Available online at http://scik.org
J. Math. Comput. Sci. 8 (2018), No. 4, 467-483
https://doi.org/10.28919/jmcs/3712
ISSN: 1927-5307

SENSITIVITY ANALYSIS FOR A PARAMETRIC GENERALIZED MULTI-VALUED IMPLICIT QUASI-VARIATIONAL-LIKE INCLUSION

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Abstract. In this paper, using proximal-point mapping technique and the property of a fixed-point set of multi-valued contractive mapping, we study the behavior and sensitivity analysis of a solution set for a parametric generalized multi-valued implicit quasi-variational-like inclusion in real Hilbert space. Further, under some suitable conditions, we discuss the Lipschitz continuity of the solution set with respect to the parameter. Our results can be viewed as a refinement and improvement of some known results in the literature.

Keywords: parametric generalized multi-valued implicit quasi-variational-like inclusion; sensitivity analysis; proximal-point mapping technique; Hausdorff metric; Hilbert space.

2010 AMS Subject Classification: 47J25, 49J40, 49J53, 47H05.

1. Introduction

Variational inequality theory has become very effective and powerful tool for studying a wide range of problems arising in mechanics, optimization, operation research, equilibrium problems

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Received March 28, 2018

and boundary value problems etc. Variational inequalities have been generalized and extended in different directions using novel and innovative techniques. A useful and important generalization of variational inequality is called the variational inclusion. Hassouni and Moudafi [9], Agarwal *et al.* [2,3], Ding [5,6], Ding and Luo [7], Fang and Huang [8], Huang [10] and Noor [17,18] have used the resolvent operator technique to obtain some important extensions and generalizations in existence results for the various classes of variational inequalities (inclusions).

In recent years, much attention has been given to develop general techniques for the sensitivity analysis of solution set of the various classes of variational inequalities (inclusions). From the mathematical and engineering point of view, sensitivity properties of various classes of variational inequalities can provide new insight concerning the problem being studied and stimulate ideas for solving problems. The sensitivity analysis of solution set for variational inequalities have been studied extensively by many authors using quite different techniques. By using the projection technique, Dafermos [4], Ding and Luo [7], Mukherjee and Verma [15] and Yen [21] studied the sensitivity analysis of solution set for some classes of variational inequalities with single-valued mappings. By using the implicit function approach that makes use of so-called normal mappings, Robinson [20] studied the sensitivity analysis of solutions for variational inequalities in finite-dimensional spaces. By using the projection and resolvent operator techniques, Adly [1], Agarwal *et al.* [2,3], Ding [5,6], Lim [13], Liu *et al.* [14], Noor [17,18], Peng and Long [19] and Zeng *et al.* [22] studied the behavior and sensitivity analysis of solution set for the various classes of parametric generalized variational inclusions involving single and multi-valued mappings.

The technique based on proximal-point mapping is a generalization of projection technique and has been widely used to study the existence of solutions and to develop iterative schemes for the various classes of variational (-like) inclusions. Recently Fang and Huang [8], Huang [10], Kazmi and Alvi [11] and Kazmi and Khan [12] has introduced the notion of η-proximal point mapping, P-proximal point mapping and P-η-proximal point mapping and used these to study the behavior and sensitivity analysis of solution set for some classes of parametric generalized variational (-like) inclusions involving single and multi-valued mappings.

Motivated by recent work going in this direction, we define strongly P-η-proximal mapping for strongly maximal P-η-monotone mapping and discuss some of its properties. Further, we consider a parametric generalized multi-valued implicit quasi-variational-like inclusion problem (in short, PGMIQVLIP) in real Hilbert space. Further, using proximal-point mapping technique and the property of a fixed-point set of multi-valued contractive mapping, we study the behavior and sensitivity analysis of a solution set for the PGMIQVLIP. Furthermore, under some suitable conditions, we discuss the Lipschitz continuity of the solution set with respect to the parameter. The results presented in this paper generalize and improve the results given by many authors, see for example [5,6,8,10-14,18,19,22].

2. Preliminaries

Let H be a real Hilbert space equipped with inner product $\langle\cdot,\cdot\rangle$ and norm $\|\cdot\|$; 2^H is the power set of H; $CB(H)$ is the family of all nonempty closed and bounded subsets of H; $C(H)$ is the family of all nonempty compact subsets of H; $\mathscr{H}(\cdot,\cdot)$ is the Hausdorff metric on $C(H)$ defined by

$$\mathscr{H}(A,B) \;=\; \max\left\{\sup_{x\in A}\inf_{y\in B} d(x,y),\; \sup_{y\in B}\inf_{x\in A} d(x,y)\right\},\; A,B\in C(H).$$

First, we review and define the following known concepts:

Definition 2.1[11]. Let $\eta : H\times H \to H$ be a mapping. Then a mapping $P : H \to H$ is said to be:

(i) η-monotone if

$$\langle P(x)-P(y),\,\eta(x,y)\rangle \;\geq\; 0,\; \forall x,y\in H;$$

(ii) strictly η-monotone if

$$\langle P(x)-P(y),\,\eta(x,y)\rangle \;>\; 0,\; \forall x,y\in H,$$

and equality holds if and only if $x=y$;

(iii) δ-strongly η-monotone if there exists a constant $\delta>0$ such that

$$\langle P(x)-P(y),\,\eta(x,y)\rangle \;\geq\; \delta\|x-y\|^2,\; \forall x,y\in H.$$

Definition 2.2[11,12]. A mapping $\eta : H \times H \to H$ is said to be τ-Lipschitz continuous if there exists a constant $\tau > 0$ such that

$$\|\eta(x,y)\| \ \leq \ \tau\|x-y\|, \ \forall x,y \in H.$$

Definition 2.3[11,12]. Let $\eta : H \times H \to H$ be a single-valued mapping. Then a multi-valued mapping $M : H \to 2^H$ is said to be:

(i) η-monotone if

$$\langle u-v, \eta(x,y)\rangle \ \geq \ 0, \ \forall x,y \in H, \ \forall u \in M(x), \ \forall v \in M(y);$$

(ii) strictly η-monotone if

$$\langle u-v, \eta(x,y)\rangle \ > \ 0, \ \forall x,y \in H, \ \forall u \in M(x), \ \forall v \in M(y),$$

and equality holds if and only if $x = y$;

(iii) γ-strongly η-monotone if there exists a constant $\gamma > 0$ such that

$$\langle u-v, \eta(x,y)\rangle \ \geq \ \gamma\|x-y\|^2, \ \forall x,y \in H, \ \forall u \in M(x), \ \forall v \in M(y);$$

(iv) maximal η-monotone if M is η-monotone and $(I+\rho M)(H) = H$ for any $\rho > 0$, where I stands for identity mapping.

