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A QUEUEING MODEL FOR SOLVING THREE LAYER SUPPLY CHAIN

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Abstract. In this paper a supply chain involving a single manufacturer and several retail outlets is considered. The manufacturer makes several products made in batches and in stored in a different warehouse after production occurs.

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1.Introduction

In this section, the supply chain management includes materials/supply management from the supplier of raw materials toltimate product and also, network of organizations that are involved, through upstream and downstream stages, in the different processes and activities that produce value in the form of products and services in the hands of the consumer. Therefore a supply chain consists of all parties involved in satisfying a customer request. nd also, the supply chain includes not only the manufacturer and suppliers, but

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also transporters, retailers, and even customers themselves [1,7 ,9 and 12]. The supply chain activities constitute various decisions are involved in their successful design and operation. Decisions regarding stocking and control of inventory of stocks are a common problem to all enterprises. Asset managers of large enterprises have the responsibility of determining the approximate inventory level in the form of components and finished goods to hold at each level of supply chain in order to guarantee specified end customer service levels.

A Given the size and complexity of the supply chain, a common problem for this asset manager is to know how to quantify the trade-off between service level and investment in inventory required to supporting these service levels[3,8 and 14]. The problem is made even more difficult because the supply chains are highly dynamic with uncertainty in demand, variability in processing times at each stage of the supply chain, multiple dimensions for customer satisfaction, finite resources, etc. The goal of a supply chain should be to maximize over supply chain profitability. Supply chain profitability is the difference between the revenue generated from the customer and the total cost incurred across all stages of the supply chain. One of the challenges in supply chain management is to control the capital in inventories. The objective of inventory control is therefore to balance conflicting goals like keeping stock levels down to have cash available for other purposes and having high stock levels for the continuity of the production and for providing a high service level to customers[4,5,6 and 10]. A good inventory management system has always been important in the workings of an effective manufacturing supply chain. Queuing systems are the natural models when dealing with problems where, the main characteristics are congestion and jams[2]. In this paper, the performance measures of the manufacturing supply chain and also, we use queue to analyze logistics processes. We review the articles of inventory management in logistics chains including single-product multi-stage systems and multi-product systems [11 and 13]. We present our model in detail, , we analyze the proposed model with an example and also, we present the results of our computational to analyze performance.

2. Preliminaries

In this section, we consider a three supply chain network including n retailers, L warehouses and a manufacturing plant. This network offers L types of product to the customers arrived to retailers' node. Customers' demands enter to the retailers and the whole demand accumulation for each product is forwarded to warehouses of that product. Then production authorization (PA) is sent to the manufacturing plant of that product with regards to the fact total number of orders should be Q_j for each products warehouse j =1,2,..., L(orders of each product from warehouse to manufacturing plant are sent in batch size). We consider a multi-item production–inventory system where, a manufacturing plant produces L type of products and separated inventory buffers are kept for each product. If it is available, an order is satisfied from buffer stock. If not, the demand is backordered. We assume that manufacturing plant serves producer of each product. After production process, products batches delivered to the warehouse of that product and fulfill the retailers demand.

2.1. Notations

The notations used in this paper are as follow:

$Q_{\mathbf{j}}$: Number of units in one bucket of product j; j = 1,2,..., L

$K_{\mathbf{j}}$: Total number of buckets at warehouse of product

$Z_{\mathbf{j}}$: Maximum inventory at warehouse of product j

$\lambda_{\alpha j}$: The retailers' demand arrival rate of product j, $\sum_{i=1}^{n} \lambda_i \, q_{ij}$

$c_{\alpha j}^2$: The retailers demand arrival rate of product, $j(\frac{1}{\lambda_{\alpha j}} \sum_{i=1}^{n} \lambda_i \, q_{i,j} C)$

μ_j : The service rate of product $j$in manufacturing plant units/unit time

A_j *:The* number of orders of product j arrived at manufacturer

c_j^2 : The probability that an arrival is for product j of service rate of product j in manufacturing plant units/unit time

