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REPRESENTATIONS BY CERTAIN OCTONARY QUADRATIC FORMS WITH COEFFICIENTS 1, 5, OR 25

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Abstract. It is an important objective to determine the number of representations of a positive integer by certain quadratic forms in number theory. Formulae for $N(1^{2i},2^{2j},3^{2k},6^{2l};n)$ for the nine octonary quadratic forms appear in the literature, whose coefficients are $1,2,3$ and 6. Moreover, the formulae for $N(1^i,3^j,9^k;n)$ for several octonary quadratic forms have been given by Alaca. Here, we determine formulae, for $N(1^i,5^j,25^k;n)$ for several octonary quadratic forms.

Keywords: Octonary quadratic forms; representations; theta functions; Dedekind eta function; Eisenstein series; modular forms; Dirichlet character.

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1. Introduction

It is interesting and important to determine explicit formulas of the representation number of positive definite quadratic forms.

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The work on representation number $card\{(x_1,x_2)\in\mathbb{Z}^2|n=x_1^2+x_2^2\}$ of quadratic form x^2+y^2 has been started by Fermat in 1640. It would be nice to obtain such simple formulas for other positive definite quadratic forms so that we would be able to understand the number of solutions of the equation $Q=n$ for any positive integer n.

Later the formula

$$card\{(x_1,x_2)\in\mathbb{Z}^2|n=x_1^2+x_2^2\}=4\left(\sum_{d|n,d\text{ is odd}}(-1)^{\frac{d-1}{2}}\right)$$

has been proved by Euler. First systematic treatment of binary quadratic forms is due to Legendre. Afterwards it was advanced by Jacobi, with the proof of

$$card\{(x_1,...,x_4)\in\mathbb{Z}^4|n=x_1^2+\cdots+x_4^2\}=8\left(\sum_{d|n,4|d}d\right)\text{ for }x^2+y^2+z^2+t^2.$$

The theory was advanced much further by Gauss in Disquisitiones Arithmetica. The research of Gauss strongly influenced both the arithmetical theory of quadratic forms in more than two variables and subsequent development of algebraic number theory. Since then, there are many more representation number formulas obtained for quadratic forms. Especially, by means of the deep theorems of Hecke [7] and Schoeneberg [19], modular forms have been used in the representation number of several quadratic forms. The generalized theta series ([9],[10], [11],[12]), quasimodular forms ([13],[14],[15], [17]). and several other methods have been also used for the representation number formulae. The formulae for $N(1^i,3^j,9^k;n)$ for several **octonary** quadratic forms have been given by Alaca [5]. Here, we determine formulae, for $N(1^i,5^j,25^k;n)$ for several octonary quadratic forms**.**

2. Preliminaries

The divisor function $\sigma_i(n)$ is defined for a positive integer i by

$$\sigma_i(n):=\begin{cases}\sum_{d\text{ positive integer},d|n}d^i,\text{if }n\text{ is a positive integer}\\ 0\text{ if n is not a positive integer.}\end{cases}\tag{1}$$

The Dedekind eta function and the theta function are defined by

$$\eta(z):=q^{1/24}\prod_{n=1}^{\infty}(1-q^n),\varphi(q):=\sum_{n\in\mathbb{Z}}q^{n^2}\tag{2}$$

where

$$q := e^{2\pi iz}, z \in H = \{x+iy : y > 0\} \tag{3}$$

and an eta quotient of level N is defined by

$$f(z) := \prod_{m|N} \eta(mz)^{a_m}, N, m \in \mathbb{N}, a_m \in \mathbb{Z}. \tag{4}$$

Here we give the following Lemma, see[[16] Theorem 1.64] about the modularity of an eta quotient.

Lemma 1. *An eta quotient of level N is a holomorphic modular form of weight* $\frac{1}{2}\sum_{m|N} a_m$ *on* $\Gamma_0(N)$, *having rational coefficients with respect to q if*

$$a) \sum_{m|N} a_m \text{ is even,}$$

$$b) \sum_{m|N} m a_m \equiv \sum_{m|N} \frac{N}{m} a_m \equiv 0 \text{ mod } 24,$$

$$c) \prod_{m|N} m^{a_m} \text{ is a square of a rational number}$$

$$d) \sum_{m|N} \frac{(\gcd(c,m))^2}{m} a_m \geq 0 \text{ for all positive divisors } c \text{ of } N.$$

For $a_1, ..., a_8 \in \mathbb{N}$ and a nonnegative integer n, we define

$$N(a_1, ..., a_8; n) := card\{(x_1, ..., x_8) \in \mathbb{Z}^8 | n = a_1 x_1^2 + \cdots + a_8 x_8^2\}.$$

Clearly $N(a_1, ..., a_8; 0) = 1$ and, without loss of generality we can assume that

$$a_1 \leq ... \leq a_8, \gcd\{a_1, \ldots, a_8\} = 1$$

Now let's consider octonary quadratic forms of the form

$$Q := x_1^2 + \cdots + x_a^2 + 5(x_{a+1}^2 + \cdots + x_{a+b}^2) + 25(x_{a+b+1}^2 + \cdots + x_{a+b+c}^2),$$

where, $a, b, c \in \mathbb{Z}, 0 \leq a \leq 8$, $0 \leq b \leq 8, 0 \leq c \leq 8$ and $a+b+c = 8$.

We write $N(1^a, 5^b, 25^c; n)$ to denote the number of representations of n by an octonary quadratic form (a,b,c). Its theta function is obviously

$$\Theta_Q = \varphi^a(q)\,\varphi^b\left(q^5\right)\varphi^c\left(q^{25}\right).$$

Formulae for $N(1^{2i}, 2^{2j}, 3^{2k}, 6^{2l}; n)$ for the nine octonary quadratic forms $(2i,2j,2k,2l) = (8,0,0,0), (2,6,0,0), (4,4,0,0), (6,2,0,0), (2,0,6,0), (4,0,4,0), (6,0,2,0), (4,0,0,4),$ and $(0,4,4,0)$ appear in the literature, [1],[2],[3],[4],[8]. Moreover, the formulae for $N(1^i, 3^j, 9^k; n)$ for twenty **octonary** quadratic forms have been given by Alaca [5]. Here, we determine formulae, for $N(1^a, 5^b, 25^c; n)$ for twenty octonary quadratic forms**.** Of course, the formula for $N(1^8, 5^0, 25^0; n)$ is well-known.

Here, we will classify all triples (a,b,c) for which Θ_Q is a modular form of weight 4 with level 100. Then we will obtain their representation numbers in terms of the coefficients of Eisenstein series and some eta quotients.

Table 1

a	b	c	a	b	c	a	b	c	a	b	c	a	b	c	a	b	c	a	b	c
1	0	7	1	6	1	2	4	2	3	2	3	4	2	2	5	2	1	7	0	1
1	2	5	2	0	6	2	6	0	3	4	1	4	4	0	6	0	2	8	0	0
1	4	3	2	2	4	3	0	5	4	0	4	5	0	3	6	2	0			

We characterize the fact that

$$\varphi^a(q)\,\varphi^b\left(q^5\right)\varphi^c\left(q^{25}\right)$$

are in $M_4(\Gamma_0(100))$.

Theorem 2. *Let*

$$Q := x_1^2 + \cdots + x_a^2 + 5(x_{a+1}^2 + \cdots + x_{a+b}^2) + 25(x_{a+b+1}^2 + \cdots + x_{a+b+c}^2)$$

where, $a,b,c \in \mathbb{Z}$, $0 \le a \le 8$, $0 \le b \le 8, 0 \le c \le 8$ and $a+b+c=8$, be an octonary quadratic form. Then its theta series is of the form

$$\begin{aligned}\Theta_Q &= \varphi^a(q)\varphi^b(q^5)\varphi^c(q^{25}) = \\ &\eta^{-2a}(q)\eta^{5a}(q^2)\eta^{-2a}(q^4)\eta^{-2b}(q^5)\eta^{5b}(q^{10})\eta^{-2b}(q^{20})\eta^{-2c}(q^{25})\eta^{5c}(q^{50})\eta^{-2c}(q^{100})\end{aligned}$$

Moreover, it is in $M_4(\Gamma_0(100))$ if and only if $b \equiv 0 \bmod 2$, i.e.,(a,b,c) is given in the Table 1.

Proof. It follows from the Lemma 1, holomorphicity criterion in [[18] Corollary 2.3,p.37] and the fact that

$$\varphi(q) = \frac{\eta^5(q^2)}{\eta^2(q)\eta^2(q^4)}.$$

The condition

$$1^{-2a}2^{5a}4^{-2a}5^{-2b}10^{5b}20^{-2b}25^{-2c}50^{5c}100^{-2c} = 2^{5a-4a+5b-4b+5c-4c}5^{-2b+5b-2b-4c+10c-4c} = 2^{a+b+c}5^{2c}5^b$$

implies that $b \equiv 0 \bmod 2$ if and only if Θ_Q is in $M_4(\Gamma_0(100))$ .Now, consider an eta quotient of the form

$$F = \eta^{a_1}(q)\eta^{a_2}(2q)\eta^{a_3}(4q)\eta^{a_4}(5q)\eta^{a_5}(10q)\eta^{a_6}(20q)\eta^{a_7}(25q)\eta^{a_8}(50q)\eta^{a_9}(100q).$$

Since

$$1^{a_1}2^{a_2}4^{a_3}5^{a_4}10^{a_5}20^{a_6}25^{a_7}50^{a_8}100^{a_9} = 2^{a_2+2a_3+a_5+2a_6+a_8+2a_9}5^{a_4+a_5+a_6+2a_7+2a_8+2a_9},$$

F is in $M_4(\Gamma_0(100))$ if and only if $a_2+a_5+a_8 \equiv 0$ mod 2, $a_4+a_5+a_6 \equiv 0$ mod 2. □

Now let ψ be the Dirichlet character mod 5 sending 2 to i, ϕ be another Dirichlet character mod 5 and

$$E_4(q) = 1+240\sum_{n=1}^{\infty}\sigma_3(n)q^n, E_k^{\psi,\phi}(z) = \sum_{n=1}^{+\infty}\sigma_{k-1}^{\psi,\phi}(n)e^{2\pi i n z}, \sigma_{k-1}^{\psi,\phi}(n) = \sum_{d|n}\psi\left(\frac{n}{d}\right)\phi(d)d^{k-1}.$$

