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LEVEL SEPARATION ON INTUITIONISTIC FUZZY T_2 SPACES

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Abstract: In this paper we give eight new notions of intuitionistic fuzzy T_2 spaces and investigate some relationship among them. Also we investigate some relations between our notions and other given notions of intuitionistic fuzzy T_2 spaces. We show that all these notions satisfy hereditary and productive property of T_2 spaces. Under some conditions it is shown that image and preimage preserve intuitionistic fuzzy T_2 spaces.

Keywords: fuzzy set; intuitionistic set; intuitionistic fuzzy set; intuitionistic topological space; intuitionistic fuzzy topological space; intuitionistic fuzzy T_2 space.

1. Introduction

Fuzzy topology is an important research field in fuzzy mathematics which has been established by Chang [1] in 1968 based on Zadeh's [2] concept of fuzzy sets. Later, the notion of an intuitionistic fuzzy set was introduced by Atanassov [3] in 1986 which take into account both the degrees of membership and nonmembership subject to the condition that their sum does not exceed 1. Coker and coworker [4][5] [6] introduced the idea of the topology of intuitionistic fuzzy sets. Since then, Coker et al[7], Amit Kumar Singh et al. [8], S. J. Lee et al. [9], Saadati et al [10], Estiaq Ahmed et al. [11] [12][13][14][15] subsequently initiated a study of intuitionistic fuzzy topological spaces by using intuitionistic fuzzy sets. Various researchers work particularly

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on intuitionistic fuzzy T_2 spaceses [15][16][8]. In this paper, we investigate the properties and features of intuitionistic fuzzy T_2 Space.

2. Notations and Preliminaries

Through this paper, X is a nonempty set, r and s are constants in $(0,1)$. T is a topology, t is a fuzzy topology, $\mathcal{T}$ is an intuitionistic topology and τ is an intuitionistic fuzzy topology. λ and μ are fuzzy sets, $A = (\mu_A, \nu_A)$ is intuitionistic fuzzy set. By $\underline{0}$ and $\underline{1}$, we denote constant fuzzy sets taking values 0 and 1 respectively.

Definition 2.1 [1]**.** Let X be a non empty set. A family t of fuzzy sets in X is called a fuzzy topology (FT, in short) on X if the following conditions hold.

(1) $\underline{0}, \underline{1} \in t$,
(2) $\lambda \cap \mu \in t$, for all $\lambda, \mu \in t$,
(3) $\cup \lambda_j \in t$, for any arbitrary family $\{\lambda_j \in t,\ j \in J\}$.

The above definition is in the sense of C. L. Chang. The pair (X, t) is called a fuzzy topological space (FTS, in short), members of t are called fuzzy open sets (FOS, in short) in X and their complements are called fuzzy closed sets (FCS, in short) in X.

Definition 2.2 [17]. Suppose X is a non empty set. An intuitionistic set A on X is an object having the form $A = (X, A_1, A_2)$ where $A_1$and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of member of A while A_2 is called the set of non-member of A. In this paper, we use the simpler notation $A = (A_1, A_2)$ instead of $A = (X, A_1, A_2)$ for an intuitionistic set.

Remark 2.1 [17]. Every subset A of a nonempty set X may obviously be regarded as an intuitionistic set having the form $A = (A, A^c)$ where $A^c = X - A$.

Definition 2.3 [17]**.** Let the intuitionistic sets A and B in X be of the forms $A = (A_1, A_2)$ and $B = (B_1, B_2)$ respectively. Furthermore, let $\{A_j, j \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_j = \left(A_j^{(1)}, A_j^{(2)}\right)$. Then

(a) $A \subseteq B$ if and only if $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$,
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(c) $\bar{A} = (A_2, A_1)$, denotes the complement of A,

(d) $\cap A_j = (\cap A_j^{(1)}, \cup A_j^{(2)})$,

(e) $\cup A_j = (\cup A_j^{(1)}, \cap A_j^{(2)})$,

(f) $\phi_{\sim} = (\phi, X)$ and $X_{\sim} = (X, \phi)$.

Definition 2.4 [18]**.** Let X be a non empty set. A family $\mathcal{T}$ of intuitionistic sets in X is called an intuitionistic topology (IT, in short) on X if the following conditions hold.

(1) $\phi_{\sim}, X_{\sim} \in \mathcal{T}$,

(2) $A \cap B \in \mathcal{T}$ for all $A, B \in \mathcal{T}$,

(3) $\cup A_j \in \mathcal{T}$ for any arbitrary family $\{A_j \in \mathcal{T}, j \in J\}$.

The pair $(X, \mathcal{T})$ is called an intuitionistic topological space (ITS, in short), members of $\mathcal{T}$ are called intuitionistic open sets (IOS, in short) in X and their complements are called intuitionistic closed sets (ICS, in short) in X.

Definition 2.5 [3]**.** Let X be a non empty set. An intuitionistic fuzzy set A (IFS, in short) in X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$, where μ_A and ν_A are fuzzy sets in X denote the degree of membership and the degree of non- membership respectively with $\mu_A(x) + \nu_A(x) \leq 1$.

Throughout this paper, we use the simpler notation $A = (\mu_A, \nu_A)$ instead of $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ for intuitionistic fuzzy sets.

Remark 2.2. Obviously every fuzzy set λ in X is an intuitionistic fuzzy set of the form $(\lambda, 1 - \lambda) = (\lambda, \lambda^c)$ and every intuitionistic set $A = (A_1, A_2)$ in X is an intuitionistic fuzzy set of the form $(1_{A_1}, 1_{A_2})$.

Definition 2.6 [3]. Let X be a nonempty set and A, B are intuitionistic fuzzy sets on X be given by (μ_A, ν_A) and (μ_B, ν_B) respectively, then

(a) $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if $A \subseteq B$ and $B \subseteq A$,

(c) $\bar{A} = (\nu_A, \mu_A)$,

(d) $A \cap B = (\mu_A \cap \mu_B, \ \nu_A \cup \nu_B)$,

(e) $A \cup B = (\mu_A \cup \mu_B, \ \nu_A \cap \nu_B)$.

Definition 2.7 [5]. Let $\{A_j = \left(\mu_{A_j}, \nu_{A_j}\right)\ , j \in J\}$ be an arbitrary family of IFSs in X. Then

(a) $\cap A_j = (\cap \mu_{A_j},\ \cup \nu_{A_j})$,

(b) $\cup A_j = (\cup \mu_{A_j},\ \cap \nu_{A_j})$,

(c) $0_\sim = (\underline{0}, \underline{1})$, $1_\sim = (\underline{1}, \underline{0})$.

Definition 2.8 [5]. An intuitionistic fuzzy topology (IFT, in short) on a nonempty set X is a family τ of IFSs in X, satisfying the following axioms:

(1) $0_\sim, 1_\sim \in \tau$,

(2) $A \cap B \in \tau$, for all $A, B \in \tau$,

(3) $\cup A_j \in \tau$ for any arbitrary family $\{A_j \in \tau,\ j \in J\}$.

The pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS, in short), members of τ are called intuitionistic fuzzy open sets (IFOS, in short) in X, and their complements are called intuitionistic fuzzy closed sets (IFCS, in short) in X.

Remark 2.3 [19]**.** Let X be a non empty set and $A \subseteq X$, then the set A may be regarded as a fuzzy set in X by its characteristic function $1_A: X \to \{0,1\} \subset [0,1]$ which is defined by

$$1_A(x) = \begin{cases} 1 & if\ x \in A \\ 0\ if\ x \notin A, i.e., & if\ x \in A^c \end{cases}$$

Again, we know that a fuzzy set λ in X may be regarded as an intuitionistic fuzzy set by $(\lambda, 1 - \lambda) = (\lambda, \lambda^c)$. So every sub set A of X may be regarded as intuitionistic fuzzy set by $(1_A, 1 - 1_A) = (1_A, 1_{A^c})$.

Theorem 2.1. Let (X, T) be a topological space. Then (X, τ) is an IFTS where $\tau = \left\{(1_{A_j}, 1_{A_j^c}), j \in J : A_j \in T\right\}$.

Proof: The proof is obvious.

Note 2.1. Above τ is the corresponding intuitionistic fuzzy topology of T.

Theorem 2.2. Let (X, t) be a fuzzy topological space. Then (X, τ) is an IFTS where $\tau = \{(\lambda,\ \lambda^c) : \lambda \in t\}$.

Proof: The proof is obvious.

Note 2.2. Above τ is the corresponding intuitionistic fuzzy topology of t.

Theorem 2.3. Let $(X, \mathcal{T})$ be an intuitionistic topological space. Then (X, τ) is an intuitionistic fuzzy topological space where $\tau = \left\{\left(1_{A_{j1}}, 1_{A_{j2}}\right), j \in J : A_j = \left(A_{j1}, A_{j2}\right) \in \mathcal{T}\right\}$.

Proof: The proof is obvious.

Note 2.3. Above τ is the corresponding intuitionistic fuzzy topology of $\mathcal{T}$.

Definition 2.9[3]**.** Let X and Y be two nonempty sets and $f: X \to Y$ be a function. If $A = \{(x, \mu_A(x), \nu_A(x)): x \in X\}$ and $B = \{(y, \mu_B(y), \nu_B(y)): y \in Y\}$ are IFSs in X and Y respectively, then the pre image of B under f, denoted by $f^{-1}(B)$ is the IFS in X defined by $f^{-1}(B) = \left\{\left(x, (f^{-1}(\mu_B))(x), (f^{-1}(\nu_B))(x)\right): x \in X\right\}$ = $\{(x, \mu_B(f(x)), \nu_B(f(x))): x \in X\}$ and the image of A under f, denoted by $f(A)$ is the IFS in Y defined by $f(A)$ = $\left\{\left(y, (f(\mu_A))(y), (f(\nu_A))(y)\right): y \in Y\right\}$, where for each $y \in Y$

$$(f(\mu_A))(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_A(x) & if\ \ f^{-1}(y) \neq \phi \\ 0 & otherwise \end{cases}$$

$$(f(\nu_A))(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \nu_A(x) & if\ \ f^{-1}(y) \neq \phi \\ 1 & otherwise \end{cases}$$

Definition 2.10 [6]. Let $A = (x, \mu_A, \nu_A)$ and $B = (y, \mu_B, \nu_B)$ be IFSs in X and Y respectively. Then the product of IFSs A and B denoted by $A \times B$ is defined by $A \times B = \{(x, y), \mu_A \times \mu_B, \nu_A \times \nu_B)\}$ where $(\mu_A \times \mu_B)(x, y) = \min\{\nu_A(x), \nu_B(y)\}$ and $(\nu_A \times \nu_B)(x, y) = \max\{\nu_A(x), \nu_B(y)\}$ for all $(x, y) \in X \times Y$.

Obviously $0 \leq (\mu_A \times \mu_B) + (\nu_A \times \nu_B) \leq 1$. This definition can be extended to an arbitrary family of IFSs.

Definition 2.11 [6]. Let (X_j, τ_j), $j = 1,2$ be two IFTSs. The product topology $\tau_1 \times \tau_2$ on $X_1 \times X_2$ is the IFT generated by $\left\{\rho_j^{-1}\left(U_j\right): U_j \in \tau_j, j = 1,2\right\}$, where $\rho_j: X_1 \times X_2 \to X_j$, $j = 1,2$ are the projection maps and IFTS $\{X_1 \times X_2, \tau_1 \times \tau_2\}$ is called the product IFTS of (X_j, τ_j), $j = 1,2$. In this case $\mathcal{S} = \left\{\rho_j^{-1}\left(U_j\right), j \in J: U_j \in \tau_j\right\}$ is a sub base and $\mathcal{B} = \left\{U_1 \times U_2: U_j \in \tau_j, j = 1,2\right\}$ is a base for $\tau_1 \times \tau_2$ on $X_1 \times X_2$.

Definition 2.12 [5]**.** Let (X,τ) and (Y,δ) be IFTSs. A function $f: X \to Y$ is called continuous if $f^{-1}(B) \in \tau$ for all $B \in \delta$ and f is called open if $f(A) \in \delta$ for all $A \in \tau$.

Definition 2.13 [20]**.** A topological space (X,T) is called T_2 if for all $x, y \in X$ with $x \neq y$, there exists $U, V \in T$ such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

Definition 2.14 [21]**.** A fuzzy topological space (X,t) is called T_2 if for any two distinct fuzzy points $x_\alpha, y_\beta \in X$, there exists $u, v \in t$ such that $x_\alpha \in u$, $y_\beta \in v$ and $u \cap v = \underline{0}$.

Definition 2.15 [8]. Let $A = (\mu_A, \nu_A)$ be a IFS in X and U be a non empty subset of X. The restriction of A to U is a IFS in U, denoted by $A|U$ and defined by $A|U = (\mu_A|U, \nu_A|U)$.

Definition 2.16. Let (X,τ) be an intuitionistic fuzzy topological space and U is a non empty sub set of X then $\tau_U = \{A|U : A \in \tau\}$ is an intuitionistic fuzzy topology on U and (U,τ_U) is called sub space of (X,τ).

3. Intuitionistic Fuzzy T_2 Spaces

Definition 3.1. Let $r \in (0,1)$. An intuitionistic fuzzy topological space (X,τ) is called.

(1) IF-T_2(r-i) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that $\mu_A(x) > r, \nu_A(x) < r$; $\mu_A(y) < r, \nu_A(y) > r$ and $\mu_B(y) > r, \nu_B(y) < r$; $\mu_B(x) < r, \nu_B(x) > r$ with $(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$.

