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Numerical Computation of Some Iterative Techniques for Solving System of Linear Equations of Multivariable

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Abstract: In this paper we introduce, numerical computation of some iterative techniques for solving system of linear simultaneous equations of 4 or more variables. Many iterative techniques is presented by the different formulae. Using Jacobi method, Seidel method and SOR method and their results are compared. The software, Matlab 2009a was used to find the solution of the linear simultaneous equations having diagonally dominant in coefficient matrix. Numerical rate of convergence of solution has been found in each calculation. It was observed that the Seidel method converges at the 12iteration while Jacobi and SOR methods converge to the exact value of X(x, y, z, t) with error level of accuracy 10^{-15} at the 22th iteration respectively. However, when we compare performance, we must compare both cost, speed of convergence. Some numerical examples are given to illustrate the efficiency and robustness of the techniques. It was then concluded that Seidel is the most effective technique.

Keywords: iterative techniques; algorithm; linear simultaneous equations on large scale; rate of convergence; numerical experiments, executing time.

1. Introduction

Solving system of linear simultaneous equations is one of the most important and challenging problems in science and engineering applications. It arises in a wide variety of practical applications in Physics, Chemistry, Biosciences, Engineering, etc. System of linear equations arises in various theoretical research fields as well as applications in science and engineering. After the availability of computers, we go to numerical methods which are suited for computer

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operations. Well known techniques of linear algebra such as Gaussian elimination and Gauss Jordon's methods are utilized to determine a common solution. Specifically, the problem of existence, uniqueness and cardinality of solution of a system of linear equations is well solved in linear algebra. In linear algebra, Iterative solver is an algorithm [1] that can be used to determine the solutions of a system of linear equations to find the rank of a matrix [3-5], and to calculate the inverse of an invertible square matrix. Another point of view, which turns out to be very useful to analyze the algorithm. The first part of the algorithm computes an LU decomposition (it is a matrix decomposition which writes a matrix as the product of a lower triangular matrix and an upper triangular matrix. A linear equation system is a set of linear equations to be solved simultaneously. This system consists of linear equations, each with coefficients, and has unknowns which have to fulfill the set of equations simultaneously. To simplify notation, it is possible to rewrite the equations in matrix notation which are diagonally dominant in coefficients matrix.[6-8] **.**Consider a system of n linear algebraic equations in n unknowns where (m=n)

$$a_{11}x_1 + a_{12}x_2 + \cdots \ldots \ldots a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots \ldots \ldots a_{2n}x_n = b_2$$
$$\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots \ldots \ldots a_{mn}x_n = b_m \qquad \text{(i)}$$

Where $a_{ij,}\ (i,j = 1(1)n$ are the known coefficient , $b_{i,}$(i = 1(1)n) are the known values and x_i ,(i= 1(1)n) are the unknowns to be determined . In matrix notation the system can be written as

$$\mathrm{A\,x} = \mathrm{b} \qquad \text{(ii)}$$

where

$$\mathrm{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{m1} & a_{m2} & a_{mn} \end{bmatrix}$$

The matrix $[\,A : b]$ is called the augmented matrix. It is formed by appending the column b to the $m \times n$ matrix. The methods of solution of the linear algebraic equations (ii) may broadly be classified into two types. (i) Direct methods: These methods produce the exact solution after a finite number of steps. (ii) Iterative methods: These methods give a sequence of approximate solutions, which converges when the number of steps tends to infinity. Here we are interested in the case when m = n; particularly when the number of equations are large. A took-kit to solve equations of this type is at the heart of the numerical analysis. Before we present the numerical

methods to solve equation (ii), we have to know the conditions under which the solutions exist. We then proceed to develop direct and iterative methods for solving large scale problems. We later discuss numerical conditioning of a matrix and its relation to errors that can arise in computing numerical solutions.[6]