Moreover, let

$$\begin{aligned}
A_1(q) &:= \frac{\eta(2z)^9\eta(5z)^6\eta(50z)\eta(100z)}{\eta(z)^5\eta(4z)\eta(10z)^2\eta(25z)}, A_2(q) := \frac{\eta(2z)^9\eta(5z)^6\eta(50z)}{\eta(z)^5\eta(10z)^2\eta(25z)},\\
A_3(q) &:= \frac{\eta(2z)^9\eta(4z)\eta(5z)^3\eta(50z)^4}{\eta(z)^5\eta(10z)\eta(25z)^2\eta(100z)}, A_4(q) := \frac{\eta(2z)10\eta(5z)^8\eta(20z)^3\eta(50z)^7}{\eta(z)^5\eta(4z)^4\eta(10z)^5\eta(25z)^3\eta(100z)^3},\\
A_5(q) &:= \frac{\eta(2z)^{10}\eta(5z)^6\eta(20z)^4\eta(50z)^2}{\eta(z)^5\eta(4z)^3\eta(10z)^4\eta(25z)\eta(100z)}, A_6(q) = \frac{\eta(2z)^{10}\eta(5z)^6\eta(20z)\eta(50z)^5}{\eta(z)^5\eta(4z)^3\eta(10z)^3\eta(25z)\eta(100z)^2},\\
A_7(q) &:= \frac{\eta(2z)^{10}\eta(5z)^4\eta(20z)^2}{\eta(z)^5\eta(4z)^2\eta(10z)^2\eta(25z)}, A_8(q) = \frac{\eta(2z)^{10}\eta(5z)^6\eta(100z)^2}{\eta(z)^5\eta(4z)^2\eta(10z)^2\eta(25z)},\\
A_9(q) &:= \frac{\eta(2z)^{10}\eta(5z)\eta(10z)^3\eta(100z)}{\eta(z)^5\eta(4z)\eta(50z)}, A_{10}(q) := \frac{\eta(2z)^{10}\eta(5z)^2\eta(25z)^3\eta(100z)}{\eta(z)^5\eta(4z)\eta(50z)^2},\\
A_{11}(q) &:= \frac{\eta(2z)^7\eta(5z)^9\eta(20z)^4\eta(50z)^2}{\eta(z)^4\eta(4z)^3\eta(10z)^5\eta(25z)\eta(100z)}, A_{12}(q) := \frac{\eta(2z)^6\eta(5z)^9\eta(50z)\eta(100z)}{\eta(z)^4\eta(4z)\eta(10z)^3\eta(25z)},\\
A_{13}(q) &:= \frac{\eta(2z)^6\eta(5z)^9\eta(50z)}{\eta(z)^4\eta(10z)^3\eta(25z)}, A_{14}(q) := \frac{\eta(2z)^6\eta(5z)^6\eta(50z)^4}{\eta(z)^4\eta(4z)\eta(10z)^2\eta(25z)^2\eta(100z)},\\
A_{15}(q) &:= \frac{\eta(2z)^7\eta(5z)^6\eta(10z)\eta(50z)^4}{\eta(z)^4\eta(4z)^2\eta(20z)\eta(25z)^2\eta(100z)}, A_{16}(q) := \frac{\eta(2z)^7\eta(5z)^7\eta(20z)^2\eta(25z)}{\eta(z)^4\eta(4z)^2\eta(10z)^3},\\
A_{17}(q) &:= \frac{\eta(2z)^7\eta(5z)^6\eta(50z)^3\eta(100z)}{\eta(z)^4\eta(4z)\eta(10z)^2\eta(25z)^2}, A_{18}(q) := \frac{\eta(2z)^7\eta(5z)^4\eta(100z)}{\eta(z)^4\eta(4z)\eta(10z)^2\eta(50z)},\\
A_{19}(q) &:= \frac{\eta(2z)^7\eta(10z)^5\eta(25z)}{\eta(z)^4\eta(4z)^2\eta(5z)\eta(20z)^2}, A_{20}(q) := \frac{\eta(2z)^7\eta(5z)^6\eta(50z)^3}{\eta(z)^4\eta(10z)^2\eta(25z)^2},\\
A_{21}(q) &:= \frac{\eta(2z)^7\eta(5z)\eta(10z)^3\eta(50z)^2}{\eta(z)^4\eta(4z)\eta(25z)\eta(100z)}, A_{22}(q) := \frac{\eta(2z)^7\eta(5z)^2\eta(25z)^2\eta(50z)}{\eta(z)^4\eta(4z)\eta(100z)},\\
A_{23}(q) &:= \frac{\eta(2z)^8\eta(5z)^6\eta(50z)^2\eta(100z)^3}{\eta(z)^4\eta(4z)^3\eta(10z)^2\eta(25z)^2}, A_{24}(q) := \frac{\eta(2z)^8\eta(5z)\eta(10z)^6\eta(50z)^2}{\eta(z)^4\eta(4z)^2\eta(20z)\eta(25z)\eta(100z)},\\
A_{25}(q) &:= \frac{\eta(4z)^2\eta(5z)^9\eta(10z)\eta(50z)^3}{\eta(z)^2\eta(20z)^2\eta(25z)^3}, A_{26}(q) := \frac{\eta(4z)^4\eta(5z)^7\eta(20z)\eta(50z)^2}{\eta(z)^2\eta(10z)^2\eta(25z)\eta(100z)},\\
A_{27}(q) &:= \frac{\eta(2z)^2\eta(4z)^2\eta(5z)^9\eta(10z)\eta(50z)}{\eta(z)^3\eta(20z)^2\eta(25z)^2}, A_{28}(q) := \frac{\eta(2z)^3\eta(5z)^9\eta(10z)\eta(100z)^2}{\eta(z)^3\eta(20z)^2\eta(25z)^2},\\
A_{29}(q) &:= \frac{\eta(2z)^3\eta(4z)\eta(5z)^9\eta(50z)^4}{\eta(z)^3\eta(10z)^3\eta(25z)^2\eta(100z)}, A_{30}(q) := \frac{\eta(4z)^5\eta(5z)^5\eta(25z)}{\eta(z)^2\eta(20z)},\\
A_{31}(q) &:= \frac{\eta(2z)^3\eta(4z)^5\eta(5z)^2\eta(10z)\eta(25z)}{\eta(z)^3\eta(20z)}, A_{32}(q) := \frac{\eta(2z)^5\eta(10z)^7\eta(20z)^2\eta(100z)}{\eta(z)\eta(4z)^3\eta(5z)^3},\\
A_{33}(q) &:= \frac{\eta(2z)^5\eta(10z)\eta(20z)^6\eta(50z)^6}{\eta(z)\eta(4z)^3\eta(5z)\eta(25z)^2\eta(100z)^3}, A_{34}(q) := \frac{\eta(2z)^6\eta(10z)^5\eta(25z)^2\eta(50z)}{\eta(4z)^2\eta(5z)^2\eta(20z)\eta(100z)},\\
A_{35}(q) &:= \frac{\eta(2z)^8\eta(5z)^2\eta(20z)\eta(50z)^5}{\eta(4z)^4\eta(10z)\eta(25z)^2\eta(100z)},\\
A_{36}(q) &:= \frac{\eta(5z)^3\eta(10z)^{10}\eta(20z)^9\eta(25z)^3\eta(50z)^8\eta(100z)^5}{\eta(z)^{10}\eta(2z)^{10}\eta(4z)^{10}}.
\end{aligned}$$

be some eta quotients of level 100 with weight 4.

Table 2

$$\Theta_Q = \sum_{i=1}^{54} \alpha_i f_i(z), f_i \text{ basis elements in Corollary 4}$$

$\Theta_{(a,b,c)}$	E_4	$E_4(2z)$	$E_4(4z)$	$E_4(5z)$	$E_4(10z)$	$E_4(20z)$	$E_4(25z)$	$E_4(50z)$	$E_4(100z)$
(1 3 4)	$\frac{1}{1170000}$	$-\frac{1}{585000}$	$\frac{1}{73125}$	$-\frac{7}{65000}$	$\frac{7}{32500}$	$-\frac{14}{8125}$	$\frac{125}{1872}$	$-\frac{125}{936}$	$\frac{125}{117}$
(1 2 5)	$\frac{1}{234000}$	$-\frac{1}{117000}$	$\frac{1}{14625}$	$-\frac{1}{9000}$	$\frac{1}{4500}$	$-\frac{2}{1125}$	$\frac{125}{1872}$	$-\frac{125}{936}$	$\frac{125}{117}$
(1 4 3)	$\frac{1}{46800}$	$-\frac{1}{23400}$	$\frac{1}{2925}$	$-\frac{1}{7800}$	$\frac{1}{3900}$	$-\frac{2}{975}$	$\frac{125}{1872}$	$-\frac{125}{936}$	$\frac{125}{117}$
(1 6 1)	$\frac{1}{9360}$	$-\frac{1}{4680}$	$\frac{1}{585}$	$-\frac{1}{4680}$	$\frac{1}{2340}$	$-\frac{2}{585}$	$\frac{125}{1872}$	$-\frac{125}{936}$	$\frac{125}{117}$
(2 0 6)	$\frac{1}{292500}$	$-\frac{1}{146250}$	$\frac{4}{73125}$	$\frac{31}{73125}$	$-\frac{62}{73125}$	$\frac{496}{73125}$	$\frac{31}{468}$	$-\frac{31}{234}$	$\frac{124}{117}$
(2 2 4)	$\frac{1}{58500}$	$-\frac{1}{29250}$	$\frac{4}{14625}$	$\frac{149}{58500}$	$-\frac{149}{29250}$	$\frac{596}{14625}$	$\frac{5}{78}$	$-\frac{5}{39}$	$\frac{40}{39}$
(2 4 2)	$\frac{1}{11700}$	$-\frac{1}{5850}$	$\frac{4}{2925}$	$\frac{77}{5850}$	$-\frac{77}{2925}$	$\frac{616}{2925}$	$\frac{25}{468}$	$-\frac{25}{234}$	$\frac{100}{117}$
(2 6 0)	$\frac{1}{2340}$	$-\frac{1}{1170}$	$\frac{4}{585}$	$\frac{31}{468}$	$-\frac{31}{234}$	$\frac{124}{117}$	0	0	0
(3 0 5)	$\frac{1}{46800}$	$-\frac{1}{23400}$	$\frac{1}{2925}$	$-\frac{1}{7800}$	$\frac{1}{3900}$	$-\frac{2}{975}$	$\frac{125}{1872}$	$-\frac{125}{936}$	$\frac{125}{117}$
(3 2 3)	$\frac{1}{9360}$	$-\frac{1}{4680}$	$\frac{1}{585}$	$-\frac{1}{4680}$	$\frac{1}{2340}$	$-\frac{2}{585}$	$\frac{125}{1872}$	$-\frac{125}{936}$	$\frac{125}{117}$
(3 4 1)	$\frac{1}{1872}$	$-\frac{1}{936}$	$\frac{1}{117}$	$-\frac{1}{1560}$	$\frac{1}{780}$	$-\frac{2}{195}$	$\frac{125}{1872}$	$-\frac{125}{936}$	$\frac{125}{117}$
(4 0 4)	$\frac{1}{9750}$	$-\frac{1}{4875}$	$\frac{8}{4875}$	$\frac{4}{1625}$	$-\frac{8}{1625}$	$\frac{64}{1625}$	$\frac{5}{78}$	$-\frac{5}{39}$	$\frac{40}{39}$
(4 2 2)	$\frac{1}{1950}$	$-\frac{1}{975}$	$\frac{8}{975}$	$\frac{149}{11700}$	$-\frac{149}{5850}$	$\frac{596}{2925}$	$\frac{25}{468}$	$-\frac{25}{234}$	$\frac{100}{117}$
(4 4 0)	$\frac{1}{390}$	$-\frac{1}{195}$	$\frac{8}{195}$	$\frac{5}{78}$	$-\frac{5}{39}$	$\frac{40}{39}$	0	0	0
(5 0 3)	$\frac{1}{1872}$	$-\frac{1}{936}$	$\frac{1}{117}$	$-\frac{1}{1560}$	$\frac{1}{780}$	$-\frac{2}{195}$	$\frac{125}{1872}$	$-\frac{125}{936}$	$\frac{125}{117}$
(5 2 1)	$\frac{5}{1872}$	$-\frac{5}{936}$	$\frac{5}{117}$	$-\frac{1}{360}$	$\frac{1}{180}$	$-\frac{2}{45}$	$\frac{125}{1872}$	$-\frac{125}{936}$	$\frac{125}{117}$
(6 0 2)	$\frac{31}{11700}$	$-\frac{31}{5850}$	$\frac{124}{2925}$	$\frac{31}{2925}$	$-\frac{62}{2925}$	$\frac{496}{2925}$	$\frac{25}{468}$	$-\frac{25}{234}$	$\frac{100}{117}$
(6 2 0)	$\frac{31}{2340}$	$-\frac{31}{1170}$	$\frac{124}{585}$	$\frac{25}{468}$	$-\frac{25}{234}$	$\frac{100}{117}$	0	0	0
(7 0 1)	$\frac{25}{1872}$	$-\frac{25}{936}$	$\frac{25}{117}$	$-\frac{7}{520}$	$\frac{7}{260}$	$-\frac{14}{65}$	$\frac{125}{1872}$	$-\frac{125}{936}$	$\frac{125}{117}$
(8 0 0)	$\frac{1}{15}$	$-\frac{2}{15}$	$\frac{16}{15}$	0	0	0	0	0	0