(2) IF-T_2(r-ii) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that $\mu_A(x) > r, \nu_A(x) < r$; $\mu_A(y) < r, \nu_A(y) > 0$ and $\mu_B(y) > r, \nu_B(y) < r$; $\mu_B(x) < r, \nu_B(x) > 0$ with $(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$.

(3) IF-T_2(r-iii) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that $\mu_A(x) > 0, \nu_A(x) < r$; $\mu_A(y) < r, \nu_A(y) > r$ and $\mu_B(y) > 0, \nu_B(y) < r$; $\mu_B(x) < r, \nu_B(x) > r$ with $(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$.

(4) IF-T_2(r-iv) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that $\mu_A(x) > 0, \nu_A(x) < r$; $\mu_A(y) < r, \nu_A(y) > 0$ and $\mu_B(y) > 0, \nu_B(y) < r$; $\mu_B(x) < r, \nu_B(x) > 0$ with $(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$.

(5) IF-T_2(r-v) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that $\mu_A(x) > r, \nu_A(x) < 1$; $\mu_A(y) < r, \nu_A(y) > r$ and $\mu_B(y) > r, \nu_B(y) < 1$; $\mu_B(x) < r, \nu_B(x) > r$ with $(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$.

(6) IF-T_2(r-vi) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that $\mu_A(x) > r, \nu_A(x) < r$; $\mu_A(y) < 1, \nu_A(y) > r$ and $\mu_B(y) > r, \nu_B(y) < r$; $\mu_B(x) < 1, \nu_B(x) > r$ with $(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$.

(7) IF-T_2(r-vii) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that $\mu_A(x) > r, \nu_A(x) < 1$; $\mu_A(y) < 1, \nu_A(y) > r$ and $\mu_B(y) > r, \nu_B(y) < 1$; $\mu_B(x) < 1, \nu_B(x) > r$ with $(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$.

(8) IF-T_2(viii) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that $\mu_A(x) > 0, \nu_A(x) < 1$; $\mu_A(y) < 1, \nu_A(y) > 0$ and $\mu_B(y) > 0, \nu_B(y) < 1$; $\mu_B(x) < 1, \nu_B(x) > 0$ with $(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$.

Theorem 3.1 Let (X, T) be a topological space and (X, τ) be its corresponding IFTS where $\tau = \left\{(1_{A_j}, 1_{A_j}^c), j \in J : A_j \in T\right\}$. Then (X, T) is T_2 $\Leftrightarrow$ (X, τ) is IF- T_2(r-k) for any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and (X, T) is $T_2 \Leftrightarrow$ (X, τ) is IF- T_2(viii)

Proof: Suppose (X, T) is T_2 space. Let $x, y \in X$ with $x \neq y$. Since (X, T) is T_2, then there exists $A, B \in T$ such that $x \in A$, $y \in B$ and $A \cap B = \emptyset$. So Clearly $x \notin B$, $y \notin A$.

By the definition of τ, we get $(1_A, 1_{A^c}), (1_B, 1_{B^c}) \in \tau$ as $A, B \in T$.

Now, $1_A(x) = 1$, $1_A(y) = 0$, $1_B(y) = 1$, $1_B(x) = 0$ as $x \in A, y \notin A$ and $y \in B, x \notin B$.

And clearly $1_{A^c}(x) = 0$, $1_{A^c}(y) = 1$, $1_{B^c}(y) = 0$, $1_{B^c}(x) = 1$

That is, $1_A(x) = 1$, $1_{A^c}(x) = 0$; $1_A(y) = 0$, $1_{A^c}(y) = 1$ and $1_B(y) = 1$, $1_{B^c}(y) = 0$; $1_B(x) = 0$, $1_{B^c}(x) = 1$

This Implies $1_A(x) > r$, $1_{A^c}(x) < r$; $1_A(y) < r$, $1_{A^c}(y) > r$ and $1_B(y) > r$, $1_{B^c}(y) < r$; $1_B(x) < r$, $1_{B^c}(x) > r$.

Again since $A \cap B = \emptyset$, then $A^c \cup B^c = X$.

So $(1_A \cap 1_B) \subset (1_{A^c} \cup 1_{B^c})$

Therefore (X, τ) is IF-T_2(r-i).

Conversely suppose (X,τ) is IF-T2(r-i). Let $x,y \in X$ with $x \neq y$. Since (X,τ) is IF-T2(r-i), then there exists $(1_A, 1_{A^c}), (1_B, 1_{B^c}) \in \tau$ such that $1_A(x) > r, 1_{A^c}(x) < r$; $1_A(y) < r$, $1_{A^c}(y) > r$ and $1_B(y) > r, 1_{B^c}(y) < r$; $1_B(x) < r, 1_{B^c}(x) > r$ with $(1_A \cap 1_B) \subset (1_{A^c} \cup 1_{B^c})$.

Since $r \in (0,1)$, we can write $1_A(x) = 1, 1_{A^c}(x) = 0$; $1_A(y) = 0, 1_{A^c}(y) = 1$ and $1_B(y) = 1, 1_{B^c}(y) = 0$; $1_B(x) = 0, 1_{B^c}(x) = 1$.

This implies $x \in A, y \notin A$ and $y \in B, x \notin B$.

And for any $z \in X$

$(1_A \cap 1_B)(z) < (1_{A^c} \cup 1_{B^c})(z)$ as $(1_A \cap 1_B) \subset (1_{A^c} \cup 1_{B^c})$.

$\Rightarrow 1_A \cap 1_B(z) = 0$, $1_{A^c} \cup 1_{B^c}(z)$ $=1 \Rightarrow z \notin A \cap B \Rightarrow A \cap B = \phi$.

That is $x \in A$, $y \in B$ and $A \cap B = \phi$

Clearly $A, B \in T$ as $(1_A, 1_{A^c}), (1_B, 1_{B^c}) \in \tau$. Therefore (X,T) is T2 Space.

Similarly we can show the other implications.

Theorem 3.2. Let (X,t) be a fuzzy topological space and (X,τ) be its corresponding IFTS where $\tau = \{(\lambda, \lambda^c) : \lambda \in t\}$. Then (X,t) is T2$\Rightarrow (X,\tau)$ is IF-T2(r-k) for $k = i, ii, iii, iv, v, vi, vii$ and (X,t) is T2 $\Rightarrow (X,\tau)$ is IF-T2(viii) where $r \in (0,1)$.

Proof: The proofs of all implications are similar. For an example we shall prove that (X,t) is T2 $\Rightarrow (X,\tau)$ is IF-T2(r-i).

Suppose (X,t) is T_2. Let $x,y \in X$ with $x \neq y$.

Consider two distinct fuzzy points x_1, y_1 in X.

Since (X,t) is T2, there exists $u, v \in t$ such that $x_1 \in u$, $y_1 \in v$ and $u \cap v = \underline{0}$.