2. Iterative Techniques

By this approach, we start with some initial guess solution, say $x^{(0)}$, for solution x and generate an improved solution estimate $x^{(k+1)}$from the previous approximation $x^{(k)}$ .This method is a very effective for solving differential equations, integral equations and related problems [4]. Let the residue vector r be defined as

$$r_i^{(k)} = b_i - \sum_{j=1}^{n} a_{ij} x_j^{(k)} \qquad \text{for i=1,2,…..n} \qquad (3)$$

ie. $r^{(k)} = b - Ax^{(k)}$ The iteration sequence $\{x^{(k)}: k = 0,1, \ldots \ldots\}$ is terminated when some norm of the residue $\|r^{(k)}\| = \|Ax^{(k)} - b\|$ becomes sufficiently small, ie.

$$\left\|\frac{r^{(k)}}{b}\right\| < \epsilon \qquad (4)$$

Whereϵ is an arbitrarily small number as $\varepsilon = 10^{-15}$ another possible termination criterion can be

$$\left\|\frac{x^{(k)} - x^{(k+1)}}{x^{(k+1)}}\right\| < \epsilon \qquad (5)$$

It may be noted that the later condition is practically equivalent to the previous termination condition. A simple way to form an iterative scheme is Richardson iterations [4]

$$x^{(k+1)} = (I - A)x^{(k)} + b \qquad (6)$$

[4] Richardson iterations preconditioned with approximate inversion

$$x^{(k+1)} = (I - MA)x^{(k)} + Mb \qquad (7)$$

Where matrix M is called approximate inverse of A if $\|I - MA\| < 1$. A question that naturally arises is will the iterations converge to the solution of Ax = b. In this section, to begin with, some well-known iterative schemes are presented. Their convergence analysis is presented next. In the derivations that follow, it is implicitly assumed that the diagonal elements of matrix A are non-zero, i.e.$a_{ii,} \neq 0$, if this is not the case, simple row exchange is often sufficient to satisfy this condition.

Theorem2.1. *A matrix A is called strictly diagonally dominant if:*

$$\sum_{j=1(j\neq i)}^{n} |a_{ij}| < |a_{ii}| \qquad \text{for i=1,2,…..n}$$

Theorem2.2. *A sufficient condition for the convergence of Jacobi and Gauss-Seidel methods is that the matrix A of linear system Ax=b is strictly diagonally dominant* [6]

Theorem.2.3. *For any arbitrary matrix A, the necessary condition for the convergence of relaxation method is 0 < w < 2*. [9]

3. Materials and Methods

3.1. Jacobi-Method: Suppose we have a guess solution, say $x^{(k)}$

$$x^{(k)} = \left[x_1{}^{(k)} \quad x_2{}^{(k)} \quad x_3{}^{(k)} \dots \dots \dots \dots x_n{}^{(k)}\right]^T$$

for, Ax = b: To generate an improved estimate starting from $x^{(k)}$;consider the first equation in the set of equations Ax = b, i.e.,

$$a_{11}x_1 + a_{12}x_2 + \cdots \dots \dots a_{1n}x_n = b_1 \tag{8}$$

Rearranging this equation, we can arrive at a iterative formula for computing, $x_1^{(k+1)}$, as

$$x_1{}^{(k+1)} = \frac{1}{a_{11}}[b_1 - a_{12}x_2{}^{(k)} \dots \dots - a_{1n}x_n{}^{(k)}] \tag{9}$$

Similarly, using second equation from Ax = b, we can derive

$$x_2{}^{(k+1)} = \frac{1}{a_{22}}[b_2 - a_{21}x_1{}^{(k)} - a_{23}x_3{}^{(k)} \dots \dots - a_{2n}x_n{}^{(k)}] \tag{10}$$

In general, using i^{th} row of Ax = b; we can generate improved guess for the i^{th} element x of as follows

$$x_i{}^{(k+1)} = \frac{1}{a_{ii}}[b_2 - a_{i1}x_1{}^{(k)} \dots - a_{i,i-1}x_{i-1}{}^{(k)} - a_{i,i+1}x_{i+1}{}^{(k)} \dots - a_{i,n}x_n{}^{(k)}] \tag{11}$$

The above equation can also be rearranged as follows

$$x_i{}^{(k+1)} = x_i{}^{(k)} + \left(\frac{r_i{}^{(k)}}{a_{ii}}\right)$$

Where $r_i^{(k)}$ is defined by equation (4). In matrix form, the method can be written as

$$x^{(k+1)} = -D^{-1}(L + U)x^{(k)} + D^{-1}b$$

The algorithm for implementing the Jacobi iteration scheme is summarized in Chart 1.