Remark 2.1. If $\eta(x,y) = x-y, \ \forall x,y \in H$, then from Definitions 2.1 and 2.3, we recover the usual definitions of monotonicity of mappings P and M.

Definition 2.4[10-12]. Let $\eta : H \times H \to H$ and $P : H \to H$ be mappings. A multi-valued mapping $M : H \to 2^H$ is said to be maximal P-η-monotone if M is η-monotone and $(P+\rho M)(H) = H$ for any $\rho > 0$.

Definition 2.5[10-12]. Let $\eta : H \times H \to H$ and $P : H \to H$ be mappings. A multi-valued mapping $M : H \to 2^H$ is said to be γ-strongly maximal P-η-monotone if M is γ-strongly η-monotone and $(P+\rho M)H = H$ for any $\rho > 0$.

Remark 2.2. Under some suitable conditions on the mappings M, P, η, we recover the usual definitions of maximal monotonicity discussed by many authors given in [8,10-12,22].

The following theorems give some important properties of γ-strongly maximal P-η-monotone mappings.

Theorem 2.1[10-12]. Let $\eta : H \times H \to H$ be a mapping and $P : H \to H$ be a strictly η-monotone mapping. Let $M : H \to 2^H$ be a γ-strongly maximal P-η-monotone multi-valued mapping. Then

(a) $\langle u-v,\, \eta(x,y)\rangle \geq 0,\ \forall (v,y) \in \text{Graph}(M)$ implies $(u,x) \in \text{Graph}(M)$, where $\text{Graph}(M) := \{(u,x) \in H \times H : u \in M(x)\}$;

(b) the mapping $(P+\rho M)^{-1}$ is single-valued for all $\rho > 0$.

By Theorem 2.1, we define strongly P-η-proximal mapping for a γ-strongly maximal η-monotone mapping M as follows:

$$R^M_{P,\eta}(z) \;=\; (P+\rho M)^{-1},\ \forall z \in H, \tag{2.1}$$

where $\rho > 0$ is a constant, $\eta : H \times H \to H$ is a mapping and $P : H \to H$ is a strictly η-monotone mapping.

Theorem 2.2[10-12]. Let $\eta : H \times H \to H$ be a τ-Lipschitz continuous mapping and $P : H \to H$ be a δ-strongly η-accretive mapping. Let $M : H \to 2^H$ be a γ-strongly maximal η-monotone multi-valued mapping. Then strongly P-η-proximal mapping $R^M_{P,\eta}$ is $\dfrac{\tau}{\delta+\rho\gamma}$-Lipschitz continuous, that is,

$$\|R^M_{P,\eta}(x) - R^M_{P,\eta}(y)\| \;\leq\; \frac{\tau}{\delta+\rho\gamma}\,\|x-y\|,\ \forall x,y \in H.$$

Lemma 2.1[16]. Let (X,d) be a complete metric space. Suppose that $T : X \to C(X)$ satisfies

$$\mathscr{H}(T(x),T(y)) \;\leq\; \nu\, d(x,y),\ \forall x,y \in X,$$

where $\nu \in (0,1)$ is a constant. Then the mapping T has fixed point in X.

Lemma 2.2[13]. Let (X,d) be a complete metric space and let $T_1,T_2 : X \to C(X)$ be θ-$\mathscr{H}$-contraction mappings, then

$$\mathscr{H}(F(T_1),F(T_2)) \;\leq\; (1-\theta)^{-1}\sup_{x\in X}\mathscr{H}(T_1(x),T_2(x)),$$

where $F(T_1)$ and $F(T_2)$ are the sets of fixed points of T_1 and T_2, respectively.

3. Formulation of problem

Let Λ and Ω be open subsets of a real Hilbert space H such that (Λ, d_1) and (Ω, d_2) are metric spaces, in which the parameters λ and μ takes values, respectively.

Let $g, m : H \times \Lambda \to H$; $P : H \to H$; $\eta : H \times H \to H$; $N : H \times H \times H \times \Omega \to H$ be single-valued mappings such that $g \neq 0$, and let $A, B, C : H \times \Omega \to C(H)$ and $F : H \times \Lambda \to C(H)$ be multi-valued mappings. Suppose that $M : H \times H \times \Lambda \to 2^H$ is a multi-valued mapping such that for each $(t, \lambda) \in H \times \Lambda$, $M(\cdot, t, \lambda) : H \to 2^H$ is strongly maximal P-η-monotone and range$(g - m)(H \times \{\lambda\}) \cap \text{domain} M(\cdot, t, \lambda) \neq \emptyset$, where $(g - m)(x, \lambda) = g(x, \lambda) - m(x, \lambda)$ for any $(x, \lambda) \in H \times \Lambda$. For each $(f, \lambda, \mu) \in H \times \Lambda \times \Omega$, we consider the following parametric generalized multi-valued implicit quasi-variational-like inclusion problem (PGMIQVLIP):