So $u(x) = 1$, $v(y) = 1$, $u(y) = 0$ and $v(x) = 0$.

Therefore $u^c(x) = 0$, $v^c(y) = 0$, $u^c(y) = 1$ and $v^c(x) = 1$.

That is, $u(x) = 1$, $u^c(x) = 0$; $v(x) = 0$, $v^c(x) = 1$ and $v(y) = 1$, $v^c(y) = 0$; $v(x) = 0$, $v^c(x) = 1$.

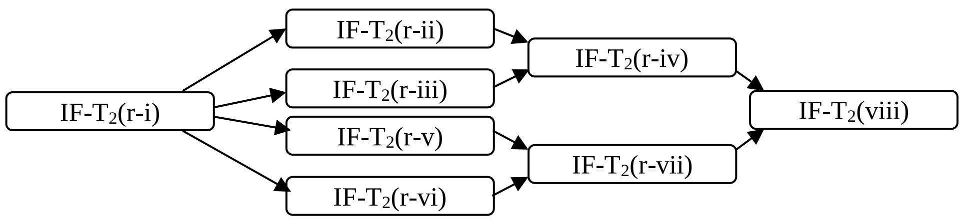
$\Rightarrow\ u(x) > r,\ u^c(x) < r;\ v(x) < r,\ v^c(x) > r$ and $v(y) > r,\ v^c(y) < r;\ v(x) < r,\ v^c(x) > r$ as $r \in (0,1)$.

Also it is clear that $(u \cap v) \subset (u^c \cup v^c)$ as $u \cap v = \underline{0}$.

Now by definition of τ; $(u, u^c), (v, v^c) \in \tau\ as\ u, v \in t$.

Therefore $\Rightarrow (X, \tau)$ is IF-T_2(r-i).

Theorem 3.3. Let (X, τ) be a IFTS. Then we have the following implications.



Proof: Suppose (X, τ) is IF-T_2(r-i). Let $x, y \in X$ with $x \neq y$. Since (X, τ) is IF-T_2(r-i), then there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that

$$\mu_A(x) > r, \nu_A(x) < r;\ \mu_A(y) < r, \nu_A(y) > r \text{ and } \mu_B(y) > r, \nu_B(y) < r;\ \mu_B(x) < r, \nu_B(x) > r \quad \ldots\ldots\ldots\ldots\ldots(1)$$

with $(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$.

Now, from (1) we can write,

$$\mu_A(x) > r, \nu_A(x) < r;\ \mu_A(y) < r, \nu_A(y) > 0 \text{ and } \mu_B(y) > r, \nu_B(y) < r;\ \mu_B(x) < r, \nu_B(x) > 0 \quad \ldots\ldots\ldots\ldots\ldots(2)$$

Again from (2) we get,

$$\mu_A(x) > 0, \nu_A(x) < r;\ \mu_A(y) < r, \nu_A(y) > 0 \text{ and } \mu_B(y) > 0, \nu_B(y) < r;\ \mu_B(x) < r, \nu_B(x) > 0 \quad \ldots\ldots\ldots\ldots\ldots(3)$$

And finally from (3),

$$\mu_A(x) > 0, \nu_A(x) < 1;\ \mu_A(y) < 1, \nu_A(y) > 0 \text{ and } \mu_B(y) > 0, \nu_B(y) < 1;\ \mu_B(x) < 1, \nu_B(x) > 0 \quad \ldots\ldots\ldots\ldots\ldots(4)$$

Therefore IF-T_2(r-i) $\Rightarrow$ IF-T_2(r-ii) $\Rightarrow$ IF-T_2(r-iv) $\Rightarrow$ IF-T_2(viii).

Again from (1) we can write

$\mu_A(x) > r, \nu_A(x) < r; \mu_A(y) < 1, \nu_A(y) > r$ and $\mu_B(y) > r, \nu_B(y) < r; \mu_B(x) < 1, \nu_B(x) > r$ …………….(5)

This implies

$\mu_A(x) > r, \nu_A(x) < 1; \mu_A(y) < 1, \nu_A(y) > r$ and $\mu_B(y) > r, \nu_B(y) < 1; \mu_B(x) < 1, \nu_B(x) > r$ ...………….(6)

This implies

$\mu_A(x) > 0, \nu_A(x) < 1; \mu_A(y) < 1, \nu_A(y) > 0$ and $\mu_B(y) > 0, \nu_B(y) < 1; \mu_B(x) < 1, \nu_B(x) > 0$ ….………….(7)

Therefore IF-T_2(r-i) ⇒ IF-T_2(r-vi) ⇒ IF-T_2(r-vii) ⇒ IF-T_2(viii).

Similarly other implications may be proved.

The reverse implications are not true in general which can be seen as the following examples:

Example 3.1. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.6, 0.1), (y, 0.2, 0.8)\}$, $B = \{(x, 0.1, 0.3), (y, 0.6, 0.3)\}$. If $r = 0.5$, then clearly (X, τ) is IF-T_2(r-ii) but not IF- T_2(r-i).

Example 3.2. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.3, 0.1), (y, 0.2, 0.8)\}$, $B = \{(x, 0.1, 0.3), (y, 0.4, 0.4)\}$. If $r = 0.5$, then clearly (X, τ) is IF-T_2(r-iv) but not IF-T_2(r-i), IF-T_2(r-ii) and IF-T_2(r-iii).

Example 3.3. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.5, 0.1), (y, 0.2, 0.8)\}$, $B = \{(x, 0.1, 0.6), (y, 0.6, 0.3)\}$. If $r = 0.5$, then clearly (X, τ) is IF-T_2(r-iii) but not IF-T_2(r-i).

Example 3.4. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.3, 0.4), (y, 0.1, 0.8)\}$, $B = \{(x, 0.1, 0.3), (y, 0.6, 0.3)\}$. If $r = 0.2$, then clearly (X, τ) is IF-T_2(r-v) but not IF-T_2(r-i).

Example 3.5. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.6, 0.1), (y, 0.5, 0.4)\}$, $B = \{(x, 0.1, 0.3), (y, 0.6, 0.1)\}$. If $r = 0.2$, then clearly (X, τ) is IF-T_2(r-vi) but not IF-T_2(r-i).

Example 3.6. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.4, 0.6), (y, 0.4, 0.6)\}$, $B = \{(x, 0.65, 0.35), (y, 0.5, 0.4)\}$. If $r = 0.3$, then clearly (X, τ) is IF-T_2(r-vii) but not IF-T_2(r-i), IF-T_2(r-v) andIF-T_2(r-vi).

Example 3.7. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.4, 0.6), (y, 0.3, 0.5)\}$, $B = \{(x, 0.1, 0.3), (y, 0.1, 0.6)\}$. If $r = 0.5$, then clearly (X, τ) is IF-T_2(viii) but not IF-T_2(r-iv) and IF-T_2(r-vii).