Chart 1: Algorithm for Jacobi Iterations

INITIALIZE: b, A, $x^{(0)}$, k_{max}, ϵ

k =0

$\delta = 100 * \epsilon$

WHILE $[(\delta > \epsilon)\ AND\ (k < kmax)]$

FOR i =1 : n

$r_i = b_i - \sum_{j=1}^{n} a_{ij} x_j$

$x_{Ni} = x_i + (\frac{r_i}{a_{ii}})$

END FOR

$\delta = \|r\|/\|b\|$

$x = x_N$

k =k+1

END WHILE

3.2. Gauss-Seidel Method: When matrix A is large, there is a practical difficulty with the Jacobi method. It is required to store all components of $x^{(k)}$ in the computer memory (as a separate variables) until calculations of $x^{(k+1)}$ is over. The Gauss-Seidel method overcomes this difficulty by using $x_i^{(k+1)}$ immediately in the next equation while computing $x_{i+1}^{(k+1)}$:This modification leads to the following set of equations

$$x_1^{(k+1)} = \frac{1}{a_{11}}[b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)} \dots \dots . a_{1n}x_n^{(k)}] \quad (12)$$

$$x_2^{(k+1)} = \frac{1}{a_{22}}\left[b_2 - \{a_{21}x_1^{(k+1))}\} - \{a_{23}x_3^{(k)} + \cdots \dots . + a_{2n}x_n^{(k)}\}\right] \quad (13)$$

$$x_3^{(k+1)} = \frac{1}{a_{33}}\left[b_3 - \{a_{31}x_1^{(k+1))} + a_{32}x_2^{(k+1))}\} - \{a_{34}x_4^{(k)} + \cdots \dots . + a_{3n}x_n^{(k)}\}\right] \quad (14)$$

In general, for i'th element of x, we have

$$x_i^{(k+1)} = \frac{1}{a_{ii}}[b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k)}]$$

To simplify programming, the above equation can be rearranged as follows

$$x_i^{(k+1)} = x_i^{(k)} + (\frac{r_i^{(k)}}{a_{ii}}) \quad (15)$$

where $r_i^{(k)} = [b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k)}]$

In matrix form, the method can be written as

$$x^{(k+1)} = -(L + D)^{-1}Ux^{(k)} + (L + D)^{-1}b \quad (16)$$

The algorithm for implementing Gauss-Siedel iteration scheme is summarized in Chart2.

Chart 2: Algorithm for Gauss Seidel Iterations

INITIALIZE: b, A, $x^{(0)}$, k_{max}, ϵ

k =0

$\delta = 100 * \epsilon$

WHILE $[(\delta > \epsilon)\ AND\ (k < kmax)]$

FOR i =1 : n

$r_i = b_i - \sum_{j=1}^{n} a_{ij} x_j$

$x_i = x_i + (\frac{r_i}{a_{ii}})$

END FOR

$\delta = \|r\| / \|b\|$

k =k+1

END WHILE

3.3. SUCESSASIVE OVER RELAXTION METHOD: Suppose we have a starting value say y, of a quantity and we wish to approach a target value, say y^* by some method. Let application of the method change the value from y to y^. If y^ is between y and y^ which is even closer to y than y^*. Then we can approach y^* faster by magnifying the change (y^ - y) [3]. In order to achieve this, we need to apply a magnifying factor w > 1 and get

$$y^* = y + w(y\hat{} - y) \quad (17)$$

This amplification process is an extrapolation and is an example of over-relaxation. If the intermediate value y^ tends to overshoot target y^*, then we may have to use w < 1; this is called under-relaxation. Application of over-relaxation to Gauss-Seidel method leads to the following set of equations

$$x_i^{(k+1)} = x_i^{(k)} + w[z_i^{(k+1)} - x_i^{(k)}] \quad \text{for i =1,2,…n} \quad (18)$$