P_j : The probability that an arrival is for product j

τ_j : The Mean of setup time of product j

η_j : The variance of setup of product j

U : The setup time

W : The process time of batch

I_j : The inventory level of product j

N_j : The number of orders of product $j$at manufacturing plant in a $GI\ /G/1$ queue

B_j : The backorders level of product j at warehouse

R_j : The number of orders arrived at warehouse of product j, but after the last batch was released for processing

h_j : The inventory holding cost for product j

b_j : The back order cost for product j

C_{sj} : The order set up cost for product j

ρ : The intensity of the manufacturing plant

L_{ji} : The lead time for logistics for retailer i to receive items from warehouse of product j

Γ_{ji} : The expected number of orders from product j at retailer i, in the queue $M^{Cj}/M/\infty$ in steady state (Actual demand).

ξ : The service rate of logistics process

W_j : The expected waiting time at warehouse of product j just due to backordering

θ_{ji} : The expected demand for product j during replenishment lead time for each item at retailer $i(\lambda_{\alpha j,L_{ji}})$

2.2. Assumptions

The assumptions of the developed model are as follow: Customers' demand includes all types of products. We assume the orders of retailer $i$is as an independent renewal process with a constant rate $x_i \geq 0$ and Squared Coefficient of Variation Ci^2. The probability vectors, $q_i = (q_{i1},\ q_{i2}\ ,...,\ q_{iL})$ define customers' demand from each kind of product at retailer i, $(\sum_{j=1}^{L} q_{ij} = 1,\ 0 \leq q_{ij} \leq 1;\ i = 0, 1, ..., n)$. Therefore, the orders of warehouse j are as the processes with a constant rate , $\lambda_{\alpha j} = \sum_{i=1}^{n} \lambda_i\ q_{ij}$ In other $c^2_{\lambda\alpha,j} = \frac{1}{\lambda_i} q_{ij} c_i^2$ words, $\lambda_{\alpha,j}$ and $c^2_{\alpha,j}$ are mean arrival rates of the aggregated input streams of product j. In our problem it is assumed that each warehouse holds one product type in batch size Q_j which maximum number of batches is j_K. Therefore, maximum inventory level of warehouse of product $j$is $Z_j = K_j \times Q_j$. We apply production authorization (**PA**) system to produce each product type. The PA system is a generalized pull based production control system. We assume that products in inventories are stored in batches for each product j, and there is a PA card attached to each batch. In this paper, we consider the case when the number of PA cards is the same as the number of batches. The PA system operates in the following way whenever Qj units are depleted from a batch in the inventory; the corresponding PA card is transmitted to the manufacturing plant and also is served as new production orders that trigger the manufacturing plant to begin its production process. In general, the manufacturing plant uses a FCFS discipline to produce these orders. Once the manufacturing plant produces Q_j units, the finished units and the PA card are sent to the warehouse. In the event when a customer places an order and there is no production inventory available, we assume that this customer wait until the product becomes available Production policy is make to stock strategy which is based on forecast and operates under (K$_{j-1}$,K$_j$) inventory control rule for warehouse of product j. The base of above assumptions, batch arrival streams in manufacturer is following a Poisson process with rate $c^2_\alpha(B) = \sum_{j=1}^{l} \frac{\lambda_{\alpha,j}}{Q_j}$following the asymptotic approach suggested by Equation[11], the batch arrival streams in manufacturer is $C^2_\alpha = \frac{\frac{\sum_{j=1}^{L} \lambda_{\alpha,j} C^2_{\alpha,j}}{Q_j}}{\lambda_\alpha(B)}$. We define the probability that an arrival in manufacturer is for product j as: $P_j = \frac{\lambda_{\alpha,j}}{Q_j \lambda_\alpha(B)}$ when it switches from producing one product type to another. We assume U and W be random variables that denote the setup time and process time experienced by a batch, respectively. The probability that a batch of product j experiences a setup (a random variable with mean τ and variance η is $1 - P_j$ we can obtain the mean of setup time experienced by an arbitrary batch as:

$$E(U) = \sum_{j=1}^{L} (1 - P_j)\tau_j. \tag{1}$$

$$C^2U = \frac{var(u)}{E^2(U)} = \frac{\sum_{j=1}^{L} P_j(1 - P_j)\eta_j}{(\sum_{j=1}^{L} P_j(1 - P_j)\tau_j)^2}. \tag{2}$$

