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SOFT $\tilde{n}$ TIMES EXPANSIVE MAPPING ABOUT FIXED POINT THEOREM

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Abstract. In this paper, we introduce the definitions about soft sets and soft reasonable expansive mapping. Then, we proved the fixed point theorems about it on complete soft metric spaces. Finally, we present some examples to validate and illustrate our approach.

Keywords: soft fixed point; complete soft metric spaces; soft contraction mapping.

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1. Introduction

The research about fixed point of expansive mapping was initiated by Machuca(see[1]). Later, Jungck discussed fixed points for other forms of expansive mapping(see[2]). In the resent, many researchers contribute many structure on soft set theory(see[3-5]). Futher, Das and Samanta[6] introduced a different notion of soft metric space by using a different concept of soft point and investigated some basic properties of these spaces.

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Recently, the study about fixed point theorem for expansive mapping and nonexpansive mapping is depply explored and has extended too many other directions. Motivated and inspired by the works(see[7-10]), in this paper, we define soft n times reasonable expansive mapping and discuss the existence of fixed point for soft n times reasonable expansive mapping.

2. Preliminaries

In this section, we gather together some definitions and known results which will be used in section 3. We use those definitions , and we dont make redundant narration. Before we state our fixed point theorems, we introduce the following basic definitions:

Definition 2.1. Let A be a nonempty subset of E. A soft set (F, A) on U is a set of the form (F, A)=$\{(F(p),p) : p \in E\}$. where F: A$\to 2^U$ is a set-valued map such that F(p)=$\varnothing$ for all $p \notin A$. F is called an approximate function of (F,A), and the collection of all soft set on U will be denoted by $S(U)$.

Definition 2.2.A soft set (F,A) on U is said to be a null soft set denoted $\varnothing$ if for all $P \in A$, F(p)=$\varnothing$.

Definition 2.3.A soft set (F,A) on U is said to be an absolute soft set, if for all $P \in A$, F(p)=U.

Definition 2.4.A soft set (F,A) on U is said to be a soft point if there is exactly one $p \in E$, such that F(p)=$\{x\}$ for some $x \in U$ and F(q)=$\varnothing$ for all $q \in E \setminus \{p\}$. It will be denoted by $\tilde{x}_p$.

Definition 2.5. Let B $(\mathbb{R})$ be the collection of all nonempty bounded subsets of $\mathbb{R}$. Then the mapping F:$E \to B(\mathbb{R})$ is called a soft real set. If (F, E) is a singleton soft set, then identifying (F, E) with the corresponding soft element, it will be called a soft real number and denoted $\tilde{r}$, $\tilde{s}$, $\tilde{t}$, ect. If all elements of E are the same singleton soft set. It denoted $\bar{r}$, $\bar{s}$, $\bar{t}$, ect.

For two soft real numbers $\tilde{r}$, $\tilde{s}$, we say that:

(N1) $\tilde{r}\tilde{\leq}\tilde{s}$, if $\tilde{r}(e)\tilde{\leq}\tilde{s}(e)$, for all $e \in A$;

(N2) $\tilde{r}\tilde{\geq}\tilde{s}$, if $\tilde{r}(e)\tilde{\geq}\tilde{s}(e)$, for all $e \in A$;

(N3) $\tilde{r}\tilde{<}\tilde{s}$, if $\tilde{r}(e)\tilde{<}\tilde{s}(e)$, for all $e \in A$;

(N4) $\tilde{r}\tilde{>}\tilde{s}$, if $\tilde{r}(e)\tilde{>}\tilde{s}(e)$, for all $e \in A$;

Definition 2.6.A be a nonempty subset of parameters and $\tilde{U}$ be the absolute soft set, i.e. $(f,\varphi)(x_\lambda) = U$ for all $\lambda \in A$, where (F,A)=$\tilde{U}$.