Where $z_i^{(k+1)}$ are generated using the Gauss-Seidel method,

$$z_i^{(k+1)} = \frac{1}{a_{ii}}[b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}] \quad \text{for i =1,2…n} \quad (19)$$

In matrix form, the method can be written as

$$x^{(k+1)} = (wL + D)^{-1}[(1 - w)D - wU]x^{(k)} + wb \quad (20)$$

Thus, in general, an iterative method can be developed by splitting matrix A. The steps in the implementation of the over-relaxation iteration scheme are summarized in Chart 3. It may be noted that w is a tuning parameter, which is chosen such that 1 < w < 2 [9]

Chart 3: Algorithm for Gauss Seidel Iterations

INITIALIZE: b, A, $x^{(0)}$, k_{max}, ϵ

k =0

$\delta = 100 * \epsilon$

WHILE $[(\delta > \epsilon)\ AND\ (k < kmax)]$

FOR i =1 : n

$r_i = b_i - \sum_{j=1}^{n} a_{ij} x_j$

$z_i = x_i + (r_i / a_{ii})$

$x_i = x_i + \omega(z_i - x_i)$

END FOR

r = b - Ax

$\delta = \|r\| / \|b\|$

k =k+1

END WHILE

3.4. Matlab Programs: Jacobi method, Gauss Seidel and SOR Methods are below as [10]

```
clc;
clear all;
%   3.112x + 0.5756y - 0.1565z - 0.0067t   =  1.571
%   0.5756x + 2.938y + 0.1103z - 0.0015t  = -0.9275
%  -0.1565x + 0.1103y + 4.127z + 0.2015t = -0.06502
%  -0.0067x - 0.0015y+ 0.2051z + 4.133t  =  -0.0177
A = [3.112 0.5756 -0.1565 -0.0067; 0.5756 2.938 0.1103 -0.0015; -0.1565 0.1103 4.127 0.2015 ;
-0.0067 -0.0015 0.2051 4.133];
b = [1.571;  -0.9275;  -0.06502; -0.0177];
% error tolerance
tol = 0.000000000000005;
%initial guess:
x0 = zeros(n,1);                    here n=4
%  Jacobi method
%---------------
xnew=x0;
error=1;
while error>tol
   xold=xnew;
      for i=1:length(xnew)
```

```
        off_diag = [1:i-1 i+1:length(xnew)];
        xnew(i) = 1/A(i,i)*( b(i)-sum(A(i,off_diag)*xold(off_diag)) );
    end
    error=norm(xnew-xold)/norm(xnew);
end
x_jacobian=xnew
%Gauss Seidel Method:
%---------------
maxiter=1000;
lambda=1;
n=length(x0);
x=x0;
error=1;
iter = 0;
while (error>tol & iter<maxiter)
    xold=x;
    for i=1:n
        I = [1:i-1 i+1:n];
        x(i) = (1-lambda)*x(i)+lambda/A(i,i)*( b(i)-A(i,I)*x(I) );
    end
    error = norm(x-xold)/norm(x);
    iter = iter+1;
end
x_siedal=x
%SOR
%---------------
maxiter=1000;
lambda=1.2;
n=length(x0);
x=x0;
error=1;
```

```
iter =0;
while (error>tol & iter<maxiter)
   xold=x;
   for i=1:n
      I = [1:i-1 i+1:n];
      x(i) = (1-lambda)*x(i)+lambda/A(i,i)*( b(i)-A(i,I)*x(I) );
   end
   error = norm(x-xold)/norm(x);
   iter = iter+1;
end
```

4. Convergence Analysis of Iterative Methods

The convergence analysis can be carried out if the above set of iterative equations are expressed in the vector-matrix notation To discuss the convergence of the iterative methods (ii) .we study the behavior of the difference between the exact solution x and an approximate $x^{(k)}$. The exact solution x will satisfy

$$x = Hx + c \tag{21}$$

Where $H = \|I - MA\| < 1$, Subtracting (21) from (6) and substituting $\epsilon^{(k)} = x^{(k)} - x$, we get

$$\epsilon^{(k+1)} = H\epsilon^{(k)} \quad , k = 0,1, 2 ,\ldots\ldots$$

From which we obtain $\epsilon^{(k)} = H^{(k)}\epsilon^{(0)}$ k = 0, 1, 2,…….