We assumed that unit production times at manufacturer for product j are i.i.d. generally distributed random variables, which is denoted by B_j, with $\frac{1}{\mu_j} = E(B_j)$, C_j^2 .Thus, mean production time for batch product j is $\frac{Q_j}{\mu_j}$ and coefficient of variation $\frac{C_j^2}{Q_j}$ similarly, we can obtain mean processing time for an arbitrary batch as:

$$E(W) = \sum_{j=1}^{L} P_j \frac{Q_j}{\mu_j}. \tag{3}$$

$$C^2W = \sum_{j=1}^{L} Q_j C_j^2. \tag{4}$$

We can obtain the mean of the effective batch service time S, $(S = U + W)$ of an arbitrary batch, from which we can then compute the corresponding mean and coefficient of variation as following as:

$$E(S) = E(U) + E(W). \tag{5}$$

$$C^2S = C^2U + C^2W. \tag{6}$$

The utilization of the manufacturing plant is given by

$$\rho = \sum_{j=1}^{l} \frac{\lambda_{\alpha,j}}{Q_j} E(s) = \lambda_\alpha(B)E(S). \tag{7}$$

The system incurs a holding cost h $_j$ per unit of inventory of product j per unit time, a backordering cost b $_j$ per unit of product j per unit time and C $_{sj}$ order setup cost for product j (\$ per set up). The goal of modeling such a supply network is to minimize supply chain total cost in order to find optimal values of K_j , Q_j .Costs contain inventory holding cost of h_j , back ordering cost of b_j, and order set up cost of C_{sj} .

3. Computing Inventory Systems

In this paper, we would like to minimize the expected total cost at the warehouses. Mathematically, we can express:

$$Min \sum_{j=1}^{L} TC(K_j, Q_j) = \sum_{j=1}^{L}(h_j E[I_j] + E[B_j] + C_{sj}(\frac{\lambda_{\alpha,j}}{Q_j}))$$

$$St. \quad K_j, Q_j \in Z^+ \tag{8}$$

for computing inventory and backorders, we use stochastic equations which capture the properties of the system as in Equation [12]. Observing that,

$$R_j = A_j - [\frac{A_j}{Q_j}]Q_j, \quad j = 1, 2, 3, ..., L, \tag{9}$$

$$B_j = max[N_j Q_j + R_j - K_j Q_j, 0] \quad j = 1, 2, 3, ..., L, \tag{10}$$

$$I_j = max[N_j Q_j - N_j Q_j - R_j, 0] \quad j = 1, 2, 3, ..., L, \tag{11}$$

the corresponding steady state probability distribution for R_j, N_j, B_j, I_j are as follow: R_j is uniformly distributed from 0 to $Q_j - 1$.

$$P\{R_j = m\} = \frac{1}{Q_j}, m = 0, 1, Q_j - 1, \tag{12}$$

thus, we use a development described in Equation[12] to approximate the probability distribution of batches using a geometric distribution of the following form:

$$P\{N = m\} = \begin{cases} 1 - \rho & if m = 0, \\ \rho & if m = 1, 2, 3, ... \end{cases} \tag{13}$$

where, $\sigma = \frac{(N-\rho)}{N}$, $N = \lambda_\alpha(B)\omega_0 + \rho$ and $\omega_0 = \{\frac{\rho^2(1+C_s^2)}{1+\rho^2 c_s^2}\}\{\frac{C_s^2+\rho^2 C_s^2}{2\lambda_\alpha(1-\rho)}\}$. From Equation[13], we also obtain the approximation of the distribution of the number of orders of product j :

$$P_j = \{N_j = m_j\} = \begin{cases} 1 - (\frac{\rho}{\sigma}) & ifm_j = 0, \\ (\frac{\rho}{\sigma})(1 - r_j)r_j^{mj} & ifm_j = 1, 2, 3, ... \end{cases} \tag{14}$$

where, $r_j = \frac{P_j\sigma}{1-\sigma(1-P_j)}$ and $P_j = \frac{\lambda_{\alpha j}}{Q_j\lambda_\alpha(B)}$ steady state probability distributions I_j, B_j are as follow:

$$P\{I_j = m\} = \frac{1}{Q_j} P_{Nj}(\frac{z_j - m}{Q_j}); \qquad m = 1, 2, 3, ..., K_j Q_j, \tag{15}$$

$$P\{B_j = m\} = \frac{1}{Q_j}(\frac{z_j - m}{Q_j}); \qquad m = 1, 2, 3, ... \quad . \tag{16}$$