A mapping d: $SP(\tilde{U}) \times SP(\tilde{U}) \to \mathbb{R}(A)^*$ is said to be a soft metric on $\tilde{U}$ if for any $U^x_\lambda, U^y_\mu, U^z_\gamma \in SP(\tilde{X})$, the following hold :

(M1) $d(U^x_\lambda, U^y_\mu) \tilde{\geq} \bar{0}$;

(M2) $d(U^x_\lambda, U^y_\mu) \tilde{=} \bar{0}$, if and only if $U^x_\lambda = U^y_\mu$;

(M3) $d(U^x_\lambda, U^y_\mu) \tilde{=} d(U^y_\mu, U^x_\lambda)$;

(M4) $d(U^x_\lambda, U^y_\mu) \tilde{\leq} d(U^x_\lambda, U^z_\gamma) + d(U^y_\mu, U^z_\gamma)$.

Given a soft metric space $(\tilde{U}, d)$, a net $\{U^{x_\alpha}_{\lambda_\alpha}\}_{\alpha\in\Lambda}$ of soft point in $\tilde{U}$ will be simply denoted by $\{U^x_{\lambda_\alpha}\}_{\alpha\in\Lambda}$. In particular, a sequence $\{U^{x_n}_{\lambda_n}\}_{n\in\mathbb{N}}$ of soft points in $\tilde{U}$ will be denoted by $\{U^x_{\lambda_n}\}_n$.

Definition 2.7. Let $\{\tilde{x}_{\lambda,n}\}_n$ be a sequence of soft points in a soft metric space $(\tilde{U}, \tilde{d}, E)$. Then the sequence $\{\tilde{x}_{\lambda,n}\}_n$ is said to be convergent in $(\tilde{U}, \tilde{d}, E)$ if there is a soft point $\tilde{y_\nu} \tilde{\in} \tilde{X}$ such that

$$\lim_{n\to\infty} \tilde{d}(x\tilde{_{\lambda,n}}, \tilde{y_\nu}) = \bar{0}$$

.

Definition 2.8. Let $\{\tilde{x}_{\lambda,n}\}_n$ be a sequence of soft points in a soft metric space $(\tilde{U}, \tilde{d}, E)$. Then the sequence $\{\tilde{x}_{\lambda,n}\}_n$ is said to be a Cauchy sequence in $(\tilde{U}, \tilde{d}, E)$ if there is a soft point $\tilde{y_\nu} \tilde{\in} \tilde{X}$ such that

$$\lim_{i,j\to\infty} \tilde{d}(x\tilde{_{\lambda,i}}, x\tilde{_{\lambda,j}}) = \bar{0}$$

.

Definition 2.9.A soft metric space $(\tilde{U}, \tilde{d}, E)$ is called complete if every Cauchy sequence in $\tilde{X}$ converges to some point of $\tilde{X}$. Throughout this paper,we use $\mathbb{N}$ to denoted the set of soft natural numbers and $[\tilde{x}]$ to denote the maximum soft integral value that is not large that $\tilde{x}$.

Definition 2.10.$f : \tilde{U} \to \tilde{U}$ is called a soft expansive mapping if there exists a soft constant $\tilde{h} \tilde{\geq} \bar{1}$ such that

$$\tilde{d}(f(\tilde{x_\lambda}), f(\tilde{y_\mu})) \tilde{\geq} \tilde{h}\tilde{d}(\tilde{x_\lambda}, \tilde{y_\mu}) \text{ ,for all } \tilde{x_\lambda}, \tilde{y_\mu} \tilde{\in} SP(\tilde{U}).$$

$f : \tilde{U} \to \tilde{U}$ is called a soft two times reasonable expansive mapping if there exists a soft constant $\tilde{h} \tilde{\geq} \bar{1}$ such that

$$\tilde{d}(\tilde{x_\lambda}, f^2(\tilde{x_\lambda})_{\varphi^2(\lambda)}) \tilde{\geq} \tilde{h}\, \tilde{d}(\tilde{x}, f(\tilde{x_\lambda})), \text{ for all } \tilde{x_\lambda} \in SP(\tilde{U}).$$