Where we have assumed that the iteration matrix H remains constant for each iteration.We given few results above which we require for providing the convergence of the iterative methods.[1]

5. Numerical Experiments and Comparative discussion

In this section, we employ the various techniques obtained in this paper to solve system of linear simultaneous equations and compare them. We use the stopping criteria $|x_{k+1} - x_k| < \varepsilon$ and $|f(x_{k+1})| < \varepsilon$, where $\varepsilon = 10^{-15}$, for computer programs. All programs are written in Matlab2009a .The results are presented in Table1to3. Example1. Let us consider the system of linear simultaneous equations of n= 4 variables.

$$3.112x + 0.5756y - 0.1565z - 0.0067t = 1.5710$$

$$0.5756x + 2.938y + 0.1103z - 0.0015t = -0.9275$$

$$-0.1565x + 0.1103y + 4.127z + 0.2015\,t = -0.06502$$

$$-0.0067x - 0.0015y + 0.2051z + 4.133\,t = -0.0177$$

Table1. Numerical Solutions of Iteration data for Jacobi method, with $\varepsilon = 10^{-15}$

Number of iterations	Numerical solutions of X(x, y, z, t)	Errors at each iteration
1	X(1) =	error =
	0.50482005141388	
	-0.31569094622192	1
	-0.01575478555852	
	-0.00428260343576	
2	X(2) =	error =
	0.56240918742365	
	-0.41400377557926	0.16792158814306
	0.01203489072131	
	-0.00279698416569	
3	X(3) =	error =
	0.58199398901909	
	-0.42632892178484	0.03278271009799
	0.01677374523566	
	-0.00411836927040	
4	X(4) =	error =
	0.58450913413709	
	-0.43034447302001	0.00671700306133
	0.01791034426032	
	-0.00432625908640	
5	X(5) =	error =
	0.58530856735578	
	-0.43088000611636	0.00135749643692
	0.01812319265143	

	-0.00438004285473	
6	X(6) =	error =
	0.58541820851009	
	-0.43104464587601	2.799561402866404e-004
	0.01817044683815	
	-0.00438950385209	
7	X(7) =	error =
	0.58545101652333	
	-0.43106790516012	5.675417343725499e-005
	0.01817946670660	
	-0.00439173085369	
8	X(8)=	error =
	0.58545576740136	
	-0.43107467252718	1.170187868812868e-005
	0.01818144119023	
	-0.00439213372092	
9	X(9) =	error =
	0.58545711753116	
	-0.43107567763107	2.376226925696227e-006
	0.01818182188583	
	-0.00439222645907	
10	X(10) =	error =
	0.58545732238184	
	-0.43107595648218	4.896841447296750e-007
	0.01818190447488	
	-0.00439224352716	
11	X(11)=	error =
	0.58545737807514	
	-0.43107599972494	9.957377583570863e-008
	0.01818192052907	

	-0.00439224739476	
12	X(12) =	error =
	0.58545738687241	
	-0.43107601124081	2.050878105103282e-008
	0.01818192398557	
	-0.00439224811686	
13	X(13) =	error =
	0.58545738917467	
	-0.43107601309447	4.175149479277625e-009
	0.01818192466221	
	-0.00439224827831	
14	X(14) =	error =
	0.58545738955120	
	-0.43107601357100	8.594936849327012e-010
	0.01818192480694	
	-0.00439224830883	
15	X(15) =	error =
	0.58545738964656	
	-0.43107601365022	1.751459502654988e-010
	0.01818192483544	
	-0.00439224831557	
16	X(16) =	error =
	0.58545738966263	
	-0.43107601366997	3.603792102737823e-011
	0.01818192484151	
	-0.00439224831686	
17	X(17) =	error =
	0.58545738966658	
	-0.43107601367335	7.349898927456906e-012
	0.01818192484271	