Find $x = x(\lambda, \mu) \in H$, $u = u(x, \mu) \in A(x, \mu)$, $v = v(x, \mu) \in B(x, \mu)$, $w = w(x, \mu) \in C(x, \mu)$ and $z = z(x, \lambda) \in F(x, \lambda)$ such that $(g - m)(x, \lambda) \in \text{domain} M(\cdot, z, \lambda)$ and

$$f \in N(u, v, w, \mu) + M((g - m)(x, \lambda), z, \lambda). \tag{3.1}$$

Some special cases:

(1) If $(\Lambda, d_1) = (\Omega, d_2)$; $N(u, v, w, \mu) = N(u, v, \lambda)$; $P = I$, an identity mapping; $\eta(x, y) = x - y$ for all $x, y \in H$; $f = 0$, and $M(\cdot, z, \lambda)$ is maximal monotone for each fixed $(z, \lambda) \in H \times \Lambda$, then the PGMIQVLIP (3.1) reduces to the following problem: Find $x = x(\lambda) \in H$, $u = u(x, \lambda) \in A(x, \lambda)$, $v = v(x, \lambda) \in B(x, \lambda)$, $z = z(x, \lambda) \in F(x, \lambda)$ such that $(g - m)(x, \lambda) \in \text{domain} M(\cdot, z, \lambda)$ and

$$0 \in N(u, v, \lambda) + M((g - m)(x, \lambda), z, \lambda). \tag{3.2}$$

Problem (3.2) was introduced and studied by Ding [5].

(2) If $m(x, \lambda) = 0$ and $A(x, \lambda) = B(x, \lambda) = F(x, \lambda) = x$ for all $(x, \lambda) \in H \times \Lambda$, then problem (3.2) reduces to the following problem: Find $x(\lambda) \in H$ such that $g(x, \lambda) \in \text{domain} M(\cdot, z, \lambda)$ and

$$0 \in N(x, x, \lambda) + M(g(x, \lambda), x, \lambda). \tag{3.3}$$

Problem (3.3) was introduced and studied by Noor [18].

(3) If $N(x, y, \lambda) = N(x, \lambda)$ and $M(x, y, \lambda) = M(x, \lambda)$ for all $(x, y, \lambda) \in H \times H \times \Lambda$, then problem (3.3) reduces to the following problem: Find $x(\lambda) \in H$ such that $g(x, \lambda) \in$

domain $M(\cdot,\lambda)$ and

$$0 \in N(x,\lambda) + M(g(x,\lambda),\lambda). \tag{3.4}$$

Problem (3.4) was introduced and studied by Adly [1].

In brief, for a suitable choice of the mappings A,B,C,F,N,M,P,g,m,η, the element f, the space H, it is easy to see that the PGMIQVLIP (3.1) includes a number of known classes of quasi-variational-like inclusions studied by many authors as special cases, see for example [1-7,11-14,17-21] and the references therein.

Now, for each fixed $(\lambda,\mu) \in \Lambda \times \Omega$, the solution set $S(\lambda,\mu)$ of the PGMIQVLIP (3.1) is denoted as

$$S(\lambda,\mu) := \Big\{ x = x(\lambda,\mu) \in H : u = u(x,\mu) \in A(x,\mu), v = v(x,\mu) \in B(x,\mu), w = w(x,\mu) \in$$

$$C(x,\mu), z = z(x,\lambda) \in F(x,\lambda) \text{ such that } f \in N(u,v,w,\mu) + M((g-m)(x,\lambda),z,\lambda) \Big\}. \tag{3.5}$$

In this paper, our main aim is to study the behavior and sensitivity analysis of the solution set $S(\lambda,\mu)$, and the conditions on these mappings A,B,C,F,N,M,P,g,m,η under which the solution set $S(\lambda,\mu)$ is nonempty and Lipschitz continuous with respect to the parameters $\lambda \in \Lambda$, $\mu \in \Omega$.

4. Sensitivity analysis of the solution set $S(\lambda,\mu)$

First, we define the following concepts.

Definition 4.1[11,12]. A mapping $g : H \times \Lambda \to H$ is said to be:

(i) (L_g, l_g)-mixed Lipschitz continuous if there exist constants $L_g, l_g > 0$ such that

$$\|g(x_1,\lambda_1) - g(x_2,\lambda_2)\| \le L_g\|x_1 - x_2\| + l_g\|\lambda_1 - \lambda_2\|, \ \forall (x_1,\lambda_1), (x_2,\lambda_2) \in H \times \Lambda;$$

(ii) s-strongly monotone if there exists a constant $s > 0$ such that

$$\langle g(x_1,\lambda) - g(x_2,\lambda), x_1 - x_2\rangle \ \ge \ s\|x_1 - x_2\|^2, \ \forall (x_1,\lambda), (x_2,\lambda) \in H \times \Lambda.$$

Definition 4.2[11,12]. A multi-valued mapping $A : H \times \Omega \to C(H)$ is said to be (L_A, l_A)-$\mathscr{H}$-mixed Lipschitz continuous if there exist constants $L_A, l_A > 0$ such that

$$\mathscr{H}(A(x_1,\mu_1),A(x_2,\mu_2)) \leq L_A\|x_1-x_2\|+l_A\|\mu_1-\mu_2\|,\ \forall(x_1,\mu_1),(x_2,\mu_2)\in H\times\Omega.$$

Definition 4.3[11,12]. Let $A,B,C : H \times \Omega \to C(H)$ be multi-valued mappings. A single-valued mapping $N : H \times H \times H \times \Omega \to H$ is said to be:

(i) α-strongly mixed monotone with respect to A, B and C if there exists a constant $\alpha > 0$ such that

$$\langle N(u_1,v_1,w_1,\mu)-N(u_2,v_2,w_2,\mu),x-y\rangle \geq \alpha\|x-y\|^2,\ \forall(x,y,\mu)\in H\times H\times\Omega,$$

$$u_1\in A(x,\mu),\ u_2\in A(y,\mu),\ v_1\in B(x,\mu),\ v_2\in B(y,\mu),\ w_1\in C(x,\mu),\ w_2\in C(y,\mu);$$

(ii) $(L_{(N,1)},L_{(N,2)},L_{(N,3)},l_N)$-mixed Lipschitz continuous if there exist constants $L_{(N,1)}, L_{(N,2)}, L_{(N,3)}$, $l_N > 0$ such that

$$\begin{aligned}\|N(x_1,y_1,z_1,\mu_1)-N(x_2,y_2,z_2,\mu_2)\| \leq\ & L_{(N,1)}\|x_1-x_2\|+L_{(N,2)}\|y_1-y_2\|\\ &+L_{(N,3)}\|z_1-z_2\|+l_N\|\mu_1-\mu_2\|,\end{aligned}$$

$$\forall(x_1,y_1,z_1,\mu_1),\ (x_2,y_2,z_2,\mu_2)\in H\times H\times H\times\Omega.$$

Now, we transfer the PGMIQVLIP (3.1) into a parametric fixed-point problem.