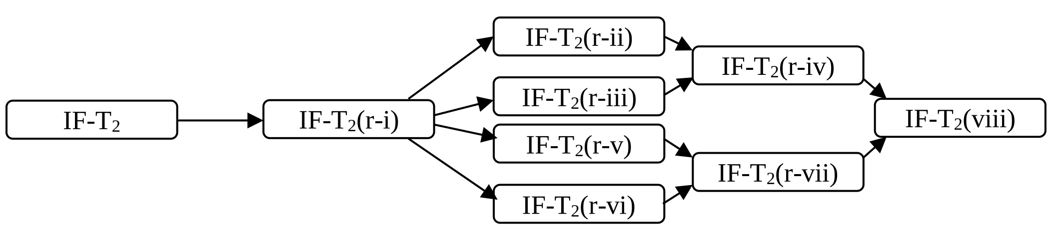
Definition 3.2[8]. Let $\alpha, \beta \in [0,1]$ and $\alpha + \beta \leq 1$. An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ of X is an intuitionistic fuzzy set in X define by

$$x_{(\alpha,\beta)}(y) = \begin{cases} (\alpha, \beta) & if\ y = x \\ (0.1) & if\ y \neq x \end{cases}$$

An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to belong to an intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ if $\alpha < \mu_A(x)$ and $\beta > \nu_A(x)$.

Definition 3.3[8]. An intuitionistic fuzzy topological space (X, τ) is called IF-T_2 if for all pair of distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}$ and $y_{(\gamma,\delta)}$ in X, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that $x_{(\alpha,\beta)} \in A$, $y_{(\gamma,\delta)} \in B$ and $A \cap B = 0_\sim$.

Theorem 3.4. Let (X, τ) be a IFTS and $r \in (0,1)$. Then we have the following implications.



Proof: To prove this theorem we have to prove that IF-$T_2 \Rightarrow$ IF-T_2(r-i). Let (X, τ) be IF-T_2 and $x, y \in X$ with $x \neq y$. Consider two distinct intuitionistic fuzzy points $x_{(r,r)}$ and $y_{(r,r)}$. Since (X, τ)

is IF-T_2, there exists $A=(\mu_A,\nu_A), B=(\mu_B,\nu_B)\in\tau$ such that $x_{(r,r)}\in A$, $y_{(r,r)}\in B$ and $A\cap B=0_\sim$.

Clearly $r<\mu_A(x), r>\nu_A(x)$ as $x_{(r,r)}\in A=(\mu_A,\nu_A)$

and $r<\mu_B(y), r>\nu_B(y)$ as $y_{(r,r)}\in B=(\mu_B,\nu_B)$

Again since $(\mu_A\cap\mu_B,\nu_A\cup\nu_B)=A\cap B=0_\sim=(\underline{0},\underline{1})$, then $(\mu_A\cap\mu_B)\subset(\nu_A\cup\nu_B)$ and clearly

$\mu_B(x)=0$, $\nu_B(x)=1$, $\mu_A(y)=0$ and $\nu_A(y)=1$.

Therefore we can write $\mu_A(x)>r,\nu_A(x)<r$; $\mu_A(y)<r,\nu_A(y)>r$ and $\mu_B(y)>r,\nu_B(y)<r$; $\mu_B(x)<r,\nu_B(x)>r$ with $(\mu_A\cap\mu_B)\subset(\nu_A\cup\nu_B)$.

So (X,τ) is IF-T_2(r-i).

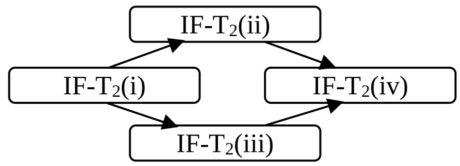
The reverse implications are not true in general which can be seen as the following example:

Example 3.8. Let $X=\{x,y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A=\{(x,0.6,0.1),(y,0.2,0.8)\}$, $B=\{(x,0.1,0.7),(y,0.6,0.3)\}$. If $r=0.5$, then clearly (X,τ) is IF-T_2(r-i) but if we consider two distinct intuitionistic fuzzy points $x_{(.1,.2)}$ and $y_{(.3,.4)}$ then clearly (X,τ) is not IF- T_2.

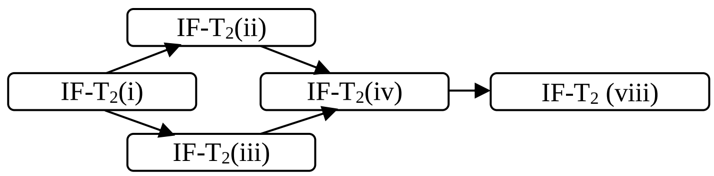
Definition 3.4[15]**.** An intuitionistic fuzzy topological space Let (X,τ) is called

(1) IF-T_2(i) if for all $x,y\in X$ with $x\neq y$, there exists $A=(\mu_A,\nu_A), B=(\mu_B,\nu_B)\in\tau$ such that $\mu_A(x)=1,\nu_A(x)=0$; $\mu_B(y)=1,\nu_B(y)=0$ and $A\cap B=0_\sim$
(2) IF-T_2(ii) if for all $x,y\in X$ with $x\neq y$, there exists $A=(\mu_A,\nu_A), B=(\mu_B,\nu_B)\in\tau$ such that $\mu_A(x)=1,\nu_A(x)=0$; $\mu_B(y)>0,\nu_B(y)=0$ and $A\cap B=(\underline{0},\gamma)$ where $\gamma\in(0,1]$
(3) IF-T_2(iii) if for all $x,y\in X$ with $x\neq y$, there exists $A=(\mu_A,\nu_A), B=(\mu_B,\nu_B)\in\tau$ such that $\mu_A(x)>0,\nu_A(x)=0$; $\mu_B(y)=1,\nu_B(y)=0$ and $A\cap B=(\underline{0},\gamma)$ where $\gamma\in(0,1]$
(4) IF-T_2(iv) if for all $x,y\in X$ with $x\neq y$, there exists $A=(\mu_A,\nu_A), B=(\mu_B,\nu_B)\in\tau$ such that $\mu_A(x)>0,\nu_A(x)=0$; $\mu_B(y)>0,\nu_B(y)=0$ and $A\cap B=(\underline{0},\gamma)$ where $\gamma\in(0,1]$

Theorem 3.5 [15]**.** Let (X, τ) be an intuitionistic fuzzy topological space. Then we have the following implications



Theorem 3.6. If (X, τ) is a IFTS, then the following implications hold.