$\Theta_{(a,b,c)}$	$E_4^{\psi^2,\psi^2}$	$E_4^{\psi^2,\psi^2}(2z)$	$E_4^{\psi^2,\psi^2}(4z)$	A_1	A_2	A_3	A_4
(1 3 4)	$\frac{1}{4875}$	$\frac{2}{4875}$	$\frac{16}{4875}$	$\frac{147902}{901875}$	$\frac{1413414629}{9018750}$	$\frac{5963}{60125}$	$\frac{217240406}{4509375}$
(1 2 5)	$\frac{1}{975}$	$\frac{2}{975}$	$\frac{16}{975}$	$\frac{4354}{60125}$	$\frac{321024749}{1803750}$	$-\frac{36299}{7215}$	$\frac{57770486}{901875}$
(1 4 3)	$\frac{1}{195}$	$\frac{2}{195}$	$\frac{16}{195}$	$\frac{68462}{36075}$	$\frac{121500149}{360750}$	$-\frac{3353}{185}$	$\frac{20894486}{180375}$
(1 6 1)	$\frac{1}{39}$	$\frac{2}{39}$	$\frac{16}{39}$	$\frac{4138}{481}$	$\frac{2571269}{2886}$	$-\frac{65959}{1443}$	$\frac{1902142}{7215}$
(2 0 6)	0	0	0	$-\frac{520868}{901875}$	$\frac{302646277}{4509375}$	$\frac{475714}{180375}$	$\frac{130316396}{4509375}$
(2 2 4)	0	0	0	$-\frac{317188}{180375}$	$\frac{26989657}{901875}$	$-\frac{58898}{12025}$	$\frac{41586236}{901875}$
(2 4 2)	0	0	0	$-\frac{54308}{36075}$	$\frac{20863237}{180375}$	$-\frac{261374}{7215}$	$\frac{21143276}{180375}$
(2 6 0)	0	0	0	$-\frac{1844}{1443}$	$\frac{1092821}{7215}$	$-\frac{75506}{481}$	$\frac{2598988}{7215}$
(3 0 5)	$\frac{1}{975}$	$\frac{2}{975}$	$\frac{16}{975}$	$-\frac{205198}{180375}$	$-\frac{1207517}{138750}$	$-\frac{37669}{36075}$	$\frac{22558306}{901875}$
(3 2 3)	$\frac{1}{195}$	$\frac{2}{195}$	$\frac{16}{195}$	$\frac{49882}{36075}$	$\frac{59331439}{360750}$	$-\frac{45567}{2405}$	$\frac{16982146}{180375}$
(3 4 1)	$\frac{1}{39}$	$\frac{2}{39}$	$\frac{16}{39}$	$\frac{18682}{1443}$	$\frac{13053611}{14430}$	$-\frac{109333}{1443}$	$\frac{2222906}{7215}$
(4 0 4)	0	0	0	$-\frac{371216}{60125}$	$-\frac{114012876}{300625}$	$\frac{4168}{12025} - \frac{40592}{555}$	$-\frac{3517648}{300625}$
(4 2 2)	0	0	0	$-\frac{89488}{36075}$	$\frac{4973944}{60125}$	$-\frac{40592}{555}$	$\frac{11166912}{60125}$
(4 4 0)	0	0	0	$\frac{2864}{481}$	$-\frac{1109676}{2405}$	$-\frac{101464}{481}$	$\frac{871472}{2405}$
(5 0 3)	$\frac{1}{195}$	$\frac{2}{195}$	$\frac{16}{195}$	$-\frac{100738}{36075}$	$-\frac{162594451}{360750}$	$\frac{23427}{2405}$	$-\frac{4484314}{180375}$
(5 2 1)	$\frac{1}{39}$	$\frac{2}{39}$	$\frac{16}{39}$	$\frac{6170}{481}$	$\frac{14411377}{14430}$	$-\frac{199375}{1443}$	$\frac{617270}{1443}$
(6 0 2)	0	0	0	$\frac{132412}{36075}$	$\frac{98698957}{180375}$	$-\frac{844526}{7215}$	$\frac{54252236}{180375}$
(6 2 0)	0	0	0	$\frac{50476}{1443}$	$-\frac{4977859}{7215}$	$-\frac{57730}{481}$	$\frac{255148}{7215}$
(7 0 1)	$\frac{1}{39}$	$\frac{2}{39}$	$\frac{16}{39}$	$\frac{6106}{1443}$	$\frac{53760059}{14430}$	$-\frac{782341}{1443}$	$\frac{10716698}{7215}$
(8 0 0)	0	0	0	0	0	0	0

$\Theta_{(a,b,c)}$	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}
(1 3 4)	$-\frac{26572553}{6012500}$	$-\frac{17071921}{360750}$	$\frac{375513917}{3607500}$	$-\frac{37134731}{4509375}$	$-\frac{808546769}{9018750}$	$\frac{28530022}{1503125}$	$-\frac{5342}{13875}$
(1 2 5)	$-\frac{1383383}{277500}$	$-\frac{3145681}{72150}$	$\frac{84507877}{721500}$	$-\frac{5409037}{300625}$	$-\frac{175193689}{1803750}$	$\frac{3248546}{901875}$	$\frac{335758}{12025}$
(1 4 3)	$-\frac{2173193}{240500}$	$-\frac{903313}{14430}$	$\frac{29889677}{144300}$	$-\frac{4355411}{180375}$	$-\frac{55470689}{360750}$	$-\frac{3180218}{60125}$	$\frac{900634}{7215}$
(1 6 1)	$-\frac{662383}{28860}$	$-\frac{497521}{2886}$	$\frac{3087089}{5772}$	$-\frac{33729}{2405}$	$-\frac{5428373}{14430}$	$-\frac{1159718}{7215}$	$\frac{159150}{481}$
(2 0 6)	$-\frac{21563947}{9018750}$	$-\frac{2180011}{60125}$	$\frac{8214097}{138750}$	$-\frac{57867046}{4509375}$	$-\frac{285267377}{4509375}$	$\frac{5467004}{115625}$	$-\frac{2923972}{60125}$
(2 2 4)	$-\frac{4970327}{1803750}$	$-\frac{143463}{12025}$	$\frac{1101077}{27750}$	$-\frac{36535886}{901875}$	$-\frac{16602519}{300625}$	$\frac{28435796}{901875}$	$-\frac{775036}{36075}$
(2 4 2)	$-\frac{2000107}{360750}$	$\frac{54037}{2405}$	$\frac{463057}{5550}$	$-\frac{15485926}{180375}$	$-\frac{12009137}{180375}$	$-\frac{5099188}{60125}$	$\frac{392508}{2405}$
(2 6 0)	$-\frac{177691}{14430}$	$\frac{100713}{481}$	$\frac{26401}{222}$	$-\frac{2145238}{7215}$	$-\frac{121307}{2405}$	$-\frac{3351932}{7215}$	$\frac{1072580}{1443}$
(3 0 5)	$-\frac{1234603}{1202500}$	$-\frac{1564531}{72150}$	$\frac{18817967}{721500}$	$-\frac{24064181}{901875}$	$-\frac{71297419}{1803750}$	$\frac{60460966}{901875}$	$-\frac{2728066}{36075}$
(3 2 3)	$-\frac{3683569}{721500}$	$-\frac{112535}{2886}$	$\frac{5969149}{48100}$	$-\frac{8189521}{180375}$	$-\frac{36806579}{360750}$	$-\frac{1898794}{180375}$	$\frac{124218}{2405}$
(3 4 1)	$-\frac{183983}{9620}$	$-\frac{443923}{2886}$	$\frac{3202867}{5772}$	$-\frac{408121}{7215}$	$-\frac{391643}{1110}$	$-\frac{126778}{555}$	$\frac{635870}{1443}$
(4 0 4)	$\frac{444218}{300625}$	$\frac{909404}{12025}$	$-\frac{781518}{4625}$	$-\frac{28776152}{300625}$	$\frac{17531676}{300625}$	$\frac{47750672}{300625}$	$-\frac{2530352}{12025}$
(4 2 2)	$-\frac{1127476}{180375}$	$\frac{165704}{2405}$	$\frac{215476}{2775}$	$-\frac{26893136}{180375}$	$-\frac{7181432}{180375}$	$-\frac{31171904}{180375}$	$\frac{2173984}{7215}$
(4 4 0)	$\frac{1098}{2405}$	$\frac{170988}{481}$	$-\frac{5118}{37}$	$-\frac{1005272}{2405}$	$\frac{354876}{2405}$	$-\frac{889328}{2405}$	$\frac{282000}{481}$
(5 0 3)	$-\frac{647393}{240500}$	$\frac{1411799}{14430}$	$-\frac{29507323}{144300}$	$-\frac{19827611}{180375}$	$\frac{16658311}{360750}$	$\frac{13584982}{60125}$	$-\frac{2209526}{7215}$
(5 2 1)	$-\frac{10007}{444}$	$-\frac{254089}{2886}$	$\frac{3565385}{5772}$	$-\frac{75013}{481}$	$-\frac{1009513}{2886}$	$-\frac{572350}{1443}$	$\frac{345470}{481}$
(6 0 2)	$-\frac{7325827}{360750}$	$\frac{157629}{2405}$	$\frac{1779577}{5550}$	$-\frac{22008886}{180375}$	$-\frac{25820057}{180375}$	$-\frac{24864468}{60125}$	$\frac{1587228}{2405}$
(6 2 0)	$\frac{146189}{14430}$	$\frac{124849}{481}$	$-\frac{58199}{222}$	$-\frac{1316998}{7215}$	$\frac{629453}{2405}$	$\frac{234148}{7215}$	$-\frac{85660}{1443}$
(7 0 1)	$-\frac{594559}{9620}$	$-\frac{511267}{2886}$	$\frac{12494851}{5772}$	$-\frac{2742313}{7215}$	$-\frac{13723247}{14430}$	$-\frac{15429202}{7215}$	$\frac{4943870}{1443}$
(8 0 0)	0	0	0	0	0	0	0