	-0.00439224831714	
18	X(18)=	error =
	0.58545738966727	
	-0.43107601367417	1.511588971059816e-012
	0.01818192484296	
	-0.00439224831720	
19	X(19) =	error =
	0.58545738966743	
	-0.43107601367431	3.084681282223669e-013
	0.01818192484301	
	-0.00439224831721	
20	X(20) =	error =
	0.58545738966746	
	-0.43107601367435	6.340530233702467e-014
	0.01818192484302	
	-0.00439224831721	
21	X(21) =	error =
	0.58545738966747	
	-0.43107601367435	1.294122602041948e-014
	0.01818192484302	
	-0.00439224831721	
22	X(22) =	error =
	0.58545738966747	
	-0.43107601367436	2.673634683604702e-015
	0.01818192484302	
	-0.00439224831721	

Table 1 shows that the iteration data obtained for Jacobi method .it is observed that the numerical Solution of the linear simultaneous equation converges at 22 iteration with error level of 0.000000000000005

Table 2. Numerical Solutions of Iteration data for Gauss Seidel method, with $\varepsilon = 10^{-15}$

Number of iterations	Numerical solutions of X(x, y, z, t)	Errors at each iteration
1	X(1) =	error =
	0.50482005141388	
	-0.41459306385086	1
	0.01446909449698	
	-0.00433273809161	
2	X(2) =	error =
	0.58222209238307	0.10916741771859
	-0.43030274901493	
	0.01803566692507	
	-0.00438995436508	
3	X(3)=	error =
	0.58530701526752	0.00436658198424
	-0.43104106091946	
	0.01817517632986	
	-0.00439214450867	
4	X(4)=	error =
	0.58545058560175	2.028635364439777e-004
	-0.43107442724245	
	0.01818161935789	
	-0.00439224361182	
5	X(5) =	error =
	0.58545708088627	
	0.43107594170821	9.179243844217245e-006
	0.01818191098059	
	-0.00439224810374	
6	X(6)=	
	0.58545737565984	error =
	-0.43107601040950	4.165695515671734e-007
	0.01818192421416	
	-0.00439224830753	
7	X(7) =	
	0.58545738903199	

	-0.43107601352624	error =
	0.01818192481449	1.889737992891211e-008
	-0.00439224831677	
8	X(8) =	
	0.58545738963864	error =
	-0.43107601366764	8.573066107410987e-010
	0.01818192484173	
	-0.00439224831719	
9	X(9)=	
	0.58545738966616	
	-0.43107601367405	error =
	0.01818192484297	3.889267014450050e-011
	-0.00439224831721	
10	X(10) =	
	0.58545738966747	
	-0.43107601367434	error =
	0.01818192484302	1.764439799124610e-012
	-0.00439224831721	
11	X(11)=	
	0.58545738966747	
	-0.43107601367436	error =
	0.01818192484302	8.002107613160653e-014
	-0.00439224831721	
12	X(12)=	
	0.58545738966747	
	-0.43107601367436	error =
	0.01818192484302	3.613063005810915e-015
	-0.00439224831721	

Table 2 shows that the iteration data obtained for Gauss Seidel method .it was observed that the numerical Solution of the linear simultaneous equation converges at the 12 iteration with error level of 0.00000000000005

Table3. Numerical Solutions of Iteration data for SOR method, with $\varepsilon = 10^{-15}$