Lemma 4.1. For each $(f,\lambda,\mu)\in H\times\Lambda\times\Omega$, (x,u,v,w,z) with $x=x(\lambda,\mu)\in H$, $u=u(x,\mu)\in A(x,\mu)$, $v=v(x,\mu)\in B(x,\mu)$, $w=w(x,\mu)\in C(x,\mu)$ and $z=z(x,\lambda)\in F(x,\lambda)$ such that $(g-m)(x,\lambda)\in \text{domain}\, M(\cdot,z,\lambda)$ is a solution of the PGMIQVLIP (3.1) if and only if the multi-valued mapping $G : H\times\Lambda\times\Omega\to 2^H$ defined by

$$\begin{aligned}G(t,\lambda,\mu)=&\bigcup_{u\in A(t,\mu),v\in B(t,\mu),w\in C(t,\mu),z\in F(t,\lambda)}\Big[t-(g-m)(t,\lambda)\\ &+R_{P,\eta}^{M(\cdot,z,\lambda)}(P\circ(g-m)(t,\lambda)-\rho N(u,v,w,\mu)+\rho f)\Big],\ t\in H,\end{aligned}\tag{4.1}$$

has a fixed point, where $P : H\to H$; $P\circ(g-m)$ denotes P composition of $(g-m)$; $R_{P,\eta}^{M(\cdot,z,\lambda)} = (P+\rho M(\cdot,z,\lambda))^{-1}$ and $\rho>0$ is a constant.

Proof. For each $(f,\lambda,\mu)\in H\times\Lambda\times\Omega$, PGMIQVLIP (3.1) has a solution (x,u,v,w,z) if and only if

$$f\in N(u,v,w,\mu)+M((g-m)(x,\lambda),z,\lambda)$$

$$\Leftrightarrow\ P\circ(g-m)(x,\lambda)-\rho N(u,v,w,\mu)+\rho f\in(P+\rho M(\cdot,z,\lambda))((g-m)(x,\lambda)).$$

Since for each $(z,\lambda)\in H\times\Lambda$, $M(\cdot,z,\lambda)$ is maximal strongly P-η-monotone, by definition of strongly P-η-proximal mapping $R_{P,\eta}^{M(\cdot,z,\lambda)}$ of $M(\cdot,z,\lambda)$, preceding inclusion holds if and only if

$$(g-m)(x,\lambda)\ =\ R_{P,\eta}^{M(\cdot,z,\lambda)}[P\circ(g-m)(x,\lambda)-\rho N(u,v,w,\mu)+\rho f],$$

that is $x\in G(x,\lambda,\mu)$. This completes the proof.

Theorem 4.1. Let the multi-valued mappings $A,B,C:H\times\Omega\to C(H)$ and $F:H\times\Lambda\to C(H)$ be $\mathscr{H}$-Lipschitz continuous in the first arguments with constant L_A,L_B,L_C and L_F, respectively. Let the mappings $\eta:H\times H\to H$ be τ-Lipschitz continuous and $P:H\to H$ be δ-strongly η-monotone. Let the mappings $g,m:H\times\Lambda\to H$ be such that $(g-m)$ is s-strongly monotone and $L_{(g-m)}$-Lipschitz continuous in the first argument and $P\circ(g-m)$ be r-strongly monotone and $L_{P\circ(g-m)}$-Lipschitz continuous in the first argument. Let the mapping $N:H\times H\times H\times\Omega\to H$ be α-strongly mixed monotone with respect to A, B and C and $(L_{(N,1)},L_{(N,2)},L_{(N,3)},l_N)$-mixed Lipschitz continuous. Suppose that the multi-valued mapping $M:H\times H\times\Lambda\to 2^H$ is such that for each $(z,\lambda)\in H\times\Lambda$, $M(\cdot,z,\lambda):H\to 2^H$ is γ-strongly maximal P-η-monotone with $\mathrm{range}\,(g-m)(H\times\{\lambda\})\cap\mathrm{domain}\,M(\cdot,z,\lambda)\neq\emptyset$. Suppose that there exist constants $k_1,k_2>0$ such that

$$\|R_{P,\eta}^{M(\cdot,x_1,\lambda_1)}(t)-R_{P,\eta}^{M(\cdot,x_2,\lambda_2)}(t)\|\ \le\ k_1\|x_1-x_2\|+k_2\|\lambda_1-\lambda_2\|,\ \forall x_1,x_2,t\in H;\ \lambda_1,\lambda_2\in\Lambda,\tag{4.2}$$

and suppose for $\rho>0$, the following condition holds:

$$\theta=q+\varepsilon(\rho)<1,\tag{4.3}$$

where $q:=k_1L_F+\sqrt{1-2s+L_{(g-m)}^2}$; $\varepsilon(\rho):=\frac{\tau}{\delta+\rho\gamma}\Big[p+\sqrt{1-2\rho\alpha+\rho^2L_N^2}\Big]$;

$$p:=\sqrt{1-2r+L_{p\circ(g-m)}^2}\ ;\ L_N:=(L_AL_{(N,1)}+L_BL_{(N,2)}+L_CL_{(N,3)}).$$

Then, for each fixed $f \in H$, the multi-valued mapping G defined by (4.1) is a compact-valued uniform θ-$\mathscr{H}$-contraction mapping with respect to $(\lambda,\mu) \in \Lambda \times \Omega$, where θ is given by (4.3). Moreover, for each $(\lambda,\mu) \in \Lambda \times \Omega$, the solution set $S(\lambda,\mu)$ of the PGMIQVLIP (3.1) is nonempty and closed.