We can obtain E(I$_j$) and E(B$_j$) as following as:

$$E(I_j) = \sum_{j=1}^{z_j} \frac{Z_j - i}{Q_j} P_{Nj}(i), \tag{17}$$

$$E(B_j) = \sum_{i=0}^{\infty} \frac{1}{Q_j} P_{Nj} \frac{Z_j + i}{Q_j} \tag{18}$$

and

$$E(B_j) = \sum_{m=1}^{\infty} \frac{m}{Q_j} P_{Nj}(K_j + \frac{m}{Q_j}). \tag{19}$$

Now, we can calculate optimal inventory level of every product. by Equation (8).

4. Model Performance Measures

In this section, the stock out probability at warehouse of product j is the fraction of time that the on-hand inventory at warehouse of product j is zero and is obtained as follows:

$P\{H_j = 0\} = P\{Z_j \leq N_j(t)Q_j + R_j(t)\}$

$= P\{\frac{Z_j - R_j(t)}{Q_j} \leq N_j(t)\} = P\{K_j - \frac{R_j(t)}{Q_j} \leq N_j(t)\}$

$= \frac{1}{Q_j} P\{K_j \leq N_j(t)\} + \frac{Q_j - 1}{Q_j} P\{K_j - \frac{R_j(t)}{Q_j} \leq N_j(t)$

$$= \frac{1}{Q_j}(\frac{\rho}{\sigma})r_j^{kj} + \frac{Q_j - 1}{Q_j}(\frac{\rho}{\sigma})r_j^{kj-1}. \tag{20}$$

Also, the fill rate at warehouse of product j is the fraction of time that the on-hand inventory at warehouse of product j is greater than zero:

$P\{I_j > 0\} = P\{Z_j > N_j Q_j + R_j\} = 1 - P\{I_j = 0\}$

$$= 1 - \frac{1}{Q_j}(\frac{\rho}{\sigma})r_j^{kj} - \frac{Q_j - 1}{Q_j} r_j^{kj-1}. \tag{21}$$

Also the lead time of product j at its manufacturing plant is given by Equation(22)

$$W_{sj} = \frac{(Q_j - 1)}{2}(\frac{1}{\lambda_{\rho j}}) + \omega_0 + (\frac{Q_j}{\mu_j}), \tag{22}$$

where, $\frac{(Q_j-1)}{2}(\frac{1}{\lambda_{\rho j}})$ is batch forming time of product j and $(\frac{Q_j}{\mu_j})$ is mean production time for product j batch.

The squared coefficient of variation of the inter- departure times which is produced from the warehouses In this section, we show how to derive the characteristics of batching departure streams from the manufacturing plant with known $\lambda_\alpha(B)$, $C_\alpha^2(B)$ which are obtained as:

$$C\rho^2(B) = (1 - \rho^2)\{\frac{c_\alpha^2(B) + \rho^2 C_s^2}{1 + \rho^2 C_s^2}\} + \rho^2 C_s^2. \tag{23}$$

We use the approximation of SCV of the inter-departure times for the batches from the manufacturing plant with batch setups in the $GI\ /G/1$ queue which is given in Equation[12] and shown in Equation (23):

$$\lambda_{\alpha j} = \sum_{j=1}^{L} q_{ij}\lambda_{\tau}. \tag{24}$$

Also, we use the following approximation of the SCV of the inter-departures of individuals from the warehouse of product j :

$$C^2_{\alpha j}(I) = Q_j C^2_{\alpha}(B) + Q_j - 1, \tag{25}$$

where, Q_j denoting the size of fixed batches of product j. When a product departs from the warehouse, there is a probability q_{ji} that the product will be routed to retailer t herefore the mean inter-arrival time for arrivals to retailer i are given by