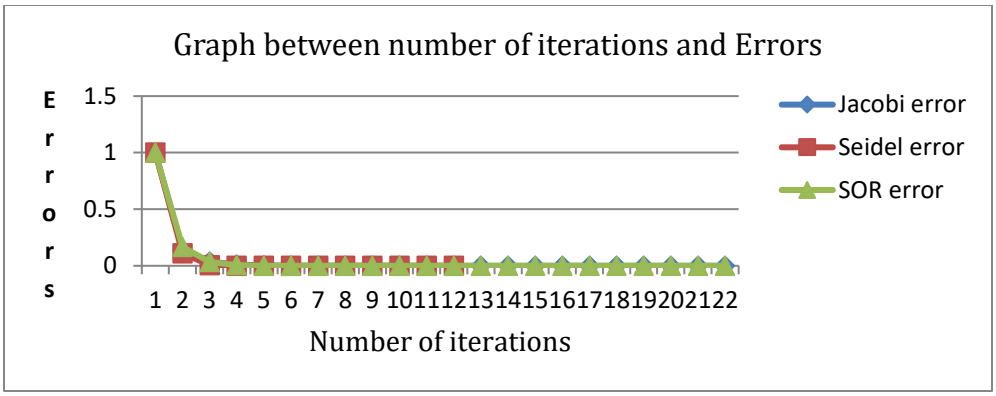
No. of iterations	Numerical Solutions of X(x,y,z,t)	Errors in each iterations
1	X(1) =	
	0.60578406169666	error =
	-0.52124818485198	1
	0.02537791532194	
	-0.00569894880129	
2	X(2) =	
	0.60183698560092	error =
	-0.41721738878662	0.14258422119967
	0.01712017300438	
	-0.00402978171698	
3	X (3)=	
	0.57904235989393	error =
	-0.43229151833930	0.03783623815512
	0.01812010442860	
	-0.00447406888318	
4	X(4) =	
	0.58700623934679	
	-0.43119431035112	error =
	0.01827335748195	0.01103642226787
	-0.00437836752937	
5	X (5)=	
	0.58517942967110	error =
	-0.43099111693600	0.00253429231144
	0.01814745363045	
	-0.00439347546181	
6	X(6) =	
	0.58549205511306	error =
	-0.43109959061077	4.589345601954924e-004

	0.01819122459500	
	-0.00439249952107	
7	X(7) =	
	0.58545625013082	error =
	-0.43107144950102	6.453339771903484e-005
	0.01817988137485	
	-0.00439207661696	
8	X(8) =	
	0.58545648166585	error =
	-0.43107144950102	7.863608435704243e-006
	0.01817988137485	
	-0.00439207661696	
9	X(9) =	
	0.58545772862405	error =
	-0.43107662087342	
	0.01818230163190	2.015299667709794e-006
	-0.00439230712583	
10	X(10) =	
	0.58545731296901	error =
	-0.43107598893356	5.789747813536638e-007
	0.01818186756164	
	-0.00439223247423	
11	X(11) =	
	0.58545740191311	error =
	-0.43107599800051	1.264805602506763e-007
	0.01818193137820	
	-0.00439225201736	
12	X(12) =	
	0.58545738860102	error=
	-0.43107601998474	2.128302335990060e-008

	0.01818192451241	
	-0.00439224753642	
13	x(13)=	
	0.58545738953652	error =
	-0.43107601214618	2.803660810121424e-009
	0.01818192476586	
	-0.00439224847019	
14	X(14) =	
	0.58545738975562	error =
	-0.43107601394582	5.189434870315965e-010
	0.01818192487017	
	-0.00439224828861	
15	X(15) =	
	0.58545738964239	error =
	-0.43107601364199	1.624058027681464e-010
	0.01818192483889	
	-0.00439224832250	
16	X(16) =	
	0.58545738967258	error =
	-0.43107601367475	4.154743530371315e-011
	0.01818192484303	
	-0.00439224831621	
17	X(17) =	
	0.58545738966671	error =
	-0.43107601367548	8.346869132251461e-012
	0.01818192484323	
	-0.00439224831742	
18	X(18) =	
	0.58545738966753	error =
	-0.43107601367396	1.317623655745480e-012

	0.01818192484295	
	-0.00439224831717	
19	X(19) =	
	0.58545738966748	error =
	-0.43107601367445	1.633847938958885e-013
	0.01818192484304	
	-0.00439224831722	
20	X(20) =	
	0.58545738966746	
	-0.43107601367434	error =
	0.01818192484302	3.043187461809806e-014
	-0.00439224831721	
21	X(21) =	
	0.58545738966747	error =
	-0.43107601367436	
	0.01818192484302	9.427901987222536e-015
	-0.00439224831721	
22	X(22)=	
	0.58545738966747	error =
	-0.43107601367436	
	0.01818192484302	2.297594335026736e-015
	-0.00439224831721	