Proof. Let (x,λ,μ) be an arbitrary element of $H \times \Lambda \times \Omega$. Since A,B,C,F are compact-valued, then for any sequences $\{u_n\} \subset A(x,\mu)$, $\{v_n\} \subset B(x,\mu)$, $\{w_n\} \subset C(x,\mu)$, $\{z_n\} \subset F(x,\lambda)$, there exist subsequences $\{u_{n_i}\} \subset \{u_n\}$, $\{v_{n_i}\} \subset \{v_n\}$, $\{w_{n_i}\} \subset \{w_n\}$, $\{z_{n_i}\} \subset \{z_n\}$ and elements $u \in A(x,\mu)$, $v \in B(x,\mu)$, $w \in C(x,\mu)$, $z \in F(x,\lambda)$ such that $u_{n_i} \to u$, $v_{n_i} \to v$, $w_{n_i} \to w$, $z_{n_i} \to z$ as $i \to \infty$. By using Theorem 2.2, (4.2) and the mixed Lipschitz continuity of N, we estimate

$$
\begin{aligned}
&\|R_{P,\eta}^{M(\cdot,z_{n_i},\lambda)}[P\circ(g-m)(x,\lambda)-\rho N(u_{n_i},v_{n_i},w_{n_i},\mu)+\rho f] \\
&\quad -R_{P,\eta}^{M(\cdot,z,\lambda)}[P\circ(g-m)(x,\lambda)-\rho N(u,v,w,\mu)+\rho f]\| \\
\le\ & \|R_{P,\eta}^{M(.,z_{n_i},\lambda)}[P\circ(g-m)(x,\lambda)-\rho N(u_{n_i},v_{n_i},w_{n_i},\mu)+\rho f] \\
&\quad -R_{P,\eta}^{M(\cdot,z,\lambda)}[P\circ(g-m)(x,\lambda)-\rho N(u_{n_i},v_{n_i},w_{n_i},\mu)+\rho f]\| \\
&\quad +\|R_{P,\eta}^{M(\cdot,z,\lambda)}[P\circ(g-m)(x,\lambda)-\rho N(u_{n_i},v_{n_i},w_{n_i},\mu)+\rho f] \\
&\quad -R_{P,\eta}^{M(\cdot,z,\lambda)}[P\circ(g-m)(x,\lambda)-\rho N(u,v,w,\mu)+\rho f]\| \\
\le\ & k_1\|z_{n_i}-z\|+\rho\,\frac{\tau}{\delta+\rho\gamma}\|N(u_{n_i},v_{n_i},w_{n_i},\mu)-N(u,v,w,\mu)\| \\
\le\ & k_1\|z_{n_i}-z\|+\rho\,\frac{\tau}{\delta+\rho\gamma}\Big[L_{(N,1)}\|u_{n_i}-u\|+L_{(N,2)}\|v_{n_i}-v\|+L_{(N,3)}\|w_{n_i}-w\|\Big] \\
\to\ & 0, \text{ as } i\to\infty.
\end{aligned}
\tag{4.4}
$$

Thus (4.1) and (4.4) yield that $G(x,\lambda,\mu) \in C(H)$.

Now, for each fixed $(\lambda,\mu) \in \Lambda \times \Omega$, we prove that $G(x,\lambda,\mu)$ is a uniform θ-$\mathscr{H}$-contraction mapping. Let (x_1,λ,μ), $(x_2,\lambda,\mu) \in H \times \Lambda \times \Omega$ and any $t_1 \in G(x_1,\lambda,\mu)$, there exist $u_1 = u_1(x_1,\mu) \in A(x_1,\mu)$, $v_1 = v_1(x_1,\mu) \in B(x_1,\mu)$, $w_1 = w_1(x_1,\mu) \in C(x_1,\mu)$ and $z_1 = z_1(x_1,\lambda) \in F(x_1,\lambda)$ such that

$$t_1 \ = \ x_1-(g-m)(x_1,\lambda)+R_{P,\eta}^{M(\cdot,z_1,\lambda)}[P\circ(g-m)(x_1,\lambda)-\rho N(u_1,v_1,w_1,\mu)+\rho f]. \tag{4.5}$$

It follows from the compactness of $A(x_2,\mu)$, $B(x_2,\mu)$, $C((x_2,\mu)$ and $F(x_2,\lambda)$ and $\mathscr{H}$-Lipschitz continuity of A,B,C,F that there exist $u_2 = u_2(x_2,\mu) \in A(x_2,\mu)$, $v_2 = v_2(x_2,\mu) \in B(x_2,\mu)$, $w_2 = w_2(x_2,\mu) \in C(x_2,\mu)$ and $z_2 = z_2(x_2,\lambda) \in F(x_2,\lambda)$ such that

$$\begin{aligned}
\|u_1 - u_2\| &\le \mathscr{H}(A(x_1,\mu), A(x_2,\mu)) \le L_A\|x_1 - x_2\|,\\
\|v_1 - v_2\| &\le \mathscr{H}(B(x_1,\mu), B(x_2,\mu)) \le L_B\|x_1 - x_2\|,\\
\|w_1 - w_2\| &\le \mathscr{H}(C(x_1,\mu), C(x_2,\mu)) \le L_C\|x_1 - x_2\|,\\
\|z_1 - z_2\| &\le \mathscr{H}(F(x_1,\lambda), F(x_2,\lambda)) \le L_F\|x_1 - x_2\|.
\end{aligned} \tag{4.6}$$

Let

$$t_2 = x_2 - (g-m)(x_2,\lambda) + R_{P,\eta}^{M(\cdot,z_2,\lambda)}[P\circ(g-m)(x_2,\lambda) - \rho N(u_2,v_2,w_2,\mu) + \rho f], \tag{4.7}$$

then we have $t_2 \in G(x_2,\lambda,\mu)$.