$$C^2_{\alpha j} = qijC^2_{\alpha j}(I) + 1 - q_{ij}. \tag{26}$$

In continue, we extend the model by adding logistics processes. We assume that there is some logistics time to supply products from warehouses to retailers. We model the logistics process of product j by using M/M$^c/\infty$ queue in continuous time, where, c_j is vehicle capacity which is deterministic and logistics time is exponential. We assume that the logistics process depends on the demands of customers for its arrival process. For the performance to analyze performance of M/M $^C/\infty$ queue, we use the results of Equation[15] and Equation[16]. We obtain mean lead time of product j from its warehouse at retailer i, by using Little's law we have:

$$W_j = \frac{E(B_j)}{\lambda_{\alpha j}}, \tag{27}$$

$$\Gamma_{ij} = \frac{q_{ij}\lambda_{\alpha j}}{\xi} C_j = \frac{1}{\xi}(q_{ij}\lambda_{\alpha j} c_j), \tag{28}$$

$$L_{ij} = \frac{\Gamma_{ij}}{q_{ij}\lambda_{\alpha j}} = \frac{1}{\xi} C_i. \tag{29}$$

We can compute expected demand of product $j$at retailer $i$during replenishment lead time as

$$\theta_{ij} = \lambda_{\alpha ji} L_{ij}. \tag{30}$$

5. Discuss Inventory Problem

In this section, the inventory problem and try to simulate it away from the mathematical treatment of the problem. The simulation method is run according to some fed in rules .by the end of running the simulated case we decide to study the consumption under the given rates of supply and demand followed, and how much money will be spent to maintain the flow through some specified length of time.

The inventory problem applied to some specified commodity or merchandize depends on some main factors, namely,

1. The time between the ordering and the arrival of the commodity. 2. The carry cost (storage, insurance, interest, deterioration…). 3. The lost of good will or trust. 4. Reordering cost. 5. The amount of demand in one day. 6. There is never more than one order outstanding (refilling). 7. We start the simulation with no waiting orders. 8. The demand for one day is as either a random number or an average demand value for that merchandize for that time of the day or the year or both. We can actually choose the refilling process as we desire .we can have different policies for this refining. This policy gives at which point we must start refilling and by which quantity of this merchandize .we should refill our store. We can actually choose the refilling process. We can have different policies for this refilling. This policy gives at which point we must start refilling. The following is a flowchart gives the cost of keeping stocks. Taking the total cost =reorder cost + Carrying (storage and insurance cost)+ lost sales cost, that is the total charges of keeping.