$\Theta_{(a,b,c)}$	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}	A_{17}
(1 3 4)	$-\frac{563549807}{18037500}$	$-\frac{1240426503}{6012500}$	$-\frac{148859039}{6012500}$	$\frac{132762329}{9018750}$	$\frac{9151007}{360750}$	$-\frac{428046227}{3607500}$
(1 2 5)	$-\frac{11786459}{277500}$	$-\frac{286859943}{1202500}$	$-\frac{96841477}{3607500}$	$\frac{23811649}{1803750}$	$\frac{786389}{24050}$	$-\frac{32481929}{240500}$
(1 4 3)	$-\frac{60289967}{721500}$	$-\frac{106653543}{240500}$	$-\frac{10092559}{240500}$	$\frac{554573}{27750}$	$\frac{189427}{2886}$	$-\frac{36942947}{144300}$
(1 6 1)	$-\frac{5947219}{28860}$	$-\frac{10855771}{9620}$	$-\frac{2616977}{28860}$	$\frac{647261}{14430}$	$\frac{154101}{962}$	$-\frac{1274853}{1924}$
(2 0 6)	$-\frac{116277631}{9018750}$	$-\frac{895168997}{9018750}$	$-\frac{47137247}{3006250}$	$\frac{57728537}{4509375}$	$\frac{394117}{60125}$	$-\frac{95846771}{1803750}$
(2 2 4)	$-\frac{36497371}{1803750}$	$-\frac{112452377}{1803750}$	$-\frac{14304881}{1803750}$	$-\frac{1738161}{300625}$	$\frac{373691}{36075}$	$-\frac{632347}{27750}$
(2 4 2)	$-\frac{26669311}{360750}$	$-\frac{69373157}{360750}$	$-\frac{896607}{120250}$	$-\frac{4228903}{180375}$	$\frac{106117}{2405}$	$-\frac{6027251}{72150}$
(2 6 0)	$-\frac{3604943}{14430}$	$-\frac{5168341}{14430}$	$\frac{378227}{14430}$	$-\frac{291853}{2405}$	$\frac{189035}{1443}$	$-\frac{337283}{2886}$
(3 0 5)	$-\frac{11793757}{3607500}$	$-\frac{52417759}{3607500}$	$-\frac{23219567}{3607500}$	$\frac{11946979}{1803750}$	$-\frac{703811}{72150}$	$\frac{1584463}{721500}$
(3 2 3)	$-\frac{36496237}{721500}$	$-\frac{13159963}{55500}$	$-\frac{14357047}{721500}$	$\frac{330539}{360750}$	$\frac{145023}{4810}$	$-\frac{17429017}{144300}$
(3 4 1)	$-\frac{526109}{2220}$	$-\frac{33100019}{28860}$	$-\frac{428951}{5772}$	$\frac{271127}{14430}$	$\frac{449437}{2886}$	$-\frac{3632077}{5772}$
(4 0 4)	$\frac{12568914}{300625}$	$\frac{128522918}{300625}$	$\frac{12782454}{300625}$	$-\frac{16764156}{300625}$	$-\frac{719588}{12025}$	$\frac{16996074}{60125}$
(4 2 2)	$-\frac{6211916}{60125}$	$-\frac{864484}{4625}$	$\frac{1766572}{180375}$	$-\frac{10762408}{180375}$	$\frac{322456}{7215}$	$-\frac{1831868}{36075}$
(4 4 0)	$-\frac{489246}{2405}$	$\frac{776598}{2405}$	$\frac{274214}{2405}$	$-\frac{503196}{2405}$	$\frac{17580}{481}$	$\frac{150074}{481}$
(5 0 3)	$\frac{33901033}{721500}$	$\frac{127222257}{240500}$	$\frac{1115757}{18500}$	$-\frac{32303551}{360750}$	$-\frac{1022809}{14430}$	$\frac{50155093}{144300}$
(5 2 1)	$-\frac{1942559}{5772}$	$-\frac{2437703}{1924}$	$-\frac{1014953}{28860}$	$-\frac{72919}{1110}$	$\frac{163053}{962}$	$-\frac{91153}{148}$
(6 0 2)	$-\frac{90810871}{360750}$	$-\frac{271660877}{360750}$	$-\frac{1126327}{120250}$	$-\frac{10870783}{180375}$	$\frac{253037}{2405}$	$-\frac{27108011}{72150}$
(6 2 0)	$-\frac{584903}{14430}$	$\frac{10487939}{14430}$	$\frac{1842827}{14430}$	$-\frac{234613}{2405}$	$-\frac{150205}{1443}$	$\frac{1379557}{2886}$
(7 0 1)	$-\frac{37488041}{28860}$	$-\frac{136553987}{28860}$	$-\frac{4329859}{28860}$	$-\frac{414725}{2886}$	$\frac{1790221}{2886}$	$-\frac{13784125}{5772}$
(8 0 0)	0	0	0	0	0	0

$\Theta_{(a,b,c)}$	A_{18}	A_{19}	A_{20}	A_{21}	A_{22}	A_{23}	A_{24}
(1 3 4)	$\frac{21519683}{1803750}$	$-\frac{83402908}{4509375}$	$\frac{134876}{901875}$	$\frac{163867}{138750}$	$\frac{49384056}{1503125}$	$-\frac{37584}{1503125}$	$-\frac{125991}{115625}$
(1 2 5)	$\frac{6177523}{360750}$	$-\frac{28518748}{901875}$	$\frac{96372}{60125}$	$-\frac{1252049}{360750}$	$\frac{35207608}{901875}$	$\frac{22088}{901875}$	$-\frac{275323}{300625}$
(1 4 3)	$\frac{3494723}{72150}$	$-\frac{12841948}{180375}$	$\frac{62876}{36075}$	$-\frac{1175969}{72150}$	$\frac{4640536}{60125}$	$-\frac{11104}{60125}$	$\frac{87877}{60125}$
(1 6 1)	$\frac{32627}{222}$	$-\frac{1205996}{7215}$	$-\frac{204}{37}$	$-\frac{109597}{2886}$	$\frac{113432}{555}$	$-\frac{5384}{7215}$	$\frac{16129}{2405}$
(2 0 6)	$\frac{635539}{901875}$	$\frac{1240024}{1503125}$	$\frac{1322936}{901875}$	$\frac{7991983}{901875}$	$\frac{60019088}{4509375}$	$\frac{197056}{1503125}$	$-\frac{1798678}{1503125}$
(2 2 4)	$\frac{3169199}{180375}$	$-\frac{3536616}{300625}$	$\frac{305192}{60125}$	$\frac{1158283}{180375}$	$\frac{2543536}{300625}$	$\frac{387488}{901875}$	$-\frac{331598}{300625}$
(2 4 2)	$\frac{2543059}{36075}$	$-\frac{4879656}{60125}$	$\frac{389816}{36075}$	$-\frac{724817}{36075}$	$\frac{6627728}{180375}$	$\frac{41536}{60125}$	$\frac{114282}{60125}$
(2 6 0)	$\frac{31519}{111}$	$-\frac{691528}{2405}$	$\frac{19976}{481}$	$-\frac{130369}{1443}$	$\frac{213488}{2405}$	$\frac{15904}{7215}$	$\frac{32746}{2405}$
(3 0 5)	$-\frac{3454567}{360750}$	$\frac{6498092}{901875}$	$\frac{258652}{60125}$	$\frac{1547887}{120250}$	$-\frac{1207832}{901875}$	$\frac{23896}{69375}$	$-\frac{338833}{300625}$
(3 2 3)	$\frac{924451}{24050}$	$-\frac{7639828}{180375}$	$\frac{150356}{36075}$	$-\frac{4421}{1850}$	$\frac{2440296}{60125}$	$\frac{55568}{180375}$	$\frac{57647}{60125}$
(3 4 1)	$\frac{38041}{222}$	$-\frac{1540868}{7215}$	$-\frac{3476}{481}$	$-\frac{51165}{962}$	$\frac{1529768}{7215}$	$\frac{2408}{7215}$	$\frac{11747}{2405}$
(4 0 4)	$-\frac{670932}{60125}$	$\frac{13845664}{300625}$	$\frac{920032}{60125}$	$\frac{2373276}{60125}$	$-\frac{23134144}{300625}$	$\frac{415616}{300625}$	$-\frac{600008}{300625}$
(4 2 2)	$\frac{4390424}{36075}$	$-\frac{8022816}{60125}$	$\frac{54752}{2775}$	$-\frac{439464}{12025}$	$\frac{7332608}{180375}$	$\frac{222688}{180375}$	$\frac{27504}{4625}$
(4 4 0)	$\frac{7036}{37}$	$-\frac{544736}{2405}$	$\frac{29792}{481}$	$-\frac{31236}{481}$	$-\frac{53184}{2405}$	$\frac{6016}{2405}$	$\frac{7352}{2405}$
(5 0 3)	$\frac{1677323}{72150}$	$\frac{13406852}{180375}$	$\frac{36332}{2775}$	$\frac{4931911}{72150}$	$-\frac{5559464}{60125}$	$\frac{89696}{60125}$	$\frac{89277}{60125}$
(5 2 1)	$\frac{70715}{222}$	$-\frac{476188}{1443}$	$-\frac{1980}{481}$	$-\frac{235861}{2886}$	$\frac{355864}{1443}$	$\frac{3032}{1443}$	$\frac{4613}{481}$
(6 0 2)	$\frac{10544299}{36075}$	$-\frac{14851816}{60125}$	$\frac{371576}{36075}$	$-\frac{2665817}{36075}$	$\frac{27486608}{180375}$	$\frac{8896}{60125}$	$\frac{1447802}{60125}$
(6 2 0)	$-\frac{10601}{111}$	$\frac{89592}{2405}$	$\frac{8776}{481}$	$\frac{36791}{1443}$	$-\frac{247952}{2405}$	$-\frac{18656}{7215}$	$-\frac{5894}{2405}$
(7 0 1)	$\frac{272713}{222}$	$-\frac{9421124}{7215}$	$-\frac{2196}{481}$	$-\frac{456397}{962}$	$\frac{6674984}{7215}$	$\frac{32744}{7215}$	$\frac{116211}{2405}$
(8 0 0)	0	0	0	0	0	0	0

