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POINTWISE SLANT SUBMERSIONS FROM SASAKIAN MANIFOLDS

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Abstract. In this paper, we study pointwise slant submersions from Sasakian manifolds onto Riemannian manifolds. We found some results on such submersions from Sasakian manifolds onto Riemannian manifolds admitting vertical and horizontal structure vector fields.

Keywords: almost contact manifolds; Riemannian submersions; pointwise slant submersions.

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1. Introduction

The theory of Riemannian submersions, started with the study of O'Neill [13] and Gray [9]. Watson [16] introduced the almost Hermitian submersions. These submersions were Riemannian submersions between almost Riemannian manifolds. Submersions between Riemannian manifolds equipped with an additional structure of almost contact type, firstly studied by Watson [17] and Chinea [6] independently.

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On the other hand, submersions have been studied by several authors. There are so many submersions. Some of submersions and related research papers are: slant submersions from almost Hermitian manifolds [14], Riemannian submersions from almost contact metric manifolds [10], On quasi-slant submanifolds of an almost Hermitian manifolds [7], Slant submanifolds in Sasakian manifolds [4], Pointwise slant submanifolds in almost Hermitian manifolds [5], Almost contact metric submersions [6], Riemannian submersions and related topics [8], Pointwise slant submersions [11], Slant submanifolds in contact geometry [12], Pointwise slant submanifolds in almost contact geometry [1], Pointwise slant submersions from Cosymplectic manifolds [15] etc.

In this paper, we focused on pointwise slant submersions from Sasakian manifolds onto Riemannian manifolds. This paper is organized as follows. We collect main notions and formulae which are needed for this paper in second section. In section 3, we obtain some results of pointwise slant submersions from Sasakian manifolds onto Riemannian manifolds admitting vertical and horizontal structure vector fields.

2. Preliminaries

An $m-$dimensional Riemannian manifold M is said to be an almost contact metric manifold if there exists on M a $(1,1)$ tensor field ϕ, a vector field ξ, a $1-$form η and a Riemannian metric g such that

$$\phi^2 = -I + \eta \otimes \xi, \phi\xi = 0, \eta \circ \xi = 0, \tag{1}$$

$$\eta(X) = g(X, \xi), \eta(\xi) = 1, \tag{2}$$

and

$$\begin{aligned} g(\phi X, \phi Y) &= g(X,Y) - \eta(X)\eta(Y), \\ g(\phi X, Y) &= -g(X, \phi Y), \end{aligned} \tag{3}$$

for any vector fileds X, Y on M.

Almost contact metric manifold is said to be contact metric manifold if $d\eta = \Phi$, where $\Phi(X,Y) = g(X,\phi Y)$ is called the fundamental $2-$form of M. On the other hand, almost contact metric structure of M is said to be normal if $[\phi,\phi](X,Y) = -2d\eta(X,Y)\xi$, for X,Y on M, where $[\phi,\phi]$ denotes the Nijenhuis tensor of ϕ given by

$$[\phi,\phi](X,Y) = \phi^2[X,Y] + [\phi X,\phi Y] - \phi[\phi X,Y] - \phi[X,\phi Y].$$

A normal contact metric manifold is called a Sasakian manifold [3] if

$$(\nabla_X \phi)Y = g(X,Y)\xi - \eta(Y)X, \tag{4}$$

for any vector fields X,Y on M. Moreover for a Sasakian manifold the following equation satisfies:

$$\begin{aligned} R(\xi,X)Y &= g(X,Y)\xi - \eta(Y)X, \\ \nabla_X \xi &= -\phi X. \end{aligned} \tag{5}$$

Let (M,g_M) be an $m-$dimensional Riemannian manifold and (N,g_N) be an $n-$dimensional Riemannian manifold. Let $F : (M,g_M) \rightarrow (N,g_N)$ be a C^∞ map. We denote the kernel space of F_* by $\ker F_*$ and consider the orthogonal complementary space $\mathscr{H} = (\ker F_*)^\perp$ to $\ker F_*$. Then the tangent bundle of M has the following decomposition, $TM = (\ker F_*) \oplus (\ker F_*)^\perp$.

We also denote the *range* of F_* by *range*F_* and consider the orthogonal complementary space $(rangeF_*)^\perp$ to $rangeF_*$ in the tangent bundle TN of N. Thus the tangent bundle TN of N has the following decomposition, $TN = (rangeF_*) \oplus (rangeF_*)^\perp$.