Table 3 shows that the iteration data obtained for SOR method .it was observed that the numerical Solution of the linear simultaneous equation converges at the 22 iteration with error level of 0.000000000000005 when optimal factor w=1.2



Example-2 Let us consider another the system of linear simultaneous equations of n=5 variables

$$10v + w + x - 2y + z = -1,\ v - 20w - 2x + y + z = 20,\ v + w + 10x - y - z = 1$$

$$-v + 2w + x + 50y + z = 2,\ v + w + x + y + 100z = -1$$

The solution of above equations are converges 16, 11, 24 iterations respectively by the above techniques up to error level of $\varepsilon = 10^{-15}$ which are given below.

v = -0.00364280090102

w = -1.01743940692132

x = 0.20949210878712

y= 0.07648785763481

z = -0.00264897758600

Thus we can solve the system of linear simultaneous equations of more variable which are diagonally dominant in coefficient matrix by the above techniques.

5.1. Executing time: The execution time of a given task is defined as the time spent by the system executing that task, including the time spent executing run time or system services on its behalf.

Table 4: Execution time comparison for iterations data of various techniques

S/No.	Techniques	Number of iterations		(Elapsed time)	
		Example1	Example2	Example1	Example2
1.	Jacobi	22	16	0.015000	0.02000
2.	Seidel	12	11	0.000140	0.01500
3.	SOR	22	24	0.016000	0.02500

Thus from the above discussions we see that the Jacobi and SOR method is taking more time in comparison to that of Seidel method to run the program. [11]. SOR method has more error than other Since the Seidel method requires less number of iterations and Jacobi and SOR method

required more iterations with error level 0.000000000000005 with optimal factor w = 1.2 Thus we see that the overall performance of the iterative techniques are in this manner.

$$\text{SOR method} \leq \text{Jacobi method} < \text{Gauss Seidel method}$$

CONCLUSION

The above plot is shows the result obtained from different algorithm. Consequently, we can see that Gauss Seidel Technique is more accurate at large scale with different parameters such as running time factor and number of iterations and error level. Based on our results and discussions, we now conclude that the Seidel method is formally the most effective of the Jacobi and SOR method, we have considered here in the study. Analysis of efficiency from the numerical computation shows that Jacobi and SOR method converges slowly. Thus these methods have great practical utilities. One can easily adopt these MATLAB codes as needed for a different type of problem linear simultaneous equations of more order like 6×6 and so on. And also can use linear simultaneous differential equation for equilibrium problem in applied mathematics.

Conflict of Interests

The authors declare that there is no conflict of interests.

REFERENCES

[1] M.K.Jain, S.R.K. Iyenger and R.K.Jain. Numerical methods for scientific and engineering computation. New Age International Publishers,147-154, 2010.

[2] Gourdin, A. and M Boumhrat; Applied Numerical Methods. Prentice Hall India, New Delhi.

[3] Strang, G.; Linear Algebra and Its Applications. Harcourt Brace Jovanovich College Publisher, New York, 1988.

[4] Kelley, C.T.; Iterative Methods for Linear and Nonlinear Equations, Philadelphia: SIAM, 1995.

[5] The MathWorks Inc. MATLAB: 7.8. or 2009a.

[6] Atkinson, K. E.; An Introduction to Numerical Analysis, John Wiley, 2001.

[7] Steven C. Chapra. Applied Numerical methods with matlab for engineers and scientist.2012

[8] Phillips, G. M. and P. J. Taylor; Theory and Applications of Numerical Analysis, Academic Press, 1996.61

[9] Sachin C. Patwardhan, Numerical Analysis Module 4 Solving Linear Algebraic Equations, Dept. of Chemical Engineering, Indian Institute of Technology, Bombay, India.

[10] Bazara, M.S., Sherali, H. D., Shetty, C. M., Nonlinear Programming, John Wiley, 1979.

[11] Hasan.A; Numerical Study of Some Iterative Methods for Solving Nonlinear Equations.www.ijesi