Proof: To prove this theorem we only have to prove that (X, τ) is IF-T_2(iv) ⇒ (X, τ) is IF-T_2(viii). Let (X, τ) is IF-T_2(iv) and $x, y \in X$ with $x \neq y$. Then there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in \tau$ such that $\mu_A(x) > 0, \nu_A(x) = 0$; $\mu_B(y) > 0, \nu_B(y) = 0$ and $A \cap B = (\underline{0}, \gamma)$ where $\gamma \in (0,1]$. Since $(\mu_A \cap \mu_B, \nu_A \cup \nu_B) = A \cap B = (\underline{0}, \gamma)$ where $\gamma \in (0,1]$, then $(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$ and clearly $\mu_B(x) = 0$, $\nu_B(x) = \gamma$, $\mu_A(y) = 0$ and $\nu_A(y) = \gamma$.

Therefore we can write $\mu_A(x) > 0, \nu_A(x) < 1$; $\mu_A(y) < 1, \nu_A(y) > 0$ and $\mu_B(y) > 0, \nu_B(y) < 1$; $\mu_B(x) < 1, \nu_B(x) > 0$ with $(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$.

So (X, τ) is IF-T_2(viii).

The reverse implications are not true in general which can be seen as the following example:

Example 3.9. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.4, 0.6), (y, 0.3, 0.5)\}$, $B = \{(x, 0.1, 0.3), (y, 0.1, 0.6)\}$. If $\gamma = 0.5$, then clearly (X, τ) is IF-T_2(viii) but not IF-T_2(iv)

Theorem 3.7. Let (X, τ) be a IFTS and $r, s \in (0,1)$ with $r < s$, then (X, τ) is IF-T_2(r-iv) ⇒ (X, τ) is IF-T_2(s-iv) and (X, τ) is IF-T_2(s-vii) ⇒ (X, τ) is IF-T_2(r-vii).

Proof: IF-T_2(r-iv) ⇒IF-T_2(s-iv): Suppose (X, τ) is IF-T_2(r-iv). Let $x, y \in X$ with $x \neq y$.

Since (X, τ) is IF-T_2(r-iv), then there exists intuitionistic fuzzy sets $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that

$\mu_A(x) > 0, \nu_A(x) < r$; $\mu_A(y) < r, \nu_A(y) > 0$ and $\mu_B(y) > 0, \nu_B(y) < r$; $\mu_B(x) < r, \nu_B(x) > 0$ with $(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$.

Since $r < s$, we can write

$\mu_A(x) > 0, \nu_A(x) < s$; $\mu_A(y) < s, \nu_A(y) > 0$ and $\mu_B(y) > 0, \nu_B(y) < s$; $\mu_B(x) < s, \nu_B(x) > 0$.

Therefore (X, τ) is IF-T_2(s-iv).

IF-T_2(s-vii) ⇒IF-T_2(r-vii): Suppose (X, τ) is IF-T_2(s-vii).

Let $x, y \in X$ with $x \neq y$. Since (X, τ) is IF-T_2(s-vii), then there exists intuitionistic fuzzy set $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that

$\mu_A(x) > s, \nu_A(x) < 1$; $\mu_A(y) < 1, \nu_A(y) > s$ and $\mu_B(y) > s, \nu_B(y) < 1$; $\mu_B(x) < 1, \nu_B(x) > s$. with $(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$.

Since $r < s$, we can write

$\mu_A(x) > r, \nu_A(x) < 1$; $\mu_A(y) < 1, \nu_A(y) > r$ and $\mu_B(y) > r, \nu_B(y) < 1$; $\mu_B(x) < 1, \nu_B(x) > r$.

So (X, τ) is IF-T_2(r-vii).

The reverse implications are not true in general which can be seen as the following examples:

Example 3.10. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.3, 0.1), (y, 0.2, 0.8)\}$, $B = \{(x, 0.1, 0.3), (y, 0.4, 0.4)\}$. If $r = 0.3$ and $s = 0.5$ then clearly (X, τ) is IF-T_2(s-iv) but not IF-T_2(r-iv).

Example 3.11. Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.4, 0.6), (y, 0.4, 0.6)\}$, $B = \{(x, 0.65, 0.35), (y, 0.5, 0.4)\}$. If $r = 0.3$ and $s = 0.5$ then clearly (X, τ) is IF-T_2(r-vii) but not IF-T_2(s-vii).

Theorem 3.8. Let (X, τ) and (Y, δ) be IFTSs and $f: X \to Y$ is one-one and continuous. Then (Y, δ) is IF-T_2(r-k)⇒ (X, τ) is IF-T_2(r-k) for any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and (Y, δ) is IF-T_2(viii)⇒ (X, τ) is IF-T_2(viii).

Proof: Suppose (Y, δ) is IF-T_2(r-i). Let $x, y \in X$ with $x \neq y$. Since f is one-one, then $f(x), f(y) \in Y$ with $f(x) \neq f(y)$. Again, since (Y, δ) is IF-T_2(r-i), there exists $A = (\mu_A, \nu_A)$,

$B = (\mu_B, \nu_B) \in \delta$ such that $\mu_A(f(x)) > r, \nu_A(f(x)) < r$; $\mu_A(f(y)) < r, \nu_A(f(y)) > r$ and $\mu_B(f(y)) > r, \nu_B(f(y)) < r; \mu_B(f(x)) < r, \nu_B(f(x)) > r$ with $(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$.

Further since f is continuous, then $f^{-1}(A) = (f^{-1}(\mu_A), f^{-1}(\nu_A)) \in \tau$ and $f^{-1}(B)$=$(f^{-1}(\mu_B), f^{-1}(\nu_B)) \in \tau$.

Now we have $f^{-1}(\mu_A)(x) = \mu_A\big(f(x)\big) > r, f^{-1}(\nu_A)(x) = \nu_A(f(x)) < r; f^{-1}(\mu_A)(y) = \mu_A\big(f(y)\big) < r, f^{-1}(\nu_A)(y) = \nu_A(f(y)) > r.$

And $f^{-1}(\mu_B)(y) = \mu_B\big(f(y)\big) > r, f^{-1}(\nu_B)(y) = \nu_B(f(y)) < r; f^{-1}(\mu_B)(x) = \mu_B\big(f(x)\big) < r, f^{-1}(\nu_B)(x) = \nu_B(f(x)) > r.$

Now for any $z \in X$, $f(z) \in Y$.

So $\big(f^{-1}(\mu_A) \cap f^{-1}(\mu_B)\big)(z)$ =min$(f^{-1}(\mu_A)(z), f^{-1}(\mu_B)(w))$=min$\Big(\mu_A\big(f(z)\big), \mu_B\big(f(z)\big)\Big)$=$(\mu_A \cap \mu_B)(f(z))$

And $\big(f^{-1}(\nu_A) \cup f^{-1}(\nu_B)\big)(z) = max(f^{-1}(\nu_A)(z), f^{-1}(\nu_B)(z))$=max$\Big(\nu_A\big(f(z)\big), \nu_B\big(f(z)\big)\Big)$=$(\nu_A \cup \nu_B)(f(z))$

Clearly $(\mu_A \cap \mu_B)\big(f(z)\big) < (\nu_A \cup \nu_B)\big(f(z)\big)$ as $(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$.