$\Theta_{(a,b,c)}$	A_{25}	A_{26}	A_{27}	A_{28}	A_{29}	A_{30}	A_{31}	A_{32}
$(1\ 3\ 4)$	$-\frac{1984}{4875}$	$\frac{7936}{8125}$	$\frac{160\,256}{24\,375}$	$-\frac{20\,992}{1625}$	$-\frac{81\,664}{4875}$	$-\frac{5632}{8125}$	$\frac{20\,093\,132}{4\,509\,375}$	$\frac{3\,142\,283}{346\,875}$
$(1\ 2\ 5)$	$-\frac{128}{325}$	$\frac{1536}{1625}$	$\frac{33\,856}{4875}$	$-\frac{13\,376}{975}$	$-\frac{5888}{325}$	$-\frac{4096}{4875}$	$\frac{5\,340\,892}{901\,875}$	$\frac{3\,840\,533}{300\,625}$
$(1\ 4\ 3)$	$-\frac{64}{195}$	$\frac{256}{325}$	$\frac{8576}{975}$	$-\frac{1152}{65}$	$-\frac{4864}{195}$	$-\frac{512}{325}$	$\frac{1\,861\,292}{180\,375}$	$\frac{4\,522\,799}{180\,375}$
$(1\ 6\ 1)$	0	0	$\frac{704}{39}$	$-\frac{1472}{39}$	$-\frac{768}{13}$	$-\frac{1024}{195}$	$\frac{13\,628}{555}$	$\frac{146\,801}{2405}$
$(2\ 0\ 6)$	$-\frac{1984}{4875}$	$\frac{7936}{8125}$	$\frac{156\,736}{24\,375}$	$-\frac{61\,504}{4875}$	$-\frac{15\,872}{975}$	$-\frac{15\,872}{24\,375}$	$\frac{2\,508\,104}{1\,503\,125}$	$\frac{18\,227\,614}{4\,509\,375}$
$(2\ 2\ 4)$	$-\frac{128}{325}$	$\frac{1536}{1625}$	$\frac{10\,112}{1625}$	$-\frac{3968}{325}$	$-\frac{1024}{65}$	$-\frac{1024}{1625}$	$\frac{2\,228\,792}{901\,875}$	$\frac{474\,398}{69\,375}$
$(2\ 4\ 2)$	$-\frac{64}{195}$	$\frac{256}{325}$	$\frac{5056}{975}$	$-\frac{1984}{195}$	$-\frac{512}{39}$	$-\frac{512}{975}$	$\frac{38\,248}{4625}$	$\frac{4\,333\,534}{180\,375}$
$(2\ 6\ 0)$	$-\frac{64}{195}$	0	0	0	0	0	$\frac{152\,536}{7215}$	$\frac{664\,942}{7215}$
$(3\ 0\ 5)$	$-\frac{128}{325}$	$\frac{1536}{1625}$	$\frac{33\,856}{4875}$	$-\frac{13\,376}{975}$	$-\frac{5888}{325}$	$-\frac{4096}{4875}$	$\frac{534\,844}{300\,625}$	$\frac{1\,971\,629}{901\,875}$
$(3\ 2\ 3)$	$\frac{256}{325}$	$\frac{256}{325}$	$\frac{8576}{975}$	$-\frac{1152}{65}$	$-\frac{4864}{195}$	$-\frac{512}{325}$	$\frac{101\,524}{13\,875}$	$\frac{2\,860\,189}{180\,375}$
$(3\ 4\ 1)$	0	0	$\frac{704}{39}$	$-\frac{1472}{39}$	$-\frac{768}{13}$	$-\frac{1024}{195}$	$\frac{76\,204}{2405}$	$\frac{502\,249}{7215}$
$(4\ 0\ 4)$	$-\frac{128}{325}$	$\frac{1536}{1625}$	$\frac{10\,112}{1625}$	$-\frac{3968}{325}$	$-\frac{1024}{65}$	$-\frac{1024}{1625}$	$-\frac{49\,312}{23\,125}$	$-\frac{2\,956\,232}{300\,625}$
$(4\ 2\ 2)$	$-\frac{64}{195}$	$\frac{256}{325}$	$\frac{5056}{975}$	$-\frac{1984}{195}$	$-\frac{512}{39}$	$-\frac{512}{975}$	$\frac{2\,593\,792}{180\,375}$	$\frac{5\,515\,424}{180\,375}$
$(4\ 4\ 0)$	0	0	0	0	0	0	$\frac{63\,584}{2405}$	$\frac{222\,648}{2405}$
$(5\ 0\ 3)$	$-\frac{64}{195}$	$\frac{256}{325}$	$\frac{8576}{975}$	$-\frac{1152}{65}$	$-\frac{4864}{195}$	$-\frac{512}{325}$	$-\frac{579\,508}{180\,375}$	$-\frac{2\,826\,601}{180\,375}$
$(5\ 2\ 1)$	0	0	$\frac{704}{39}$	$-\frac{1472}{39}$	$-\frac{768}{13}$	$-\frac{1024}{195}$	$\frac{65\,596}{1443}$	$\frac{36\,533}{481}$
$(6\ 0\ 2)$	$-\frac{64}{195}$	$\frac{256}{325}$	$\frac{5056}{975}$	$-\frac{1984}{195}$	$-\frac{512}{39}$	$-\frac{512}{975}$	$\frac{1\,552\,264}{60\,125}$	$\frac{8\,956\,174}{180\,375}$
$(6\ 2\ 0)$	0	0	0	0	0	0	$\frac{101\,656}{7215}$	$\frac{333\,982}{7215}$
$(7\ 0\ 1)$	0	0	$\frac{704}{39}$	$-\frac{1472}{39}$	$-\frac{768}{13}$	$-\frac{1024}{195}$	$\frac{328\,812}{2405}$	$\frac{1\,763\,737}{7215}$
$(8\ 0\ 0)$	0	0	0	0	0	0	0	0

$\Theta_{(a,b,c)}$	A_{33}	A_{34}	A_{35}	A_{36}
(1 3 4)	$-\frac{13569693}{1503125}$	$-\frac{51167}{1202500}$	$\frac{637753}{120250}$	0
(1 2 5)	$-\frac{11380999}{901875}$	$\frac{6419}{721500}$	$\frac{165033}{24050}$	0
(1 4 3)	$-\frac{1460733}{60125}$	$-\frac{927}{48100}$	$\frac{63833}{4810}$	0
(1 6 1)	$-\frac{412283}{7215}$	$\frac{823}{5772}$	$\frac{31945}{962}$	0
(2 0 6)	$-\frac{16821614}{4509375}$	$\frac{24959}{138750}$	$\frac{389147}{180375}$	0
(2 2 4)	$-\frac{1587058}{300625}$	$\frac{82549}{120250}$	$\frac{105167}{36075}$	0
(2 4 2)	$-\frac{2927534}{180375}$	$\frac{119027}{72150}$	$\frac{74267}{7215}$	0
(2 6 0)	$-\frac{127914}{2405}$	$\frac{2497}{962}$	$\frac{49055}{1443}$	0
(3 0 5)	$-\frac{1579429}{901875}$	$\frac{185849}{721500}$	$\frac{1969}{72150}$	0
(3 2 3)	$-\frac{822663}{60125}$	$\frac{25603}{48100}$	$\frac{97729}{14430}$	0
(3 4 1)	$-\frac{423809}{7215}$	$\frac{11749}{5772}$	$\frac{101905}{2886}$	0
(4 0 4)	$\frac{3844232}{300625}$	$\frac{79702}{60125}$	$-\frac{99636}{12025}$	0
(4 2 2)	$-\frac{2851424}{180375}$	$\frac{79636}{36075}$	$\frac{28384}{2405}$	0
(4 4 0)	$-\frac{45048}{2405}$	$\frac{1862}{481}$	$\frac{10620}{481}$	0
(5 0 3)	$\frac{1107467}{60125}$	$\frac{71673}{48100}$	$-\frac{56367}{4810}$	0
(5 2 1)	$-\frac{89767}{1443}$	$\frac{24415}{5772}$	$\frac{44865}{962}$	0
(6 0 2)	$-\frac{5774174}{180375}$	$\frac{116747}{72150}$	$\frac{176627}{7215}$	0
(6 2 0)	$\frac{100806}{2405}$	$\frac{1097}{962}$	$-\frac{4585}{1443}$	0
(7 0 1)	$-\frac{1791857}{7215}$	$\frac{35797}{5772}$	$\frac{500065}{2886}$	0
(8 0 0)	0	0	0	0

Table 3

$A_i = \sum_{i=1}^{36} \beta_i g_i(z), g_i$ basis elements in the proof of Theorem 5