A Riemannian submersion F is a differentiable map from (M,g_M) onto (N,g_N) satisfying the following conditions:

(i) F has the maximal rank,

(ii) The differential map F_* preserves the lengths of horizontal vectors.

For each $x \in N$, $F^{-1}(x)$ is an $(m-n)$ dimensional submanifold of M so called fiber. If a vector field on M is always tangent (resp. horizontal) to fibres, then it is called vertical (rep. horizontal). A vector field X on M is said to be basic if it is horizontal and $F-$related to a vector field X_* on N, i.e., $F_* X_q = X_{*F(q)}$ for all $q \in M$. We denote the projection morphisms on the distributions $\ker F_*$ and $(\ker F_*)^\perp$ by $\mathscr{V}$ and $\mathscr{H}$ respectively.

A Riemannian submersion is characterized via O'Neill tensors $\mathscr{T}$ and $\mathscr{A}$ by the formulae

(6) $$\mathscr{T}_E F = \mathscr{H}\nabla_{\mathscr{V}E}\mathscr{V}F + \mathscr{V}\nabla_{\mathscr{V}E}\mathscr{H}F,$$

(7) $$\mathscr{A}_E F = \mathscr{V}\nabla_{\mathscr{H}E}\mathscr{H}F + \mathscr{H}\nabla_{\mathscr{H}E}\mathscr{V}F,$$

for arbitrary vector fields E and F on M, where ∇ is the Levi-Civita connection of (M, g_M) [13].

Lemma 2.1. Let $F : (M, g_M) \to (N, g_N)$ be Riemannian submersion between Riemannian manifolds. If X and Y are basic vector fields on M, then

(i) $g_M(X,Y) = g_N(X_*, Y_*) \circ F$,

(ii) the horizontal part $[X,Y]^{\mathscr{H}}$ of $[X,Y]$ is a basic vector field and corresponds to $[X_*, Y_*]$ i.e., $F_*([X,Y]^{\mathscr{H}}) = [X_*, Y_*]$,

(iii) $[V,X]$ is vertical for any vector field V of $\ker F_*$,

(iv) $(\nabla^M_X Y)^{\mathscr{H}}$ is vertical for any vector field corresponding to $\nabla^N_{X_*} Y_*$, where ∇^M and ∇^N are the Riemannian connection on M and N, respectively.

On the other hand, from equations (6) and (7), we have

(8) $$\nabla_X Y = \mathscr{T}_X Y + \widehat{\nabla}_X Y,$$

(9) $$\nabla_X V = \mathscr{H}\nabla_X V + \mathscr{T}_X V,$$

(10) $$\nabla_V X = \mathscr{A}_V X + \mathscr{V}\nabla_V X,$$

(11) $$\nabla_V W = \mathscr{H}\nabla_V W + \mathscr{A}_V W,$$

for any $X, Y \in \Gamma(\ker F_*)$ and $V, W \in \Gamma(\ker F_*)^{\perp}$, where $\widehat{\nabla}_X Y = \mathscr{V}\nabla_X Y$. Moreover, if V is basic then $\mathscr{H}\nabla_X V = \mathscr{A}_V X$.

On the other hand, for any $E \in \Gamma(TM)$, it is seen that $\mathscr{T}$ is vertical $\mathscr{T}_E = \mathscr{T}_{\mathscr{V}E}$ and $\mathscr{A}$ is horizontal $\mathscr{A}_E = \mathscr{A}_{\mathscr{H}E}$.

The tensor fields $\mathscr{T}$ and $\mathscr{A}$ satisfy the equations:

(12) $$\mathscr{T}_X Y = \mathscr{T}_Y X,$$

(13) $$\mathscr{A}_V W = -\mathscr{A}_W V = \frac{1}{2}\mathscr{V}[V,W],$$

for any $X,Y \in \Gamma(\ker F_*)$ and $V,W \in \Gamma(\ker F_*)^\perp$.

It can be easily seen that a Riemannian submersion $F : (M, g_M) \to (N, g_N)$ has totally geodesic fibers if and only if $\mathscr{T}$ identically vanishes.