Definition 2.11. $f : \tilde{U} \to \tilde{U}$ is called a soft twenty-one type expansive mapping if there exists a soft constant $\tilde{h} \tilde{\geq} \bar{1}$ such that

$$(1)\quad \tilde{d}(f(\tilde{x_\lambda}), f(\tilde{y_\mu})) \tilde{\geq} \tilde{h} min\{\tilde{d}(\tilde{x_\lambda}, \tilde{y_\mu}), \tilde{d}(\tilde{x_\lambda}, f(\tilde{x_\lambda})), \tilde{d}(\tilde{y_\mu}, f(\tilde{y_\mu})), \tilde{d}(\tilde{x_\lambda}, f(\tilde{y_\mu})), \tilde{d}(\tilde{y_\mu}, f(\tilde{x_\lambda}))\}, \quad for\ all\ \tilde{x_\lambda}, \tilde{y_\mu} \tilde{\in} SP(\tilde{U}).$$

$f : \tilde{U} \to \tilde{U}$ is called a soft twenty-three type expansive mapping if there exists a soft constant $\tilde{h} \tilde{\geq} \bar{1}$ such that

$$(2)\quad \tilde{d}^2(f(\tilde{x_\lambda}), f^2(\tilde{y_\mu})_{\varphi^2(\mu)}) \tilde{\geq} \tilde{h} min\{\tilde{d}^2(\tilde{x_\lambda}, \tilde{y_\mu}), \tilde{d}(\tilde{x_\lambda}, \tilde{y_\mu}) \cdot \tilde{d}(\tilde{x_\lambda}, f(\tilde{x_\lambda})), \tilde{d}(\tilde{x_\lambda}, f(\tilde{x_\lambda})) \cdot \tilde{d}(\tilde{y_\mu}, f(\tilde{y_\mu})),$$
$$\tilde{d}^2(\tilde{x_\lambda}, f(\tilde{x_\lambda}), \tilde{d}(\tilde{y_\mu}, f(\tilde{y_\mu})) \cdot \tilde{d}(\tilde{x_\lambda}, f(\tilde{y_\lambda})), \tilde{d}(\tilde{y_\mu}, f(\tilde{y_\mu})) \cdot \tilde{d}(\tilde{y_\mu}, f(\tilde{x_\lambda}))\}, forall \tilde{x_\lambda}, \tilde{y_\mu} \tilde{\in} SP(\tilde{U}).$$

3. Main results

Definition 3.1. Let $(\tilde{U}, \tilde{d}, E)$ be a complete soft metric space, and let $(f, \varphi) : (\tilde{U}, \tilde{d}, E) \to (\tilde{U}, \tilde{d}, E)$ is called a soft n times reasonable expansive mapping if there exists a soft constant $\tilde{h} \tilde{>} \bar{1}$ such that

$$(3)\qquad \tilde{d}(\tilde{x_\lambda}, f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}}) \tilde{\geq} \tilde{h}\, \tilde{d}(\tilde{x_\lambda}, (f, \varphi)\tilde{x_\lambda}), \quad for\ all\ \tilde{x} \tilde{\in} \tilde{U} \ \ (n \geq 2, n \in \mathbb{N})$$

Definition 3.2. Let $(\tilde{U}, \tilde{d}, E)$ be a complete soft metric space, and let $(f, \varphi) : (\tilde{U}, \tilde{d}, E) \to (\tilde{U}, \tilde{d}, E)$ is called a soft H_1-type n times reasonable expansive mapping if there exists a soft constant $\tilde{h} \tilde{\geq} \bar{1}$ such that

$$(4)\quad \tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}}, f^{n-1}(\tilde{y_\mu})_{\varphi^{n-1}_{(\mu)}}) \tilde{\geq} \tilde{h} min\{\tilde{d}(\tilde{x_\lambda}, \tilde{y_\mu}), \tilde{d}(\tilde{x_\lambda}, (f, \varphi)(\tilde{x_\lambda})), \tilde{d}(f^{n-2}(\tilde{y_\mu})_{\varphi^{n-2}_{(\mu)}},$$
$$f^{n-1}(\tilde{y_\mu})_{\varphi^{n-1}_{(\mu)}}), \tilde{d}(\tilde{x_\lambda}, f^{n-1}(\tilde{y_\mu})_{\varphi^{n-1}_{(\mu)}})\}$$