Therefore $\big(f^{-1}(\mu_A) \cap f^{-1}(\mu_B)\big)(z) < \big(f^{-1}(\nu_A) \cup f^{-1}(\nu_B)\big)(z)$ for all $z \in X$

That is, $\big(f^{-1}(\mu_A) \cap f^{-1}(\mu_B)\big) \subset \big(f^{-1}(\nu_A) \cap f^{-1}(\nu_B)\big)$

So (X, τ) is IF-T_2(r-i).

Similarly we can show other implications.

Theorem 3.9. Let (X, τ) and (Y, δ) be IFTSs and $f: X \to Y$ is one-one, onto and open. Then (X, τ) is IF-T_2(r-k) $\Rightarrow$ (Y, δ) is IF-T_2(r-k) for any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and (X, τ) is IF-T_2(viii) $\Rightarrow$ (Y, δ) is IF-T_2(viii).

Proof: Suppose (X, τ) is IF-T_2(r-i).

Let $x, y \in Y$ with $x \neq y$. Since f is onto, then there exists some $p, q \in X$ such that $f(p) = x$ and $f(q) = y$. Again since f is one-one, then these p and q are unique and $p \neq q$. i.e., $f^{-1}(x) = \{p\}$ and $f^{-1}(y) = \{q\}$.

Now since (X, τ) is IF-T_2(r-i), then there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that $\mu_A(p) > r, \nu_A(p) < r$; $\mu_A(q) < r, \nu_A(q) > r$ and $\mu_B(q) > r, \nu_B(q) < r$; $\mu_B(p) < r, \nu_B(p) > r$ with $(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$.

Further since f is open, so $f(A) = \big(f(\mu_A), f(\nu_A)\big) \in \delta, f(B) = \big(f(\mu_B), f(\nu_B)\big) \in \delta$

Now we have

$$f(\mu_A)(x) = \sup_{a\in f^{-1}(x)} \mu_A(a) = \mu_A(p) > r, f(\nu_A)(x) = \inf_{a\in f^{-1}(x)} \nu_A(a) = \nu_A(p) < r;$$

$$f(\mu_A)(y) = \sup_{a\in f^{-1}(y)} \mu_A(a) = \mu_A(q) < r, f(\nu_A)(y) = \inf_{a\in f^{-1}(y)} \nu_A(a) = \nu_A(q) > r.$$

And $f(\mu_B)(y) = \sup_{a\in f^{-1}(y)} \mu_B(a) = \mu_B(q) > r, f(\nu_B)(y) = \inf_{a\in f^{-1}(y)} \nu_B(a) = \nu_B(q) < r;$

$$f(\mu_B)(x) = \sup_{a\in f^{-1}(x)} \mu_B(a) = \mu_B(p) < r, f(\nu_B)(x) = \inf_{a\in f^{-1}(x)} \nu_A(a) = \nu_A(p) > r.$$

Now for any $w \in Y$ there exists a unique $z \in X$ such that $f(z) = w$ as f is one-one and onto.

So $\big(f(\mu_A) \cap f(\mu_B)\big)(w)$=min$(f(\mu_A)(w), f(\mu_B)(w))$=min$\Big(\sup_{a\in f^{-1}(w)} \mu_A(a), \sup_{a\in f^{-1}(w)} \mu_B(a)\Big)$

=min$\big(\mu_A(z), \mu_B(z)\big)$=$(\mu_A \cap \mu_B)(z)$

And

$\big(f(\nu_A) \cup f(\nu_B)\big)(w)$=max$(f(\nu_A)(w), f(\nu_B)(w))$=max$\Big(\inf_{a\in f^{-1}(w)} \nu_A(a), \inf_{a\in f^{-1}(w)} \nu_B(a)\Big)$

=max$\big(\nu_A(z), \nu_B(z)\big)$=$(\nu_A \cup \nu_B)(z)$

Clearly $(\mu_A \cap \mu_B)(z) < (\nu_A \cup \nu_B)(z)$ as$(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$

That is, $\big(f(\mu_A) \cap f(\mu_B)\big) \subset \big(f(\nu_A) \cup f(\nu_B)\big)$

Therefore (Y, δ) is IF-T_2(r-i).

Similarly we can show other implications.

From theorem 3.8 and theorem 3.9 we have the following corollary.

Corollary 3.1. If (X, τ) and (Y, δ) are IFTSs and $f: X \to Y$ is a homeomorphism then (X, τ) is IF-T_2(r-k) if and only if (Y, δ) is IF-T_2(r-k) for any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and (X, τ) is IF-T_2(viii) if and only if (Y, δ) is IF-T_2(viii).

Remark 3.1. IF-T_2(r-k) for k= i, ii, iii, iv, v, vi, vii and IF-T_2(viii) are topological property.

Theorem 3.10. Let (X, τ) be an intuitionistic fuzzy topological space and U is a non empty sub set of X. Then (X, τ) is IF-T_2(r-k) $\Rightarrow$ (U, τ_U) is IF-T_2(r-k) for any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and (X, τ) is IF-T_2(viii) $\Rightarrow$ (U, τ_U) is IF-T_2(viii).

Proof: Suppose (X, τ) is IF-T_2(r-i). Let $x, y \in U$ with $x \neq y \Rightarrow x, y \in X$ with $x \neq y$ as $U \subseteq X$.

Since (X, τ) is IF-T_2(r-i), then there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in \tau$ such that

$\mu_A(x) > r, \nu_A(x) < r; \mu_A(y) < r, \nu_A(y) > r$ and $\mu_B(y) > r, \nu_B(y) < r; \mu_B(x) < r, \nu_B(x) > r$

with $(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$.

Clearly $A|U = (\mu_A|U, \nu_A|U) \in \tau_U$ and $B|U = (\mu_B|U, \nu_B|U) \in \tau_U$

Now we have $\mu_A|U(x) = \mu_A(x) > r$, $\nu_A|U(x) = \nu_A(x) < r$; $\mu_A|U(y) = \mu_A(y) < r$, $\nu_A|U(y) = \nu_A(y) > r$.

And $\mu_B|U(y) = \mu_B(y) > r$, $\nu_B|U(y) = \nu_B(y) < r$; $\mu_B|U(x) = \mu_B(x) < r$, $\nu_B|U(x) = \nu_A(x) > r$.

Clearly $(\mu_A|U \cap \mu_B|U) \subset (\nu_A|U \cup \nu_B|U)$

Therefore (U, τ_U) is IF-T_2(r-i).

Similarly, we can show others implications.

Hence the properties IF-T_2(r-k) for $k = i, ii, iii, iv, v, vi$, vii and IF-$T_2$(viii) are hereditary.