Now, we consider the notion of harmonic maps between Riemannian manifolds. Let (M, g_M) and (N, g_N) be Riemannian manifolds and suppose that F is a C^∞ mapping between them. Then the differential F_* of F can be considered as a section of the bundle $Hom(TM, F^{-1}TN) \to M$, where $F^{-1}TN$ is the pullback bundle that has fibers $(F^{-1}TN)_q = T_{F(q)}N, q \in M$. If $Hom(TM, F^{-1}TN)$ has a connection ∇ induced from the Riemannian connection ∇^M, then the second fundamental form of F is given by

$$(\nabla F_*)(X,Y) = \nabla^F_X F_* Y - F_*(\nabla^M_X Y), \tag{14}$$

for any $X,Y \in \Gamma(TM)$, where ∇^F is the pullback connection.

For a Riemannian submersion F, we can easily seen

$$(\nabla F_*)(V,W) = 0, \tag{15}$$

for all $V,W \in \Gamma(\ker F_*)^\perp$ [2].

3. Pointwise slant submersion from almost contact metric manifolds

Let F be a Riemannian submersion from a Sasakian manifold $(M, \phi, \xi, \eta, g_M)$ onto a Riemannian manifold (N, g_N). If for each $q \in M$ the angle $\theta(X)$ between ϕX and the space $\ker F_*$ is independent of the choice of the non-zero vector field $X \in \Gamma(\ker F_*) - <\xi>$, then F is called a pointwise slant submersion. The angle θ is said to be slant function of the pointwise slant submersion [11].

A pointwise slant submersion F is called slant if its slant function θ is independent of the choice of the point on $(M, \phi, \xi, \eta, g_M)$. Then the constant angle θ is called the slant angle of the slant submersion.

3.1. Pointwise slant submersion for $\xi \in \Gamma(\ker F_*)$

Let F be a Riemannian submersion from a Sasakian manifold (M,ϕ,ξ,η,g_M) onto a Riemannian manifold (N,g_N).

For $X\in\Gamma(\ker F_*)$, we get

$$\phi X=\psi X+\omega X, \tag{16}$$

where ψX and ωX are vertical and horizontal components of ϕX, respectively.

For $V\in\Gamma(\ker F_*)^{\perp}$, we have

$$\phi V=BV+CV, \tag{17}$$

where BV and CV are vertical and horizontal components of ϕV, respectively

By using equations $(3),(16)$ and (17), we get

$$g_M(\psi X,Y)=-g_M(X,\psi Y), \tag{18}$$

$$g_M(\omega X,V)=-g_M(X,BV), \tag{19}$$

for all $X,Y\in\Gamma(\ker F_*)$ and $V\in\Gamma(\ker F_*)^{\perp}$.

Again the using equations $(5),(8),(10),(16)$ and (17), we get

$$\begin{aligned}\widehat{\nabla}_X\xi &= \psi X, \mathscr{T}_X\xi=-\omega X,\\ \mathscr{V}\nabla_X\xi &= -BX, \mathscr{A}_X\xi=-CX,\end{aligned} \tag{20}$$

for all $X\in\Gamma(\ker F_*)$ and $V\in\Gamma(\ker F_*)^{\perp}$.

For $X,Y\in\Gamma(\ker F_*)$, define

$$(\nabla_X\psi)Y=\widehat{\nabla}_X\psi Y-\psi\widehat{\nabla}_X Y, \tag{21}$$

and

$$(\nabla_X\omega)Y=\mathscr{H}\nabla_X\omega Y-\omega\widehat{\nabla}_X Y, \tag{22}$$

where ∇ is the Levi-Civita connection on M. We say that the ω is parallel if

$$(\nabla_X\omega)Y=0. \tag{23}$$

Lemma 3.1. Let (M,ϕ,ξ,η,g_M) be a Sasakian manifold and (N,g_N) be a Riemannian manifold. Let $F:(M,\phi,\xi,\eta,g_M)\to(N,g_N)$ be a pointwise slant submersion. Then,

$$(\nabla_X\psi)Y = B\mathscr{T}_XY - \mathscr{T}_X\omega Y + R(\xi,X)Y, \tag{24}$$

and

$$(\nabla_X\omega)Y = C\mathscr{T}_XY - \mathscr{T}_X\psi Y, \tag{25}$$

for any $X,Y\in\Gamma(\ker F_*)$.

