Zaltman, Targeting Prospects for a New Product, J. Advertising Res. 16 (1976): 7-18.

[8] V. Mahajan, M. Eitan, Innovation Diffusion and New-Product Growth Models in Marketing, J. Market. 43 (1979): 55-68.

[9] E. M. Rogers, M. Everett, Diffusion Of Innovation (4th ed.), New York: Free Press (1995).

[10] G. Feichtinger, Chaotic Behavior in an Advertising Diffusion Model, Int. J. Bifurcation Chao 1 (1995): 255-263.

[11] H. Singh, J. Dhar, H. S. Bhatti, Sumit Chandok, An epidemic model of childhood disease dynamics with maturation delay and latent period of infection, Model. Earth Syst. Environ. 2(2016): Article ID 79.

[12] A. Calsinna, J. P. Palmada, J. Ripoll, Optimal latent period in a bacteriophage population model structured by infection-age, Math. Models Methods Appl. Sci. 21(4) (2011): 693-718.

[13] A. Kaddar, A. Abta, H. Alaoui, Stability analysis in a delayed SIR epedemic model with a saturated incidence rate, Nonlinear Anal., Model. Cantrol 15(3) (2010), 299-306.

[14] X. Liu, Y. Liu, Y. Zhang, Z. Chen, Z. Tang, Q. Xu, Y. Wang, P. Zahoo, Z. Qi, Pre-existing immunity with high neutralizing activity to 2009 pandemic h1n1 influenza virus in shanghai population, Plos One 8(3) (2013), Article ID e58810.

[15] G. P. Sahu, J. Dhar, Analysis of an sveis epedemic model with partial temporary immunity with saturation incidence rate, Appl. Math. Model. 36(3) (2012), 908-923.

[16] C. Sun, W. Yang, J. Arino, K. Khan, Effect of media-induced social distancing on disease transmission in a two patch setting, Math. Biosci. 230(2) (2011), 87-95.

[17] S. Funk and E. Gilad and C. Watkins and V. A. Jansen, The spread of awreness and its impact on epedemic outbreaks, Proc. Nat. Acad. Sci. 106(16) (2009), 6872-6877.

[18] J. M. Tchuenche, N. Dube, C. P. Bhunu, C. T. Bauch, The impact of media coverage on the transmission dynamics of human influenza, BMC Public Health 11(Suppl 1) (2011), Article ID S5.

[19] H. Singh, J. Dhar, H. S. Bhatti, Bifurcation in disease dynamics with latent period of infection and media awreness, Int. J. Bifurcation Chaos 26(2016), Article ID 1650097.

[20] J. Dhar, M.Tyagi, P. Sinha, Dynamical Behavior in an Innovation Diffusion Marketing Model with Thinker Class of Population, Int. J. Buss. Mgt. Eco. Res. 1(1) (2010): 79-84.

[21] P. Vanden Driwssche, J. Watmough, Reproduction Numbers and Subthreshold Endemic Equilibria For Compartmental Models Of Disease Transmission, Math. Biosci. 180 (2002): 29-48.

[22] S.Ruan, Absolute Stabilty, Conditional Stability and Bifurcation in Kolmogrov-Type Predator-Prey Systems with Discrete Delays, Q. Appl. Math. 59(1) (2001): 159-174.

[23] H. Singh, J. Dhar, H. S. Bhatti, Dynamics of prey-generalized predator system with disease in prey and gestation delay for predator, Model. Earth Syst. Environ. 2(2016), Article ID 52.

[24] J. Guckenheimer, P. Holmes, Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields, Berlin, Springer (1983).

[25] Y. Song, M. Han, J. Wei, Stability and Hopf Bifurcation Analysis on a Simplified Bem Neural Network With Delays, Phys. D, Nonlinear Phenoma 200(3) (2005): 184-204.