Definition 3.3. Let $(\tilde{U}, \tilde{d}, E)$ be a complete soft metric space, and let $(f, \varphi) : (\tilde{U}, \tilde{d}, E) \to (\tilde{U}, \tilde{d}, E)$ is called a soft H_2-type n times reasonable expansive mapping if there exists a soft constant $\tilde{h} \tilde{\geq} \bar{1}$

such that

$$\tilde{d}^2(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}}, f^{n-1}(\tilde{y_\mu})_{\varphi^{n-1}_{(\mu)}}) \tilde{\geq} \tilde{h} min\{\tilde{d}^2(\tilde{x_\lambda},\tilde{y_\mu}), \tilde{d}(\tilde{x_\lambda},\tilde{y_\mu})\cdot\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})), \tilde{d}(\tilde{x_\lambda}, (f,\varphi)(\tilde{x_\lambda}))\cdot\tilde{d}(f^{n-2}(\tilde{y_\mu})_{\varphi^{n-2}_{(\mu)}}, f^{n-1}(\tilde{y_\mu})_{\varphi^{n-1}_{(\mu)}}), \tilde{d}^2(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})), \tilde{d}(f^{n-2}(\tilde{y_\mu})_{\varphi^{n-2}_{(\mu)}}, f^{n-1}(\tilde{y_\mu})_{\varphi^{n-1}_{(\mu)}})\cdot\tilde{d}(\tilde{x_\lambda}, f^{n-1}(\tilde{y_\mu})_{\varphi^{n-1}_{(\mu)}})\} \tag{5}$$

Lemma 3.4. Let $(\tilde{U},\tilde{d},E)$ be a complete soft metric space, let $\tilde{U_1}$ be a subset of $\tilde{U}$, and left the soft mappings $(f,\varphi_1),(g,\varphi_2) : \tilde{U_1}\to\tilde{U}$ satisfy the following conditions:
(i) (f,φ_1) is a surjective soft mapping $(f,\varphi)(\tilde{U_1}) = (\tilde{U})$;
(ii) There is a functional $\phi : \tilde{U}\to\mathbb{R}^+$ which is lower semicontinuous bounded from below such that $\tilde{d}((f,\varphi_1)(\tilde{x_{\lambda 1}}),(g,\varphi_2)(\tilde{x_{\lambda 2}})) \tilde{\leq} \phi((f,\varphi_1)(\tilde{x_{\lambda 1}})) - \phi((g,\varphi_2)(\tilde{x_{\lambda 2}}))$ for all $\tilde{x_{\lambda i}} \tilde{\in} \tilde{U_1}$.
Then, (f,φ_1) and (g,φ_2) have a coincidence point, that is, there exists at least an $\tilde{x_\lambda}\tilde{\in}\tilde{U_1}$ such that $(f,\varphi_1)(\tilde{x_\lambda}) = (g,\varphi_2)(\tilde{x_\lambda})$.

Theorem 3.5. Let $(\tilde{U},\tilde{d},E)$ be a complete soft metric space and (f,φ) be a continuous and soft surjective mapping if there exists a constant $\tilde{h}\tilde{>}\bar{1}$ such that

$$\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)}, f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)}) \tilde{\geq} \tilde{h}\,\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})) \quad for\ all\ \tilde{x_\lambda}\tilde{\in} SP(\tilde{U}) \tag{6}$$

Then, (f,φ) has a fixed point in $(\tilde{U},\tilde{d},E)$.