Theorem 3.1. Let (M,ϕ,ξ,η,g_M) be a Sasakian manifold and (N,g_N) be a Riemannain manifold. Let $F:(M,\phi,\xi,\eta,g_M)\to(N,g_N)$ be a map. The map F is pointwise slant submersion if and only if

$$\psi^2 = \cos^2\theta(-I+\eta\otimes\xi).$$

Corollary 3.1. Let (M,ϕ,ξ,η,g_M) be a Sasakian manifold and (N,g_N) be a Riemannian manifold. Let $F:(M,\phi,\xi,\eta,g_M)\to(N,g_N)$ be a pointwise slant submersion. Then,

$$g_M(\psi X,\psi Y) = \cos^2\theta(g_M(X,Y)-\eta(X)\eta(Y)),$$

$$g_M(\omega X,\omega Y) = \sin^2\theta(g_M(X,Y)-\eta(X)\eta(Y)),$$

for any $X,Y\in\Gamma(\ker F_*)$.

Theorem 3.2. Let (M,ϕ,ξ,η,g_M) be a Sasakian manifold and (N,g_N) be a Riemannian manifold. Let $F:(M,\phi,\xi,\eta,g_M)\to(N,g_N)$ be a pointwise slant submersion with slant function θ. If ω is parallel, then

$$\mathscr{T}_{\psi X}\psi X = -\cos^2\theta\,\mathscr{T}_X(X-\eta(X)\psi X),$$

for any $X\in\Gamma(\ker F_*)$.

Let F is a C^∞ map between Riemannian manifolds (M,g_M) and (N,g_N), then the adjoint ${}^*F_*$ map of F_* is defined by

$$g_M(X,{}^*F_{*q}Y) = g_N(F_{*q}X,Y), \tag{26}$$

for any $X\in T_qM, Y\in T_xN$ and $q\in M$. For each $q\in M$, F_*^h is a map characterized by

$$F_*^h : ((\ker F_*)^\perp(q), g_{M(q)}(\ker F_*)^\perp(q)) \to (rangeF_*(x), g_{N_x}(rangeF_*)(x)),$$

denote the adjoint of F_*^h by ${}^*F_*^h$. Let ${}^*F_*$ be the adjoint of F_* that is defined by $F_{*q} : (T_qM, g_M) \to (T_xN, g_N)$.

Then the linear transformation $({}^*F_*)^h : rangeF_*(x) \to (\ker F_*)^\perp(q)$, defined as $({}^*F_*)^h Y ={}^* F_*Y$, where $Y \in \Gamma(rangeF_*), F(q) = x$, is an isomorphism and $(F_{*q}^h)^{-1} = ({}^*F_{*q})^h ={}^* (F_{*q}^h)$.

Theorem 3.3. Let $(M, \phi, \xi, \eta, g_M)$ be a Sasakian manifold and (N, g_N) be a Riemannain manifold. Let $F : (M, \phi, \xi, \eta, g_M) \to (N, g_N)$ be a pointwise slant submersion with non-zero slant function θ. Then the fibers are totally geodesic submanifolds in M if and only if

$$\begin{aligned}
& g_N(\nabla^N_{V'} F_*(\omega X), F_*(\omega Y)) \\
= \; & -\sin^2\theta g_M([X,V],Y) + \sin 2\theta V(\theta) g_M(\phi X, \phi Y) + g_M(\mathscr{A}_V \omega\psi X, Y) \\
& -g_M(\mathscr{A}_V \omega X, \psi Y) + \sin^2\theta \eta(X) g_M(Y, BV) - \cos^2\theta \eta(Y) g_M(X, BV) \\
& -\sin^2\theta \eta(\nabla_V X)\eta(Y),
\end{aligned}$$

for any $X, Y \in \Gamma(\ker F_*)$ and $V \in \Gamma(\ker F_*)^\perp$, where V and V' are $F-$related vector fields and ∇^N is the Riemannian connection on N.

Proof. For $X, Y \in \Gamma(\ker F_*)$ and $V \in \Gamma(\ker F_*)^\perp$, using equations $(3), (4), (8),$ $(11), (16), (17)$ and (14), theorem (3.1), we get

$$\begin{aligned}
& \sin^2\theta g_M(\mathscr{T}_X Y, V) \\
= \; & -\sin^2\theta g_M([X,V],Y) + \sin 2\theta V(\theta) g_M(\phi X, \phi Y) + g_M(\mathscr{A}_V \omega\psi X, Y) \\
& -g_N(\nabla^N_{V/} F_*(\omega X), F_*(\omega Y)) - g_M(\mathscr{A}_V \omega X, \psi Y) + \sin^2\theta \eta(X) g_M(Y, BV) \\
& -\cos^2\theta \eta(X) g_M(Y, BV) - \sin^2\theta \eta(\nabla_V X)\eta(Y)).
\end{aligned}$$