Theorem 3.11. Let (X_j, τ_j), $j = 1{,}2$ be two IFTSs and $(X, \tau) = (X_1 \times X_2, \tau_1 \times \tau_2)$. If each (X_j, τ_j), $j = 1{,}2$ are IF-T_2(r-k), then (X, τ) is IF-T_2(r-k) for any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and if each (X_j, τ_j), $j = 1{,}2$ are IF-T_2(viii), then (X, τ) is IF-T_2(viii).

Proof: The proof of all implications are similar. For an example, we shall prove that if each (X_j, τ_j), $j = 1,2$ are IF-T_2(r-i), then (X, t) is IF-T_2(r-i).

Let each (X_j, τ_j), $j = 1,2$ are IF-T_2(r-i).

Suppose $x, y \in X$ with $x \neq y$ where $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Then at least $x_1 \neq y_1$ or $x_2 \neq y_2$.

Consider $x_1 \neq y_1$. Clearly $x_1, y_1 \in X_1$. Since (X_1, τ_1) is IF-T_2(r-i), then there exists $A_1 = (\mu_{A_1}, \nu_{A_1})$, $B_1 = (\mu_{B_1}, \nu_{B_1}) \in \tau_1$ such that $\mu_{A_1}(x_1) > r, \nu_{A_1}(x_1) < r; \mu_{A_1}(y_1) < r, \nu_{A_1}(y_1) > r$ and $\mu_{B_1}(y_1) > r, \nu_{B_1}(y_1) < r; \mu_{B_1}(x_1) < r, \nu_{B_1}(x_1) > r$ with $(\mu_{A_1} \cap \mu_{B_1}) \subset (\nu_{A_1} \cup \nu_{B_1})$.

Choose $A_2 = B_2 = 1_\sim = (\underline{1}, \underline{0})$ and Clearly $A_2, B_2 \in \tau_2$

Let $A = A_1 \times A_2 = (\mu_{A_1} \times \underline{1}, \nu_{A_1} \times \underline{0}) = (\mu_A, \nu_A)$ (say)

and $B = B_1 \times B_2 = (\mu_{B_1} \times \underline{1}, \nu_{B_1} \times \underline{0}) = (\mu_B, \nu_B)$ (say)

By the definition of product IFT; $A, B \in \tau$.

Now we have $\mu_A(x) = (\mu_{A_1} \times \underline{1})(x_1, x_2) = min(\mu_{A_1}(x_1), \underline{1}(x_2)) = min(\mu_{A_1}(x_1), 1) > r$ as $\mu_{A_1}(x_1) > r$,

$\nu_A(x) = (\nu_{A_1} \times \underline{0})(x_1, x_2) = max(\nu_{A_1}(x_1), \underline{0}(x_2)) = max(\nu_{A_1}(x_1), 0) < r$ as $\nu_{A_1}(x_1) < r$;

$\mu_A(y) = (\mu_{A_1} \times \underline{1})(y_1, y_2) = min(\mu_{A_1}(y_1), \underline{1}(y_2)) = min(\mu_{A_1}(y_1), 1) < r$ as $\mu_{A_1}(y_1) < r$,

$\nu_A(y) = (\nu_{A_1} \times \underline{0})(y_1, y_2) = max(\nu_{A_1}(y_1), \underline{0}(y_2)) = max(\nu_{A_1}(y_1), 0) > r$ as $\nu_{A_1}(y_1) > r$

And $\mu_B(y) = (\mu_{B_1} \times \underline{1})(y_1, y_2) = min(\mu_{B_1}(y_1), \underline{1}(y_2)) = min(\mu_{B_1}(y_1), 1) > r$ as $\mu_{B_1}(y_1) > r$,

$\nu_B(y) = (\nu_{B_1} \times \underline{0})(y_1, y_2) = max(\nu_{B_1}(y_1), \underline{0}(y_2)) = max(\nu_{B_1}(y_1), 0) < r$ as $\nu_{B_1}(y_1) < r$;

$\mu_B(x) = (\mu_{B_1} \times \underline{1})(x_1, x_2) = min(\mu_{B_1}(x_1), \underline{1}(x_2)) = min(\mu_{B_1}(x_1), 1) < r$ as $\mu_{B_1}(x_1) < r$,

$\nu_B(x) = (\nu_{B_1} \times \underline{0})(x_1, x_2) = max(\nu_{B_1}(x_1), \underline{0}(x_2)) = max(\nu_{B_1}(x_1), 0) > r$ as $\nu_{B_1}(x_1) > r$;

Again for any $z = (z_1, z_1) \in X$

$(\mu_A \cap \mu_B)(z) = \min (\mu_A(z), \mu_B(z)) = min\left((\mu_{A_1} \times \underline{1})(z_1, z_2), (\mu_{B_1} \times \underline{1})(z_1, z_2)\right)$

$= min\left(min(\mu_{A_1}(z_1), \underline{1}(z_2)), min\,(\mu_{B_1}(z_1), \underline{1}(z_2))\right) = min\left(\mu_{A_1}(z_1), \mu_{B_1}(z_1)\right) = \left(\mu_{A_1} \cap \mu_{B_1}\right)(z_1).$

And $(\nu_A \cup \nu_B)(z) = \max(\nu_A(z), \nu_B(z)) = max\left((\nu_{A_1} \times \underline{0})(z_1, z_2), (\nu_{B_1} \times \underline{0})(z_1, z_2)\right)$

$= max\left(max(\nu_{A_1}(z_1), \underline{0}(z_2)), max(\nu_{B_1}(z_1), \underline{0}(z_2))\right)$

$= max\left(\mu_{A_1}(z_1), \mu_{B_1}(z_1)\right) = \left(\nu_{A_1} \cup \nu_{B_1}\right)(z_1).$

Clearly $\left(\mu_{A_1} \cap \mu_{B_1}\right)(z_1) < \left(\nu_{A_1} \cup \nu_{B_1}\right)(z_1)$ as $\left(\mu_{A_1} \cap \mu_{B_1}\right) \subset \left(\nu_{A_1} \cup \nu_{B_1}\right)$.

So $(\mu_A \cap \mu_B)(z) < (\nu_A \cup \nu_B)(z)$

That is, $(\mu_A \cap \mu_B) \subset (\nu_A \cup \nu_B)$.

Therefore (X, τ) is is IF-T_2(r-iv).

Remark 3.2. The properties IF-T_2(r-k) for $k = i, ii, iii, iv, v, vi, vii$ and IF-T_2(viii) are Productive.

4. Conclusion

In this paper we see that our eight notions are more general than that of Amit Kumar Singh et al.In particular our notion (viii) is more general that of Estiaq Ahmed et al and Amit Kumar Singh et al. Also we see that our notions satisfy hereditary and productive properties. Moreover the notions preserved under one-one and open maping. As far we know our notion (viii) is the most general among all given notions of intuitionistic fuzzy T_2 topological spaces.

Conflict of Interests

The authors declare that there is no conflict of interests.

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