Proof. By (2.4) we have

$$\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)}, f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)}) - \tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})) \tilde{\geq} \tilde{h}\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})) - \tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})) \tag{7}$$

Thus,

$$\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})) \tilde{\leq} \frac{\bar{1}}{\tilde{h}-\bar{1}}[\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)}, f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)}) - \tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda}))] \tag{8}$$

Let $\phi(\tilde{x_\lambda}) = \frac{\bar{1}}{\tilde{h}-\bar{1}}[\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)}, f^{n-2}(\tilde{x_\lambda})_{\varphi^{n-2}(\lambda)}) + \tilde{d}(f^{n-2}(\tilde{x_\lambda})_{\varphi^{n-2}(\lambda)}, f^{n-3}(\tilde{x_\lambda})_{\varphi^{n-3}(\lambda)}) + ... + \tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})]$

So we can get $\phi((f,\varphi)(\tilde{x_\lambda}))$. Then we have $\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda}) \tilde{\leq} \phi((f,\varphi)(\tilde{x_\lambda}))-\phi(\tilde{x_\lambda})$. From the continuity of $\tilde{d}$, we know that $\phi(\tilde{x_\lambda})$ is continuous. Thus $\phi(\tilde{x_\lambda})$ is lower semicontinuous bounded from below. Therefore the conclusion follows immediately from the above Lemma 2.4. This completes the proof.

Theorem 3.6. Let $(\tilde{U},\tilde{d},E)$ be a complete soft metric space and (f,φ) be a continuous and soft surjective $\tilde{n}$ ($\tilde{n}\tilde{\geq}2$, $\tilde{n} \tilde{\in} \tilde{\mathbb{N}}$)times reasonable expansive mapping. Assume that either (i)or(ii)holds:

(i)(f,φ) is an H_1-type $\tilde{n}$ times reasonable expansive mapping;

(ii)(f,φ) is an H_2-type $\tilde{n}$ times reasonable expansive mapping.

Then,(f,φ)has a fixed point in $\tilde{U}$.

Proof. In the case of (i),taking $\tilde{y_\mu} = (f,\varphi)(\tilde{x_\lambda})$in (2.2),we have

$$\begin{aligned}\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)},f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)}) &\tilde{\geq} \tilde{h}\, min\{\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})),\ \tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})),\\ &\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)},f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)}),\ \tilde{d}(\tilde{x_\lambda},f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)})\}\\ &=\tilde{h}\, min\ \{\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})),\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)},\\ &f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)}),\ \tilde{d}(\tilde{x_\lambda},f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)})\}\end{aligned} \tag{9}$$

Because (f,φ) is an $\tilde{n}$times reasonable expansive mapping, we have

$$\tilde{d}(\tilde{x_\lambda},f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)}) \tilde{\geq} \tilde{h}\, \tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})) \tilde{>} \tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})) \tag{10}$$

Thus,we obtain

$$\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)},f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)}) \tilde{\geq} \tilde{h}\, min\{\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})),\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)},f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)})\} \tag{11}$$

If $\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)},f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)}) = min\{\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})),\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)},f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)})\}$, then $\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)},f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)}) \tilde{\geq} \tilde{h}\ \{\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)},f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)})\}$. Hence, $\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)},f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)}) = 0$. (otherwise, $\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)},f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)}) \tilde{\geq}$ $\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)},f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)})$, which is a contradiction). Therefore, $f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)}=f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)}$, that is $f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)} = f(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)})$, which implies that $f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)}$ is a fixed point of (f,φ) in $\tilde{U}$.

If $\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})) = min\,\{\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})),\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)},f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)})\}$, then $\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}(\lambda)},f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)}) \tilde{\geq} h\,\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda}))$.

By Theorem 2.5, we obtain that (f,φ) has a fixed point in $\tilde{U}$. In the case of (ii),taking $\tilde{y_\mu} = (f,\varphi)(\tilde{x_\lambda})$in (2.3),we have

(12)
$$\begin{aligned}\tilde{d}^2(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}},f^n(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}}) \tilde{\geq} \tilde{h}min\{&\tilde{d}^2(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})),\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda}))\cdot\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})),\\ &\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda}))\cdot\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}},f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}}),\tilde{d}^2(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})),\\ &\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}},f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}})\cdot\tilde{d}(\tilde{x_\lambda},f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}})\}\\ = h\,min\{&\tilde{d}^2(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})),\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda}))\cdot\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}},\\ &f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}}),\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}},f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}})\cdot\tilde{d}(\tilde{x_\lambda},f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}})\}\end{aligned}$$