By considering the fibers as totally geodesic, we derive the formula given the hypothesis. Conversely, it can be directly verified. □

Theorem 3.4. Let $(M, \phi, \xi, \eta, g_M)$ be a Sasakian manifold and (N, g_N) be a Riemannain manifold. Let $F : (M, \phi, \xi, \eta, g_M) \to (N, g_N)$ be a pointwise slant submersion with non-zero slant function θ. Then totally geodesic map if and only if

$$\begin{aligned} & g_N(\nabla^N_{V'} F_*(\omega X), F_*(\omega Y)) \\ = \;& -\sin^2\theta g_M([X,V],Y) + \sin 2\theta V(\theta) g_M(\phi X, \phi Y) + g_M(\mathscr{A}_V \omega\psi X, Y) \\ & -g_M(\mathscr{A}_V \omega X, \psi Y) + \sin^2\theta \eta(X) g_M(Y, BV) - \cos^2\theta \eta(Y) g_M(X, BV) \\ & -\sin^2\theta \eta(\nabla_V X)\eta(Y), \end{aligned}$$

and

$$\begin{aligned} g_M(\mathscr{A}_V \omega V, BW) &= g_N(\nabla^F_V F_*(\omega\psi X), F_*(W)) - g_N(\nabla^F_V F_*(\omega X), F_*(CW)) \\ & \quad -\sin^2\theta \eta(X) g_M(V, CW), \end{aligned}$$

for any $X,Y \in \Gamma(\ker F_*)$ and $V,W \in \Gamma(\ker F_*)^\perp$, where V and V' are $F-$related vector fields and ∇^F is the pullback connection along F.

Proof. By definition, it follows that F is totally geodesic if and only if $(\nabla F_*)(X,Y) = 0$, for any $X,Y \in \Gamma(TM)$.

From theorem (3.3), we obtain the first equation. On the other hand, for any $X,Y \in \Gamma(\ker F_*)$ and $V,W \in \Gamma(\ker F_*)^\perp$, using equations $(3),(4),(10),(11),(16),(17)$ and (14) theorem (3.1), we obtain

$$\begin{aligned} \sin^2\theta g_N((\nabla F_*)(V,X), F_*(W)) &= g_N(\nabla^F_V F_*\omega X, F_* W) - g_N(\nabla^F_V F_*\omega X, F_* CW) \\ & \quad -g_M(\mathscr{A}_V \omega X, BW) - \sin^2\theta \eta(X) g_M(V, CW). \end{aligned}$$

Conversely it is easily proved. □

3.2. Pointwise slant submersions for $\xi \in \Gamma((\ker F_*)^\perp)$

In this section, we give the basic equations of pointwise slant submersion from a Sasakian manifolds onto a Riemannian manifolds for $\xi \in \Gamma(\ker F_*)^\perp$.

From equations (1) and (2), we have

$$\phi^2 X = -X, \tag{27}$$

and

$$g(\phi X, \phi Y) = g(X, Y), \tag{28}$$

for any $X, Y \in \Gamma(\ker F_*)$. Moreover, from equations $(9), (11), (5), (16)$ and (17), we get

$$\mathscr{T}_X \xi = -\psi X, \tag{29}$$

$$\mathscr{A}_V \xi = -BV, \tag{30}$$

and

$$\eta(\nabla_X Y) = 0, \tag{31}$$

for any $X, Y \in \Gamma(\ker F_*)$ and $V \in \Gamma(\ker F_*)^\perp$.

Theorem 3.5. Let $(M, \phi, \xi, \eta, g_M)$ be a Sasakian manifold and (N, g_N) be a Riemannain manifold. Let $F : (M, \phi, \xi, \eta, g_M) \to (N, g_N)$ be a map. The map F is pointwise slant submersion if and only if

$$\psi^2 = -(\cos^2\theta) I.$$

Corollary 3.2. Let $(M, \phi, \xi, \eta, g_M)$ be a Sasakian manifold and (N, g_N) be a Riemannain manifold. Let $F : (M, \phi, \xi, \eta, g_M) \to (N, g_N)$ be a pointwise slant submersion. Then,

$$g_M(\psi X, \psi Y) = \cos^2\theta\, g_M(X, Y),$$

$$g_M(\omega X, \omega Y) = \sin^2\theta\, g_M(X, Y),$$

for any $X, Y \in \Gamma(\ker F_*)$.