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}
$\Delta_{5,4}$	$\frac{1}{48}$	$\frac{5}{24}$	$\frac{41}{240}$	$\frac{1}{10}$	$\frac{1}{16}$	$\frac{1}{15}$	$\frac{1}{24}$	$\frac{1}{8}$	$\frac{1}{48}$	$-\frac{1}{240}$	$\frac{1}{120}$
$\Delta_{5,4}(2z)$	$\frac{11}{30}$	$\frac{17}{12}$	$\frac{151}{120}$	$\frac{17}{10}$	$\frac{41}{60}$	$\frac{23}{30}$	$\frac{1}{6}$	$\frac{13}{12}$	$\frac{3}{8}$	$\frac{11}{60}$	$\frac{9}{20}$
$\Delta_{5,4}(4z)$	$\frac{43}{15}$	0	$\frac{46}{15}$	$\frac{104}{15}$	$\frac{49}{15}$	$\frac{8}{3}$	0	$\frac{8}{3}$	$\frac{10}{3}$	$\frac{5}{3}$	$\frac{12}{5}$
$\Delta_{5,4}(5z)$	$\frac{125}{48}$	$\frac{325}{24}$	$\frac{425}{48}$	$\frac{25}{2}$	$\frac{125}{16}$	$\frac{25}{3}$	$\frac{125}{24}$	$\frac{25}{8}$	$\frac{125}{48}$	$\frac{95}{48}$	$\frac{85}{24}$
$\Delta_{5,4}(10z)$	$\frac{125}{6}$	$\frac{1225}{12}$	$\frac{1675}{24}$	$\frac{125}{2}$	$\frac{725}{12}$	$\frac{395}{6}$	$\frac{275}{6}$	$\frac{125}{12}$	$\frac{275}{8}$	$\frac{455}{12}$	$\frac{145}{4}$
$\Delta_{5,4}(20z)$	$\frac{475}{3}$	0	$\frac{550}{3}$	$\frac{200}{3}$	$\frac{625}{3}$	$\frac{520}{3}$	200	$-\frac{200}{3}$	$\frac{650}{3}$	$\frac{625}{3}$	140
$\Delta_{10,4}$	$-\frac{1}{144}$	$\frac{1}{24}$	$\frac{37}{720}$	$-\frac{7}{45}$	$-\frac{11}{144}$	$-\frac{2}{45}$	$\frac{1}{12}$	$-\frac{1}{72}$	$-\frac{1}{48}$	$\frac{17}{720}$	$-\frac{23}{360}$
$\Delta_{10,4}(2z)$	$-\frac{41}{180}$	0	$-\frac{8}{45}$	$\frac{8}{45}$	$-\frac{13}{180}$	$-\frac{1}{9}$	$-\frac{1}{3}$	$\frac{1}{9}$	0	$-\frac{5}{36}$	$-\frac{11}{90}$
$\Delta_{10,4}(5z)$	$-\frac{1075}{144}$	$\frac{175}{24}$	$-\frac{625}{144}$	$\frac{25}{9}$	$\frac{175}{144}$	$\frac{20}{9}$	$-\frac{25}{12}$	$-\frac{175}{72}$	$-\frac{125}{16}$	$-\frac{785}{144}$	$\frac{155}{72}$
$\Delta_{10,4}(10z)$	$\frac{1625}{36}$	0	$\frac{500}{9}$	$\frac{100}{9}$	$\frac{925}{36}$	$\frac{185}{9}$	25	$\frac{175}{9}$	$\frac{100}{3}$	$\frac{625}{36}$	$\frac{395}{18}$
$\Delta_{20,4}(z)$	$\frac{1}{36}$	0	$\frac{1}{36}$	$\frac{2}{9}$	$\frac{1}{72}$	$\frac{1}{36}$	$-\frac{1}{8}$	$\frac{5}{36}$	0	$-\frac{5}{72}$	$\frac{1}{72}$
$\Delta_{20,4}(5z)$	$\frac{175}{36}$	0	$\frac{125}{18}$	$\frac{125}{9}$	$\frac{475}{72}$	$\frac{175}{36}$	$\frac{175}{24}$	$-\frac{25}{36}$	$\frac{125}{12}$	$\frac{625}{72}$	$\frac{475}{72}$
$\Delta_{25,4,1}(z)$	$\frac{5}{42}$	$-\frac{13}{42}$	$\frac{1}{28}$	$\frac{4}{21}$	$\frac{11}{42}$	$\frac{16}{105}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{5}{28}$	$\frac{23}{210}$	$\frac{1}{6}$
$\Delta_{25,4,1}(2z)$	$-\frac{11}{21}$	$\frac{17}{7}$	$\frac{1}{4}$	$\frac{5}{7}$	$-\frac{2}{21}$	$\frac{13}{42}$	$-\frac{11}{21}$	$\frac{37}{42}$	$-\frac{13}{12}$	$-\frac{197}{210}$	$-\frac{1}{42}$
$\Delta_{25,4,1}(4z)$	$\frac{8}{3}$	0	$\frac{24}{7}$	$\frac{80}{21}$	$\frac{24}{7}$	$\frac{8}{3}$	$\frac{32}{7}$	$-\frac{8}{7}$	$\frac{136}{21}$	$\frac{128}{21}$	$\frac{40}{21}$
$\Delta_{25,4,2}(z)$	$-\frac{11}{42}$	$\frac{1}{6}$	$-\frac{11}{42}$	0	$-\frac{3}{14}$	$-\frac{16}{105}$	$-\frac{1}{6}$	$-\frac{1}{14}$	$-\frac{5}{42}$	$-\frac{1}{210}$	$-\frac{5}{42}$
$\Delta_{25,4,2}(2z)$	$\frac{4}{21}$	$-\frac{8}{3}$	$-\frac{59}{42}$	$-\frac{4}{7}$	$-\frac{19}{21}$	$-\frac{251}{210}$	$-\frac{11}{21}$	$-\frac{23}{42}$	$-\frac{1}{14}$	$\frac{1}{42}$	$-\frac{1}{2}$
$\Delta_{25,4,2}(4z)$	$-\frac{152}{21}$	0	$-\frac{200}{21}$	$-\frac{64}{21}$	$-\frac{104}{21}$	$-\frac{376}{105}$	$-\frac{32}{7}$	$-\frac{8}{3}$	$-\frac{136}{21}$	$-\frac{352}{105}$	$-\frac{24}{7}$

	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}	A_{17}	A_{18}	A_{19}	A_{20}	A_{21}	A_{22}
$\Delta_{5,4}$	0	$\frac{23}{120}$	$\frac{19}{120}$	$\frac{2}{15}$	$\frac{1}{20}$	$\frac{1}{40}$	$\frac{1}{60}$	$-\frac{1}{30}$	$\frac{13}{120}$	$\frac{3}{16}$	$\frac{13}{120}$
$\Delta_{5,4}(2z)$	$\frac{19}{60}$	$\frac{16}{15}$	$\frac{16}{15}$	$\frac{14}{15}$	$\frac{2}{15}$	$\frac{4}{15}$	$\frac{13}{30}$	$-\frac{7}{40}$	$\frac{49}{60}$	$\frac{31}{24}$	$\frac{13}{20}$
$\Delta_{5,4}(4z)$	$\frac{12}{5}$	$\frac{275}{24}$	$\frac{43}{15}$	$\frac{32}{15}$	0	$\frac{5}{3}$	3	$\frac{2}{5}$	0	$\frac{10}{3}$	$\frac{5}{3}$
$\Delta_{5,4}(5z)$	$\frac{5}{2}$	$\frac{250}{3}$	$\frac{175}{24}$	$\frac{20}{3}$	$\frac{15}{4}$	$\frac{5}{8}$	$\frac{25}{12}$	$\frac{25}{3}$	$\frac{25}{24}$	$\frac{175}{16}$	$\frac{265}{24}$
$\Delta_{5,4}(10z)$	$\frac{235}{12}$	0	$\frac{175}{3}$	$\frac{140}{3}$	$\frac{110}{3}$	$-\frac{5}{3}$	$\frac{175}{6}$	$\frac{525}{8}$	$\frac{325}{12}$	$\frac{1775}{24}$	$\frac{265}{4}$
$\Delta_{5,4}(20z)$	1400	0	$\frac{475}{3}$	$\frac{320}{3}$	160	$\frac{25}{3}$	175	250	0	$\frac{650}{3}$	$\frac{625}{3}$
$\Delta_{10,4}$	$-\frac{7}{30}$	$\frac{7}{120}$	$\frac{23}{360}$	$\frac{4}{45}$	$\frac{11}{180}$	$-\frac{11}{360}$	$-\frac{1}{60}$	$\frac{19}{720}$	$\frac{1}{10}$	$\frac{1}{48}$	$\frac{1}{90}$
$\Delta_{10,4}(2z)$	$-\frac{35}{6}$	0	$-\frac{41}{180}$	$-\frac{11}{45}$	$-\frac{2}{9}$	$\frac{1}{36}$	$-\frac{1}{12}$	$-\frac{23}{45}$	0	0	$-\frac{5}{36}$
$\Delta_{10,4}(5z)$	$\frac{215}{6}$	$\frac{125}{24}$	$-\frac{275}{72}$	$-\frac{40}{9}$	$-\frac{65}{36}$	$-\frac{145}{72}$	$-\frac{25}{4}$	$-\frac{775}{144}$	0	$-\frac{25}{48}$	$\frac{10}{9}$
$\Delta_{10,4}(10z)$	$\frac{1}{24}$	0	$\frac{1625}{36}$	$\frac{335}{9}$	$\frac{190}{9}$	$\frac{475}{36}$	$\frac{325}{12}$	$\frac{275}{9}$	0	$\frac{100}{3}$	$\frac{625}{36}$
$\Delta_{20,4}(z)$	$\frac{25}{8}$	0	$\frac{1}{36}$	$\frac{1}{36}$	$-\frac{1}{9}$	$\frac{1}{18}$	0	$-\frac{23}{144}$	0	0	$-\frac{5}{72}$
$\Delta_{20,4}(5z)$	$\frac{1}{14}$	0	$\frac{175}{36}$	$\frac{175}{36}$	$\frac{50}{9}$	$\frac{25}{18}$	$\frac{25}{3}$	$\frac{1375}{144}$	0	$\frac{125}{12}$	$\frac{625}{72}$
$\Delta_{25,4,1}(z)$	$-\frac{17}{42}$	$-\frac{4}{21}$	$\frac{1}{21}$	0	$\frac{1}{7}$	$\frac{2}{35}$	$\frac{1}{7}$	$\frac{1}{6}$	$-\frac{5}{21}$	$\frac{19}{84}$	$\frac{19}{70}$
$\Delta_{25,4,1}(2z)$	$\frac{40}{21}$	$\frac{40}{21}$	$\frac{4}{21}$	$\frac{5}{21}$	$-\frac{17}{42}$	$-\frac{1}{7}$	$-\frac{13}{14}$	$-\frac{1}{14}$	$\frac{5}{7}$	$\frac{23}{84}$	$\frac{37}{210}$
$\Delta_{25,4,1}(4z)$	$-\frac{3}{14}$	0	$\frac{8}{3}$	$\frac{16}{7}$	$\frac{24}{7}$	$\frac{8}{21}$	$\frac{16}{3}$	$\frac{40}{7}$	0	$\frac{136}{21}$	$\frac{128}{21}$
$\Delta_{25,4,2}(z)$	$\frac{5}{42}$	$\frac{4}{21}$	$-\frac{4}{21}$	$-\frac{4}{21}$	$-\frac{1}{7}$	$-\frac{3}{35}$	$-\frac{2}{21}$	$-\frac{1}{6}$	$\frac{2}{21}$	$-\frac{2}{7}$	$-\frac{43}{210}$
$\Delta_{25,4,2}(2z)$	$-\frac{40}{7}$	$-\frac{44}{21}$	$-\frac{23}{21}$	$-\frac{19}{21}$	$-\frac{17}{42}$	$-\frac{4}{105}$	$\frac{1}{42}$	$-\frac{1}{14}$	$-\frac{8}{21}$	$-\frac{31}{42}$	$-\frac{31}{70}$
$\Delta_{25,4,2}(4z)$	$\frac{2}{5}$	0	$-\frac{152}{21}$	$-\frac{16}{3}$	$-\frac{24}{7}$	$-\frac{232}{105}$	$-\frac{32}{7}$	$-\frac{40}{7}$	0	$-\frac{136}{21}$	$-\frac{352}{105}$