Next, using Theorem 2.2 and (4.1), we estimate

$$\begin{aligned}
\|t_1 - t_2\| \le\ & \|x_1 - x_2 - ((g-m)(x_1,\lambda) - (g-m)(x_2,\lambda))\|\\
& + \|R_{P,\eta}^{M(\cdot,z_1,\lambda)}[P\circ(g-m)(x_1,\lambda) - \rho N(u_1,v_1,w_1,\mu) + \rho f]\\
& - R_{P,\eta}^{M(\cdot,z_2,\lambda)}[P\circ(g-m)(x_1,\lambda) - \rho N(u_1,v_1,w_1,\mu) + \rho f]\|\\
& + \|R_{P,\eta}^{M(\cdot,z_2,\lambda)}[P\circ(g-m)(x_1,\lambda) - \rho N(u_1,v_1,w_1,\mu) + \rho f]\\
& - R_{P,\eta}^{M(\cdot,z_2,\lambda)}[P\circ(g-m)(x_2,\lambda) - \rho N(u_2,v_2,w_2,\mu) + \rho f]\|\\
\le\ & \|x_1 - x_2 - ((g-m)(x_1,\lambda) - (g-m)(x_2,\lambda))\| + k_1\|z_1 - z_2\|\\
& + \frac{\tau}{\delta + \rho\gamma}\Big[\|x_1 - x_2 - (P\circ(g-m)(x_1,\lambda) - P\circ(g-m)(x_2,\lambda))\|\\
& + \|x_1 - x_2 - \rho(N(u_1,v_1,w_1,\mu) - N(u_2,v_2,w_2,\mu))\|\Big].
\end{aligned} \tag{4.8}$$

Since $(g-m)$ is s-strongly monotone and $L_{(g-m)}$-Lipschitz continuous, we have

$$\begin{aligned}
& \|x_1 - x_2 - ((g-m)(x_1,\lambda) - (g-m)(x_2,\lambda))\|^2\\
\le\ & \|x_1 - x_2\|^2 - 2\langle (g-m)(x_1,\lambda) - (g-m)(x_2,\lambda), x_1 - x_2\rangle + \|(g-m)(x_1,\lambda) - (g-m)(x_2,\lambda)\|^2\\
\le\ & (1 - 2s + L_{(g-m)}^2)\|x_1 - x_2\|^2.
\end{aligned} \tag{4.9}$$

Similarly, since $P\circ(g-m)$ is r-strongly monotone and $L_{P\circ(g-m)}$-Lipschitz continuous, we have

$$\|x_1-x_2-(P\circ(g-m)(x_1,\lambda)-P\circ(g-m)(x_2,\lambda))\|^2 \le (1-2r+L^2_{P\circ(g-m)})\|x_1-x_2\|^2. \quad (4.10)$$

Since N is $(L_{(N,1)},L_{(N,2)},L_{(N,3)},l_N)$-mixed Lipschitz continuous; A,B,C are $\mathscr{H}$-Lipschitz continuous, we have

$$\begin{aligned}
&\|N(u_1,v_1,w_1,\mu)-N(u_2,v_2,w_2,\mu)\| \le L_{(N,1)}\|u_1-u_2\|+L_{(N,2)}\|v_1-v_2\|+L_{(N,3)}\|w_1-w_2\| \\
&\le L_{(N,1)}\mathscr{H}(A(x_1,\mu),A(x_2,\mu))+L_{(N,2)}\mathscr{H}(B(x_1,\mu),B(x_2,\mu))+L_{(N,3)}\mathscr{H}(C(x_1,\mu),C(x_2,\mu)) \\
&\le (L_AL_{(N,1)}+L_BL_{(N,2)}+L_CL_{(N,3)})\|x_1-x_2\|. \qquad (4.11)
\end{aligned}$$

Further, since N is α-strongly mixed monotone with respect to A, B and C and using (4.11), we have

$$\begin{aligned}
&\|x_1-x_2-\rho(N(u_1,v_1,w_1,\mu)-N(u_2,v_2,w_2,\mu))\|^2 \\
&\le \|x_1-x_2\|^2-2\rho\langle N(u_1,v_1,w_1,\mu)-N(u_2,v_2,w_2,\mu),\, x_1-x_2\rangle \\
&\quad +\rho^2\|N(u_1,v_1,w_1,\mu)-N(u_2,v_2,w_2,\mu)\|^2 \\
&\le \|x_1-x_2\|^2-2\rho\alpha\|x_1-x_2\|^2+\rho^2(L_AL_{(N,1)}+L_BL_{(N,2)}+L_CL_{(N,3)})^2\|x_1-x_2\|^2 \\
&\le \Big(1-2\rho\alpha+\rho^2(L_AL_{(N,1)}+L_BL_{(N,2)}+L_CL_{(N,3)})^2\Big)\|x_1-x_2\|^2. \qquad (4.12)
\end{aligned}$$

Now, from (4.8)-(4.12), we have

$$\|t_1-t_2\| \le \theta\,\|x_1-x_2\|, \qquad (4.13)$$

where $\theta := q+\varepsilon(\rho)$; $q := k_1L_F+\sqrt{1-2s+L^2_{(g-m)}}$;

$\varepsilon(\rho) := \dfrac{\tau}{\delta+\rho\gamma}\Big[\sqrt{1-2r+L^2_{P\circ(g-m)}}+\sqrt{1-2\rho\alpha+\rho^2L_N^2}\Big]$;

$L_N := (L_AL_{(N,1)}+L_BL_{(N,2)}+L_CL_{(N,3)})$.