Theorem 3.6. Let $(M, \phi, \xi, \eta, g_M)$ be a Sasakian manifold and (N, g_N) be a Riemannain manifold. Let $F : (M, \phi, \xi, \eta, g_M) \to (N, g_N)$ be a pointwise slant submersion with slant function θ. If ω is parellel, then

$$\mathscr{T}_{\psi X} \psi X = -\cos^2\theta\, \mathscr{T}_X X,$$

for any $X \in \Gamma(\ker F_*)$.

Theorem 3.7. Let (M,ϕ,ξ,η,g_M) be a Sasakian manifold and (N,g_N) be a Riemannain manifold. Let $F:(M,\phi,\xi,\eta,g_M)\to(N,g_N)$ be a pointwise slant submersion with non-zero slant function θ. Then the fibers are totally geodesic submanifolds in M if and only if

$$\begin{aligned} g_N(\nabla^N_{V'}F_*(\omega X),F_*(\omega Y)) &= -\sin^2\theta g_M([X,V],Y)+\sin 2\theta V(\theta)g_M(X,Y) \\ &\quad +g_M(\mathscr{A}_V\omega\psi X,Y)-g_M(\mathscr{A}_V\omega X,\psi Y), \end{aligned}$$

for any $X,Y\in\Gamma(\ker F_*)$ and $V\in\Gamma(\ker F_*)^{\perp}$, where V and V' are $F-$related vector fields and ∇^N is the Riemannian connection on N.

Proof. For any $X,Y\in\Gamma(\ker F_*)$ and $V\in\Gamma(\ker F_*)^{\perp}$, using equations (2), (3), (4), (8), (11), (16), (17), and (14), theorem (3.6), we get

$$\begin{aligned} &\sin^2\theta g_M(\mathscr{T}_X Y,V) \\ = &-\sin^2\theta g_M([X,V],Y)+\sin 2\theta V(\theta)g_M(X,Y)+g_M(\mathscr{A}_V\omega\psi X,Y) \\ &-g_N(\nabla^N_{V'}F_*(\omega X),F_*(\omega Y))-g_M(\mathscr{A}_V\omega X,\psi Y). \end{aligned}$$

By considering the fibers as totally geodesic, we derive the formula given the hypothesis. Conversely, it can be directly verified. □

Theorem 3.8. Let (M,ϕ,ξ,η,g_M) be a Sasakian manifold and (N,g_N) be a Riemannain manifold. Let $F:(M,\phi,\xi,\eta,g_M)\to(N,g_N)$ be a pointwise slant submersion with non-zero slant function θ. Then F is totally geodesic map if and only if

$$\begin{aligned} g_N(\nabla^N_{V'}F_*(\omega X),F_*(\omega Y)) &= -\sin^2\theta g_M([X,V],Y)+\sin 2\theta V(\theta)g_M(X,Y) \\ &\quad +g_M(\mathscr{A}_V\omega\psi X,Y)-g_M(\mathscr{A}_V\omega X,\psi Y), \end{aligned}$$

and

$$g_M(\mathscr{A}_V\omega X,BW)=g_N(\nabla^F_V F_*(\omega\psi X),F_*(W))-g_N(\nabla^F_V F_*(\omega X),F_*(CW)),$$

for any $X,Y\in\Gamma(\ker F_*)$ and $V,W\in\Gamma(\ker F_*)^{\perp}$, where V and V' are $F-$related vector fields and ∇^F is the pullback connection along F.

Proof. By definition, it follows that F is totally geodesic if and only if $(\nabla F_*)(X,Y)=0$, for any $X,Y\in\Gamma(TM)$.

From theorem 3.3, we obtain the first equation. On the other hand, for any $X,Y \in \Gamma(\ker F_*)$ and $V,W \in \Gamma(\ker F_*)^{\perp}$, using equations $(2),(3),(4),(10),(11),(16),(17)$ and (14), theorem (3.6), we obtain

$$\sin^2\theta g_N((\nabla F_*)(V,X),F_*(W)) = g_N(\nabla^F_V F_*\omega X, F_*W) - g_N(\nabla^F_V F_*\omega X, F_*CW) - g_M(\mathscr{A}_V\omega X, BW).$$

Conversely it is easily proved. □

Conflict of Interests

The authors declare that there is no conflict of interests.

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