	A_{23}	A_{24}	A_{25}	A_{26}	A_{27}	A_{28}	A_{29}	A_{30}	A_{31}	A_{32}	A_{33}
$\Delta_{5,4}$	$\frac{1}{120}$	$\frac{1}{6}$	$\frac{2}{15}$	$\frac{41}{240}$	$\frac{1}{3}$	$-\frac{1}{6}$	$\frac{3}{20}$	$-\frac{1}{24}$	$-\frac{7}{240}$	0	$\frac{7}{240}$
$\Delta_{5,4}(2z)$	$-\frac{1}{15}$	$\frac{19}{15}$	$\frac{5}{6}$	$\frac{29}{30}$	$\frac{19}{6}$	$-\frac{29}{30}$	$\frac{53}{60}$	$-\frac{1}{12}$	$-\frac{17}{240}$	$\frac{1}{20}$	$\frac{1}{10}$
$\Delta_{5,4}(4z)$	$\frac{1}{5}$	$\frac{44}{15}$	$-\frac{32}{15}$	$\frac{46}{15}$	$\frac{16}{3}$	$-\frac{32}{15}$	$\frac{12}{5}$	0	0	$\frac{2}{15}$	$\frac{7}{15}$
$\Delta_{5,4}(5z)$	$\frac{13}{24}$	$\frac{25}{3}$	$\frac{25}{6}$	$\frac{425}{48}$	$\frac{175}{6}$	$-\frac{10}{3}$	$\frac{25}{4}$	$\frac{175}{24}$	$\frac{425}{48}$	$-\frac{5}{2}$	$-\frac{65}{48}$
$\Delta_{5,4}(10z)$	$\frac{2}{3}$	$\frac{175}{3}$	$\frac{145}{6}$	$\frac{175}{3}$	$\frac{1175}{6}$	$-\frac{125}{6}$	$\frac{605}{12}$	$\frac{625}{12}$	$\frac{3175}{48}$	$-\frac{45}{4}$	-10
$\Delta_{5,4}(20z)$	17	$\frac{500}{3}$	$\frac{160}{3}$	$\frac{550}{3}$	$\frac{2000}{3}$	$\frac{160}{3}$	140	200	250	$-\frac{70}{3}$	$-\frac{65}{3}$
$\Delta_{10,4}$	$-\frac{11}{360}$	$\frac{1}{24}$	$-\frac{17}{90}$	$\frac{1}{60}$	$-\frac{1}{18}$	$-\frac{2}{9}$	$\frac{7}{120}$	$\frac{1}{72}$	$\frac{3}{160}$	$-\frac{1}{72}$	$-\frac{11}{720}$
$\Delta_{10,4}(2z)$	$\frac{11}{180}$	$-\frac{1}{10}$	$\frac{41}{45}$	$-\frac{3}{20}$	$\frac{7}{9}$	$\frac{71}{45}$	$-\frac{7}{30}$	$-\frac{5}{18}$	$-\frac{5}{12}$	$-\frac{1}{45}$	$\frac{1}{180}$
$\Delta_{10,4}(5z)$	$-\frac{1}{72}$	$-\frac{25}{24}$	$-\frac{50}{9}$	0	$-\frac{200}{9}$	$-\frac{85}{18}$	$-\frac{25}{8}$	$-\frac{275}{72}$	$-\frac{425}{96}$	$\frac{95}{72}$	$-\frac{145}{144}$
$\Delta_{10,4}(10z)$	$\frac{37}{36}$	$\frac{175}{6}$	$\frac{355}{9}$	$\frac{325}{12}$	$\frac{1225}{9}$	$\frac{205}{9}$	$\frac{215}{6}$	$\frac{475}{18}$	$\frac{125}{4}$	$-\frac{35}{9}$	$\frac{95}{36}$
$\Delta_{20,4}(z)$	$-\frac{1}{90}$	0	$\frac{11}{36}$	$\frac{1}{48}$	$\frac{5}{36}$	$\frac{5}{36}$	$\frac{1}{24}$	$-\frac{5}{36}$	$-\frac{5}{32}$	$-\frac{1}{36}$	$-\frac{1}{72}$
$\Delta_{20,4}(5z)$	$-\frac{5}{18}$	$\frac{25}{3}$	$\frac{125}{36}$	$\frac{125}{16}$	$\frac{875}{36}$	$\frac{275}{36}$	$\frac{25}{8}$	$\frac{125}{18}$	$\frac{875}{96}$	$\frac{25}{18}$	$-\frac{25}{72}$
$\Delta_{25,4,1}(z)$	$\frac{1}{15}$	$\frac{4}{21}$	$-\frac{2}{21}$	$\frac{19}{84}$	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{1}{21}$	$\frac{17}{168}$	$\frac{13}{84}$	$-\frac{3}{56}$	$-\frac{1}{168}$
$\Delta_{25,4,1}(2z)$	$-\frac{11}{105}$	$\frac{3}{7}$	$\frac{5}{6}$	$\frac{5}{28}$	$\frac{4}{7}$	$-\frac{5}{6}$	$\frac{1}{6}$	$-\frac{25}{168}$	$-\frac{17}{84}$	$-\frac{1}{56}$	$-\frac{23}{168}$
$\Delta_{25,4,1}(4z)$	$\frac{8}{21}$	$\frac{16}{3}$	$-\frac{8}{7}$	$\frac{100}{21}$	$\frac{256}{21}$	$\frac{8}{7}$	$\frac{40}{21}$	$\frac{30}{7}$	$\frac{40}{7}$	$-\frac{8}{21}$	$\frac{8}{21}$
$\Delta_{25,4,2}(z)$	$-\frac{2}{105}$	$-\frac{5}{21}$	$-\frac{10}{21}$	$-\frac{13}{84}$	$-\frac{25}{21}$	$-\frac{10}{21}$	$-\frac{1}{7}$	$-\frac{17}{168}$	$-\frac{13}{84}$	$\frac{3}{56}$	$-\frac{1}{24}$
$\Delta_{25,4,2}(2z)$	$-\frac{2}{105}$	$-\frac{8}{21}$	$\frac{1}{42}$	$-\frac{59}{84}$	$-\frac{20}{21}$	$\frac{1}{42}$	$-\frac{5}{6}$	$-\frac{25}{168}$	$-\frac{17}{84}$	$-\frac{1}{56}$	$\frac{3}{56}$
$\Delta_{25,4,2}(4z)$	$-\frac{24}{35}$	$-\frac{16}{3}$	$-\frac{248}{21}$	$-\frac{92}{21}$	$-\frac{640}{21}$	$-\frac{248}{21}$	$-\frac{40}{7}$	$-\frac{30}{7}$	$-\frac{40}{7}$	$\frac{8}{21}$	$-\frac{8}{21}$

	A_{34}	A_{35}	A_{36}
$\Delta_{5,4}$	$-\frac{1}{30}$	$-\frac{1}{5}$	$\frac{1}{48}$
$\Delta_{5,4}(2z)$	$-\frac{3}{10}$	$-\frac{4}{5}$	$\frac{11}{30}$
$\Delta_{5,4}(4z)$	$-\frac{4}{15}$	$-\frac{8}{5}$	$\frac{43}{15}$
$\Delta_{5,4}(5z)$	$\frac{35}{6}$	-15	$\frac{125}{48}$
$\Delta_{5,4}(10z)$	$\frac{65}{2}$	-90	$\frac{125}{6}$
$\Delta_{5,4}(20z)$	$\frac{380}{3}$	-200	$\frac{475}{3}$
$\Delta_{10,4}$	$\frac{1}{18}$	0	$-\frac{1}{144}$
$\Delta_{10,4}(2z)$	$-\frac{2}{45}$	$-\frac{2}{5}$	$-\frac{41}{180}$
$\Delta_{10,4}(5z)$	$-\frac{5}{18}$	$\frac{20}{3}$	$-\frac{1075}{144}$
$\Delta_{10,4}(10z)$	$\frac{110}{9}$	0	$\frac{1625}{36}$
$\Delta_{20,4}(z)$	$\frac{1}{36}$	$\frac{50}{3}$	$\frac{1}{36}$
$\Delta_{20,4}(5z)$	$\frac{175}{36}$	$\frac{25}{3}$	$\frac{175}{36}$
$\Delta_{25,4,1}(z)$	$-\frac{4}{105}$	$-\frac{8}{35}$	$\frac{5}{42}$
$\Delta_{25,4,1}(2z)$	$\frac{52}{105}$	$-\frac{8}{35}$	$-\frac{11}{21}$
$\Delta_{25,4,1}(4z)$	$-\frac{64}{105}$	$-\frac{32}{21}$	$\frac{8}{3}$
$\Delta_{25,4,2}(z)$	$\frac{4}{21}$	$\frac{8}{35}$	$-\frac{11}{42}$
$\Delta_{25,4,2}(2z)$	$\frac{8}{35}$	$\frac{8}{5}$	$\frac{4}{21}$
$\Delta_{25,4,2}(4z)$	$\frac{256}{105}$	$\frac{96}{35}$	$-\frac{152}{21}$