Hence, we have

$$d(t_1,G(x_2,\lambda,\mu)) = \inf_{t_2\in G(x_2,\lambda,\mu)}\|t_1-t_2\| \le \theta\|x_1-x_2\|.$$

Since $t_1\in G(x_1,\lambda,\mu)$ is arbitrary, we obtain

$$\sup_{t_1\in G(x_1,\lambda,\mu)} d(t_1,G(x_2,\lambda,\mu)) \le \theta\|x_1-x_2\|.$$

By using same argument, we can prove

$$\sup_{t_2 \in G(x_2,\lambda,\mu)} d(G(x_1,\lambda,\mu),t_2) \ \le\ \theta\|x_1 - x_2\|.$$

By the definition of the Hausdorff metric $\mathscr{H}$ on $C(H)$, we have

$$\mathscr{H}\Big(G(x_1,\lambda,\mu), G(x_2,\lambda,\mu)\Big) \ \le\ \theta\|x_1 - x_2\|, \tag{4.14}$$

that is, $G(x,\lambda,\mu)$ is a uniform θ-$\mathscr{H}$-contraction mapping with respect to $(\lambda,\mu) \in \Lambda \times \Omega$. Also, it follows from condition (4.3) that $\theta < 1$ and hence $G(x,\lambda,\mu)$ is a multi-valued contraction mapping which is uniform with respect to $(\lambda,\mu) \in \Lambda \times \Omega$. By Lemma 2.1 for each $(\lambda,\mu) \in \Lambda \times \Omega$, $G(x,\lambda,\mu)$ has a fixed point $x = x(\lambda,\mu) \in H$, that is, $x = x(\lambda,\mu) \in G(x,\lambda,\mu)$ and hence Lemma 4.1 ensure that $S(\lambda,\mu) \neq \emptyset$. Further, for any sequence $\{x_n\} \subset S(\lambda,\mu)$ with $\lim_{n\to\infty} x_n = x_0$, we have $x_n \in G(x_n,\lambda,\mu)$ for all $n \ge 1$. By virtue of (4.14), we have

$$\begin{aligned} d(x_0, G(x_0,\lambda,\mu)) \ &\le\ \|x_0 - x_n\| + \mathscr{H}\Big(G(x_n,\lambda,\mu), G(x_0,\lambda,\mu)\Big) \\ &\le\ (1+\theta)\|x_n - x_0\| \to 0 \text{ as } n \to \infty, \end{aligned}$$

that is, $x_0 \in G(x_0,\lambda,\mu)$ and hence $x_0 \in S(\lambda,\mu)$. Thus $S(\lambda,\mu)$ is closed in H.

Now, we prove that the solution set $S(\lambda,\mu)$ of the PGMIQVLIP (3.1) is $\mathscr{H}$-Lipschitz continuous for each $(\lambda,\mu) \in \Lambda \times \Omega$.

Theorem 4.2. Let the multi-valued mappings A,B,C and F be $\mathscr{H}$-mixed Lipschitz continuous with pairs of constants (L_A,l_A), (L_B,l_B), (L_C,l_C) and (L_F,l_F), respectively. Let the mappings η,P be the same as in Theorem 4.1. Let the mappings $(g-m)$ be s-strongly monotone and $(L_{(g-m)},l_{(g-m)})$-mixed Lipschitz continuous, and $P\circ(g-m)$ be r-strongly monotone and $(L_{P\circ(g-m)},l_{P\circ(g-m)})$-mixed Lipschitz continuous. Let the mapping N be α-strongly mixed monotone with respect to A, B and C, and $(L_{(N,1)},L_{(N,2)},l_{(N,3)},l_N)$-mixed Lipschitz continuous. Suppose that the multi-valued mapping M is same as in Theorem 4.1 and condition (4.3) holds, then for each $(\lambda,\mu) \in \Lambda \times \Omega$, the solution set $S(\lambda,\mu)$ of the PGMIQVLIP (3.1) is a $\mathscr{H}$-Lipschitz continuous mapping from $\Lambda \times \Omega$ into H.

Proof. For each (λ,μ), $(\bar{\lambda},\bar{\mu}) \in \Lambda \times \Omega$, it follows from Theorem 4.1, $S(\lambda,\mu)$ and $S(\bar{\lambda},\bar{\mu})$ are both nonempty and closed subsets of H. Also by Theorem 4.1, $G(x,\lambda,\mu)$ and $G(x,\bar{\lambda},\bar{\mu})$ both are multi-valued θ-$\mathscr{H}$-contraction mappings with same contractive constant $\theta \in (0,1)$. By Lemma 2.2, we obtain

$$\mathscr{H}(S(\lambda,\mu),S(\bar{\lambda},\bar{\mu})) \le \left(\frac{1}{1-\theta}\right)\sup_{x\in H}\mathscr{H}(G(x,\lambda,\mu),G(x,\bar{\lambda},\bar{\mu})), \tag{4.15}$$

where θ is given by (4.3).

Now, for any $a \in G(x,\lambda,\mu)$, there exist $u = u(x,\mu) \in A(x,\mu)$, $v = v(x,\mu) \in B(x,\mu)$, $w = w(x,\mu) \in C(x,\mu)$ and $z = z(x,\lambda) \in F(x,\lambda)$ satisfying

$$a = x-(g-m)(x,\lambda)+R_{P,\eta}^{M(\cdot,z,\lambda)}[P\circ(g-m)(x,\lambda)-\rho N(u,v,w,\mu)+\rho f]. \tag{4.16}$$