Because $\tilde{U}$ is an $\tilde{n}$ times reasonable expansive mapping, we have

$$\tilde{d}(\tilde{x_\lambda},f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)}) \tilde{\geq} \tilde{h}\,\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})) \tilde{>} \tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})) \tag{13}$$

Hence, $\tilde{d}(\tilde{x_\lambda},f^n(\tilde{x_\lambda})_{\varphi^n(\lambda)})\cdot\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}},f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}}) \tilde{>} \tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda}))$
$\cdot\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}},f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}})$.

Therefore, we have

$$\begin{aligned}\tilde{d}^2(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}},f^n(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}}) \tilde{\geq} \tilde{h}min\{&\tilde{d}^2(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})),\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda}))\\ &\cdot\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}},f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}})\}\end{aligned} \tag{14}$$

If $\tilde{d}^2(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})) = min\,\{\tilde{d}^2(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})),\tilde{d}(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda}))\cdot\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}},f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}})\}$, then

$$\tilde{d}^2(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}},f^n(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}}) \tilde{\geq} \tilde{h}\{\tilde{d}^2(\tilde{x_\lambda},(f,\varphi)(\tilde{x_\lambda})) \tag{15}$$

that is, $\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}}, f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}}) \tilde{\geq} \sqrt{\tilde{h}}\, \tilde{d}(\tilde{x_\lambda}, (f,\varphi)(\tilde{x_\lambda}))$.

Because $\sqrt{\tilde{h}} \tilde{\geq} 1$, by Theorem 2.5, we obtain that $\tilde{U}$ has a fixed point in $\tilde{U}$.

If $\tilde{d}(\tilde{x_\lambda}, (f,\varphi)(\tilde{x_\lambda})) \cdot \tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}}, f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}}) = min\,\{\tilde{d}^2(\tilde{x_\lambda}, (f,\varphi)(\tilde{x_\lambda})), \tilde{d}(\tilde{x_\lambda}, (f,\varphi)(\tilde{x_\lambda})) \cdot \tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}}, f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}})\}$, then $\tilde{d}^2(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}}, f^n(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}}) \tilde{\geq} \tilde{h}\tilde{d}(\tilde{x_\lambda}, (f,\varphi)(\tilde{x_\lambda})) \cdot \tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}}, f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}})$, that is

$$\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}}, f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}}) \cdot (\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}}, f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}}) - h\tilde{d}(\tilde{x_\lambda}, (f,\varphi)(\tilde{x_\lambda}))) \tilde{\geq} \tilde{0}. \tag{16}$$

If $\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}}, f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}}) = \tilde{0}$, then $f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}} = f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}}$, that is $f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}} = f(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}})$, which implies that $f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}}$ is a fixed point of (f,φ) in $\tilde{U}$.

If $\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}}, f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}}) \neq \tilde{0}$, then $\tilde{d}(f^{n-1}(\tilde{x_\lambda})_{\varphi^{n-1}_{(\lambda)}}, f^n(\tilde{x_\lambda})_{\varphi^n_{(\lambda)}}) \tilde{\geq} h\tilde{d}(\tilde{x_\lambda}, (f,\varphi)(\tilde{x_\lambda}))) \tilde{\geq} \tilde{0}$.

By Theorem 2.5, we obtain that (f,φ) has a fixed point in $\tilde{U}$.

Therefore,by induction we derive that (f,φ) has a fixed point in $\tilde{U}$.

This completes the proof.

Corollary 3.7. Let $(\tilde{U},\tilde{d},E)$ be a complete soft metric space and (f,φ) be a continuous and soft surjective twenty-three type expansive mapping and (f,φ) is a two times reasonable expansive mapping, then (f,φ) has a fixed point in $\tilde{U}$.

Conflict of Interests

The authors declare that there is no conflict of interests.

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