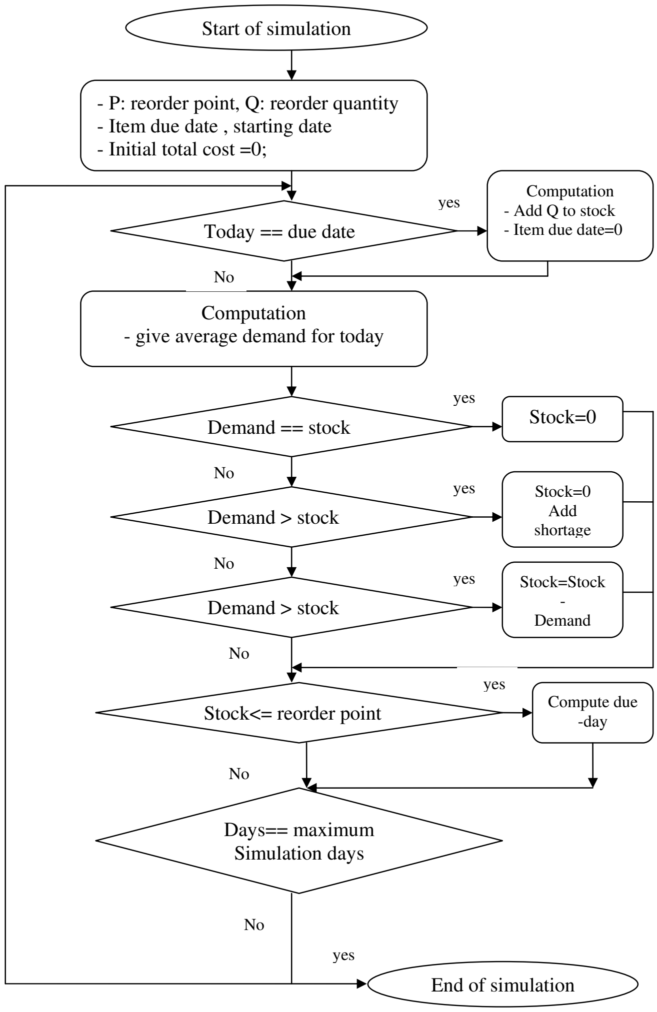

FIGURE 1. The flow chart of the inventory system

6. Main results

In this section, we analyze the model by an example. We consider a supply chain network which produces three product types. The supply chain includes a manufacturing plant, three warehouses and two retailers where, the demands for products are characterized by $\lambda_1 = 0.6$ and $\lambda_2 = 0.8$ the probability vectors $C_{\alpha 1}^2 = C_{\alpha 2}^2 = 1$ define customers' demand for three products at two retailers. Information of the manufacturing plant and costs of three warehouses are showed in Table 1.

Table1: Information of the manufacturing plant

Product type	μ_j	C_j^2	τ_j	ξ_j	h_j	b_j	C_{sj}
1	0.5	0.7	0.1	1	8	80	6
2	0.6	0.8	0.3	0.9	10	100	10
3	0.8	0.6	0.15	0.99	14	120	12

We obtain optimum value K_j by variety values of batch sizing Q_j, for three products. In condition that, we increase Q_j, and optimum number of batches (K_j^*) and optimum maximum inventory level is variable but there is not any trend. Furthermore, total cost of three products is increasing in Q_j of Table2.

Table2: Total cost variation by increasing Q_j if $\frac{b_j}{h_j} = 10$

Product Type	Q_j	K_j^*	ρ_j	$E[I_j]$	$E[B_j]$	W_j
1	*2*	*40*	0.6478	58.4301	100.715	296.6715
	4	*4*	0.0543	509.4270	70.3479	70.0239
	6	*5*	0.1830	1.2278e+003	45.9475	53.4018
	8	*7*	0.0907	2.1918e+003	33.811	57.2372
	10	*8*	0.0563	3.3783e+003	26.9097	69.6334
	14	*13*	0.0394	6.3245e+005	19.0833	110.3028
	26	*24*	0.0235	1.8941e+004	11.0573	327.4489
	40	*36*	00098	3.1748e+004	6.6081	745.0176

2	*3*	*3*	0.0056	3.1748e+004	98.1773	105.2937
	6	*5*	0.3066	183.3429	51.4060	50.8385
	9	*8*	0.0907	893.3124	34.4818	56. 3591
	15	*14*	0.0465	1.9782e+003	21.5401	104.6691
	24	*22*	0.0109	4.9555e+003	15.3412	233.0138
	30	*27*	0.0081	1.0238e+003	11.3783	351.4637
3	*5*	*5*	0.1238	1.3367e+003	50.1774	54.5614
	15	*14*	0.0212	9.3725e+003	15.3870	98.9385
	30	*27*	0.0081	3.1185e+004	8.1782	334.1556
	50	*1*	0.0043	6.2078e+004	4.4188	888.8527

6. Conclusion

In this paper, we presented a model to analyze performance of a three-layer supply chain which produces more than one product. We used $GI/GI/1$ queue operating under $(K_j{-}1,\ K_j)$ inventory control policy to analyze the performance of warehouses. We obtained performance of measures such as stock-out probability, fill-rate and lead time of warehouses in proposed model. In the model, we used $M/M^c/\infty$ queue to analyze logistics process. In this paper, we surveyed the effect of order batching in multi-product, multi-supply chains. In future researches, we can consider a center warehouse that in the stock-out condition in each warehouse; customers' demands are satisfied (adding transmittal cost). Also, the pricing concept can be added to our model as an attractive aspect of future research.

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