It is easy to see that there exist $\bar{u} = u(x,\bar{\mu}) \in A(x,\bar{\mu})$, $\bar{v} = v(x,\bar{\mu}) \in B(x,\bar{\mu})$, $\bar{w} = w(x,\bar{\mu}) \in C(x,\bar{\mu})$ and $\bar{z} = z(x,\bar{\lambda}) \in F(x,\bar{\lambda})$ such that

$$\begin{aligned}
\|u-\bar{u}\| &\le \mathscr{H}(A(x,\mu),A(x,\bar{\mu})) \le l_A\|\mu-\bar{\mu}\|,\\
\|v-\bar{v}\| &\le \mathscr{H}(B(x,\mu),B(x,\bar{\mu})) \le l_B\|\mu-\bar{\mu}\|,\\
\|w-\bar{w}\| &\le \mathscr{H}(C(x,\lambda),C(x,\bar{\mu})) \le l_C\|\mu-\bar{\mu}\|,\\
\|z-\bar{z}\| &\le \mathscr{H}(F(x,\lambda),F(x,\bar{\lambda})) \le l_F\|\lambda-\bar{\lambda}\|.
\end{aligned} \tag{4.17}$$

Let

$$b = x-(g-m)(x,\bar{\lambda})+R_{P,\eta}^{M(\cdot,\bar{z},\bar{\lambda})}[P\circ(g-m)(x,\bar{\lambda})-\rho N(\bar{u},\bar{v},\bar{w},\bar{\mu})+\rho f]. \tag{4.18}$$

Clearly, $b \in G(x,\bar{\lambda},\bar{\mu})$.

Since N is mixed Lipschitz continuous and in view of (4.2), (4.16)-(4.18) and with $t = P\circ(g-m)(x,\bar{\lambda})-\rho N(\bar{u},\bar{v},\bar{w},\bar{\mu})+\rho f$, we have

$$\begin{aligned}
\|a-b\| &\le \|(g-m)(x,\lambda)-(g-m)(x,\bar{\lambda})\|\\
&\quad+\|R_{P,\eta}^{M(\cdot,z,\lambda)}[P\circ(g-m)(x,\lambda)-\rho N(u,v,w,\mu)+\rho f]-R_{P,\eta}^{M(\cdot,z,\lambda)}(t)\|\\
&\quad+\|R_{P,\eta}^{M(\cdot,z,\lambda)}(t)-R_{P,\eta}^{M(\cdot,\bar{z},\lambda)}(t)\|+\|R_{P,\eta}^{M(\cdot,\bar{z},\lambda)}(t)-R_{P,\eta}^{M(\cdot,\bar{z},\bar{\lambda})}(t)\|\\
&\le \|(g-m)(x,\lambda)-(g-m)(x,\bar{\lambda})\|\\
&\quad+\frac{\tau}{\delta+\rho\gamma}\Big[\|P\circ(g-m)(x,\lambda)-P\circ(g-m)(x,\bar{\lambda})\|\\
&\quad+\rho\|N(u,v,w,\mu)-N(\bar{u},\bar{v},\bar{w},\bar{\mu})\|\Big]+k_1\|z-\bar{z}\|+k_2\|\lambda-\bar{\lambda}\|
\end{aligned}$$

$$\leq\ l_{(g-m)}\|\lambda-\bar{\lambda}\|+\frac{\tau}{\delta+\rho\gamma}\Big[l_{P\circ(g-m)}\|\lambda-\bar{\lambda}\|$$
$$+\rho(l_A L_{(N,1)}+l_B L_{(N,2)}+l_C L_{(N,3)}+l_N)\|\mu-\bar{\mu}\|\Big]+k_1 l_F\|\lambda-\bar{\lambda}\|+k_2\|\lambda-\bar{\lambda}\|$$
$$\leq\ \theta_1\Big(\|\lambda-\bar{\lambda}\|+\|\mu-\bar{\mu}\|\Big), \tag{4.19}$$

where

$$\theta_1:=\max\Big\{(l_{(g-m)}+k_1 l_F+k_2+\frac{\tau}{\delta+\rho\gamma}L_{P\circ(g-m)}),\ \frac{\tau}{\delta+\rho\gamma}(l_A L_{(N,1)}+l_B L_{(N,2)}+l_C L_{(N,3)}+l_N)\Big\}.$$

Hence, we obtain

$$\sup_{a\in G(x,\lambda,\bar{\mu})} d(a,G(x,\bar{\lambda},\mu))\ \leq\ \theta_1\|(\lambda,\mu)-(\bar{\lambda},\bar{\mu})\|_*,$$

where $\|(\lambda,\mu)\|_*=\|\lambda\|+\|\mu\|$.

By using similar argument, we have

$$\sup_{b\in G(x,\bar{\lambda},\bar{\mu})} d(G(x,\lambda,\mu),b)\ \leq\ \theta_1\|(\lambda,\mu)-(\bar{\lambda},\bar{\mu})\|_*.$$

Hence, for all $(x,\lambda,\mu),(x,\bar{\lambda},\bar{\mu})\in H\times\Lambda\times\Omega$, it follows that

$$\mathscr{H}(G(x,\lambda,\mu),G(x,\bar{\lambda},\bar{\mu}))\leq\theta_1\|(\lambda,\mu)-(\bar{\lambda},\bar{\mu})\|_*.$$

By Lemma 2.2, we obtain

$$\mathscr{H}(S(\lambda,\mu),S(\bar{\lambda},\bar{\mu}))\ \leq\ \Big(\frac{\theta_1}{1-\theta}\Big)\|(\lambda,\mu)-(\bar{\lambda},\bar{\mu})\|_*,$$

which implies that $S(\lambda,\mu)$ is $\mathscr{H}$-Lipschitz continuous in $(\lambda,\mu)\in\Lambda\times\Omega$. This completes the proof.

Remark 4.1. Since the PGMIQVLIP (3.1) includes many known classes of parametric generalized quasi-variational-like inclusions as special cases, Theorems 4.1-4.2 improve and generalize the known results given in [5,6,8,10-14,18,19,22].

Conflict of Interests

The authors declare that there is no conflict of interests.

Acknowledgements

This work has been done under a Research Project (Project Grant Number: S-0114-1438) sanctioned by the Deanship of Scientific Research Unit of Tabuk University, Ministry of Higher Education, Kingdom of Saudia Arabia.

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