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	
$\Delta_{25,4,3}(z)$	$\frac{23}{48}$	$-\frac{23}{24}$	$\frac{1}{16}$	$-\frac{7}{30}$	$\frac{5}{48}$	$-\frac{1}{15}$	$\frac{5}{24}$	$-\frac{5}{24}$	
$\Delta_{25,4,3}(2z)$	$-\frac{17}{6}$	$\frac{13}{4}$	$-\frac{13}{8}$	$\frac{3}{10}$	$-\frac{13}{12}$	$-\frac{11}{30}$	$-\frac{11}{6}$	$\frac{7}{12}$	
$\Delta_{25,4,3}(4z)$	$\frac{25}{3}$	0	0	$\frac{104}{15}$	9	$\frac{104}{15}$	8	0	
$\Delta_{50,4,1}(z)$	$-\frac{11}{48}$	$\frac{1}{2}$	$-\frac{1}{16}$	$\frac{1}{3}$	$-\frac{1}{48}$	$\frac{1}{15}$	$-\frac{5}{24}$	$\frac{1}{6}$	
$\Delta_{50,4,1}(2z)$	$\frac{13}{12}$	0	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{17}{12}$	$\frac{19}{15}$	$\frac{5}{3}$	$-\frac{1}{3}$	
$\Delta_{50,4,2}(z)$	$-\frac{8}{63}$	$\frac{1}{3}$	$-\frac{11}{252}$	$\frac{2}{9}$	$-\frac{4}{63}$	$-\frac{1}{315}$	$-\frac{1}{6}$	$\frac{8}{63}$	
$\Delta_{50,4,2}(2z)$	$\frac{40}{63}$	0	$\frac{46}{63}$	$\frac{92}{63}$	$\frac{62}{63}$	$\frac{236}{315}$	$\frac{6}{7}$	$-\frac{4}{9}$	
$\Delta_{50,4,3}(z)$	$-\frac{1}{48}$	-1	$-\frac{9}{16}$	$-\frac{1}{3}$	$-\frac{19}{48}$	$-\frac{7}{15}$	$-\frac{5}{24}$	$-\frac{1}{6}$	
$\Delta_{50,4,3}(2z)$	$-\frac{23}{12}$	0	$-\frac{5}{2}$	$-\frac{2}{3}$	$-\frac{23}{12}$	$-\frac{23}{15}$	$-\frac{5}{3}$	$-\frac{1}{3}$	
$\Delta_{50,4,4}(z)$	$\frac{1}{21}$	$\frac{10}{21}$	$\frac{89}{252}$	$\frac{16}{63}$	$\frac{5}{21}$	$\frac{17}{63}$	$\frac{1}{6}$	$\frac{1}{9}$	$-$
$\Delta_{50,4,4}(2z)$	$\frac{4}{3}$	0	$\frac{100}{63}$	$-\frac{4}{63}$	$\frac{10}{21}$	$\frac{4}{9}$	$\frac{6}{7}$	$\frac{32}{63}$	
$\Delta_{50,4,5}(z)$	$-\frac{1}{48}$	$\frac{13}{24}$	$\frac{25}{144}$	$\frac{13}{45}$	$\frac{5}{48}$	$\frac{8}{45}$	$\frac{1}{12}$	$\frac{7}{72}$	
$\Delta_{50,4,5}(2z)$	$\frac{5}{12}$	0	$\frac{5}{9}$	$\frac{62}{45}$	$\frac{13}{12}$	$\frac{37}{45}$	1	$-\frac{5}{9}$	
$\Delta_{100,4,1}(z)$	$\frac{2}{9}$	0	$\frac{11}{36}$	$\frac{1}{9}$	$\frac{5}{18}$	$\frac{23}{90}$	$\frac{1}{3}$	$-\frac{1}{18}$	
$\Delta_{100,4,2}(z)$	$-\frac{1}{3}$	0	$-\frac{7}{18}$	$-\frac{4}{9}$	$-\frac{1}{2}$	$-\frac{7}{18}$	$-\frac{1}{3}$	$\frac{1}{18}$	
$\Delta_{100,4,3}(z)$	$-\frac{5}{12}$	0	$-\frac{35}{72}$	$-\frac{1}{18}$	$-\frac{5}{24}$	$-\frac{7}{36}$	$-\frac{7}{24}$	$-\frac{5}{36}$	
$\Delta_{100,4,4}(z)$	$\frac{1}{4}$ $-\frac{7}{228}\sqrt{19}$	0	$\frac{5}{16}$ $-\frac{13}{304}\sqrt{19}$	$\frac{5}{228}\sqrt{19}$ $+\frac{1}{4}$	$\frac{5}{24}$	$\frac{1}{570}\sqrt{19}$ $+\frac{3}{20}$	$\frac{5}{24}$	$-\frac{2}{57}\sqrt{19}$	
$\Delta_{100,4,5}(z)$	$\frac{7}{228}\sqrt{19}$ $\frac{1}{4}$	0	$\frac{13}{304}\sqrt{19}$ $\frac{5}{16}$	$-\frac{5}{228}\sqrt{19}$ $\frac{1}{4}$	$\frac{5}{24}$	$-\frac{1}{570}\sqrt{19}$ $\frac{3}{20}$	$\frac{5}{24}$	$\frac{2}{57}\sqrt{19}$	

	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}
$\Delta_{25,4,3}(z)$	$\frac{5}{16}$	$\frac{53}{240}$	$-\frac{1}{24}$	$-\frac{139}{60}$	$-\frac{19}{24}$	$\frac{1}{24}$	0	$\frac{3}{20}$
$\Delta_{25,4,3}(2z)$	$-\frac{73}{24}$	$-\frac{31}{12}$	$-\frac{7}{12}$	$\frac{20}{3}$	$\frac{8}{3}$	$-\frac{4}{3}$	$-\frac{16}{15}$	$-\frac{22}{15}$
$\Delta_{25,4,3}(4z)$	$\frac{26}{3}$	$\frac{107}{15}$	$\frac{20}{3}$	$-\frac{1}{6}$	0	$\frac{25}{3}$	$\frac{32}{5}$	$\frac{32}{5}$
$\Delta_{50,4,1}(z)$	$-\frac{25}{48}$	$-\frac{119}{240}$	$\frac{1}{24}$	$\frac{5}{6}$	$\frac{3}{8}$	1	0	$-\frac{1}{6}$
$\Delta_{50,4,1}(2z)$	$\frac{13}{6}$	$\frac{23}{12}$	$\frac{7}{6}$	$-\frac{5}{42}$	0	$\frac{13}{12}$	1	$\frac{4}{3}$
$\Delta_{50,4,2}(z)$	$-\frac{25}{84}$	$-\frac{86}{315}$	$\frac{1}{126}$	$\frac{4}{7}$	$\frac{13}{42}$	$-\frac{2}{63}$	$-\frac{2}{63}$	$-\frac{17}{126}$
$\Delta_{50,4,2}(2z)$	$\frac{22}{21}$	$\frac{62}{63}$	$\frac{44}{63}$	0	0	$\frac{40}{63}$	$\frac{4}{9}$	$\frac{44}{63}$
$\Delta_{50,4,3}(z)$	$-\frac{5}{48}$	$-\frac{7}{80}$	$-\frac{5}{24}$	$-\frac{3}{2}$	$-\frac{7}{8}$	$-\frac{11}{24}$	$-\frac{1}{3}$	$-\frac{1}{6}$
$\Delta_{50,4,3}(2z)$	$-\frac{11}{6}$	$-\frac{23}{20}$	$-\frac{7}{6}$	0	0	$-\frac{23}{12}$	$-\frac{5}{3}$	$-\frac{4}{3}$
$\Delta_{50,4,4}(z)$	$\frac{25}{252}$	$\frac{5}{63}$	$\frac{1}{6}$	$\frac{5}{126}$	$\frac{5}{14}$	$\frac{2}{7}$	$\frac{2}{9}$	$\frac{17}{126}$
$\Delta_{50,4,4}(2z)$	$\frac{80}{63}$	$\frac{58}{63}$	$\frac{4}{21}$	$\frac{68}{63}$	0	$\frac{4}{3}$	$\frac{76}{63}$	$\frac{44}{63}$
$\Delta_{50,4,5}(z)$	$\frac{5}{144}$	$\frac{71}{720}$	$\frac{1}{24}$	$-\frac{1}{90}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{13}{180}$
$\Delta_{50,4,5}(2z)$	$\frac{13}{9}$	$\frac{259}{180}$	$\frac{5}{6}$	$\frac{5}{18}$	0	$\frac{5}{12}$	$\frac{13}{45}$	$\frac{38}{45}$
$\Delta_{100,4,1}(z)$	$\frac{5}{12}$	$\frac{17}{45}$	$\frac{5}{18}$	$\frac{1}{6}$	0	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{5}{18}$
$\Delta_{100,4,2}(z)$	$-\frac{5}{18}$	$-\frac{2}{9}$	$-\frac{1}{2}$	$-\frac{5}{18}$	0	$-\frac{1}{3}$	$-\frac{2}{9}$	$-\frac{5}{18}$
$\Delta_{100,4,3}(z)$	$-\frac{25}{72}$	$-\frac{17}{72}$	$-\frac{5}{24}$	$-\frac{25}{72}$	0	$-\frac{5}{12}$	$-\frac{13}{36}$	$-\frac{2}{9}$
$\Delta_{100,4,4}(z)$	$\frac{5}{16}$	$\frac{4}{285}\sqrt{19}$	$\frac{5}{24}$	$\frac{5}{24}$	0	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{6}$
	$-\frac{5}{912}\sqrt{19}$	$\frac{29}{120}$		$-\frac{5}{228}\sqrt{19}$		$-\frac{7}{228}\sqrt{19}$	$-\frac{5}{228}\sqrt{19}$	
$\Delta_{100,4,5}(z)$	$\frac{5}{16}$	$-\frac{4}{285}\sqrt{19}$	$\frac{5}{24}$	$\frac{5}{24}$	0	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{6}$
	$\frac{5}{912}\sqrt{19}$	$\frac{29}{120}$		$\frac{5}{228}\sqrt{19}$		$\frac{7}{228}\sqrt{19}$	$\frac{5}{228}\sqrt{19}$	

	A_{17}	A_{18}	A_{19}	A_{20}	A_{21}	A_{22}	A_{23}	A_{24}
$\Delta_{25,4,3}(z)$	$\frac{3}{40}$	$\frac{1}{4}$	$\frac{1}{3}$	$-\frac{5}{24}$	$\frac{11}{48}$	$\frac{9}{40}$	$\frac{7}{120}$	$\frac{1}{6}$
$\Delta_{25,4,3}(2z)$	$-\frac{2}{5}$	$-\frac{5}{2}$	$-\frac{21}{8}$	$\frac{1}{4}$	$-\frac{31}{24}$	$-\frac{59}{60}$	$-\frac{1}{15}$	-1
$\Delta_{25,4,3}(4z)$	$\frac{23}{15}$	$\frac{23}{3}$	10	0	$\frac{26}{3}$	$\frac{107}{15}$	$\frac{1}{3}$	$\frac{20}{3}$
$\Delta_{50,4,1}(z)$	$-\frac{1}{120}$	$-\frac{5}{12}$	$-\frac{13}{48}$	0	$-\frac{1}{48}$	$\frac{1}{60}$	$-\frac{1}{40}$	$-\frac{1}{24}$
$\Delta_{50,4,1}(2z)$	$\frac{7}{60}$	$\frac{19}{12}$	$\frac{13}{6}$	0	$\frac{13}{6}$	$\frac{23}{12}$	$-\frac{1}{20}$	$\frac{11}{6}$
$\Delta_{50,4,2}(z)$	$-\frac{8}{315}$	$-\frac{5}{21}$	$-\frac{1}{18}$	$\frac{1}{14}$	$\frac{1}{28}$	$\frac{17}{630}$	$-\frac{8}{315}$	$\frac{1}{14}$
$\Delta_{50,4,2}(2z)$	$\frac{22}{315}$	$\frac{6}{7}$	$\frac{64}{63}$	0	$\frac{22}{21}$	$\frac{62}{63}$	$\frac{16}{315}$	$\frac{2}{3}$
$\Delta_{50,4,3}(z)$	$\frac{1}{120}$	$-\frac{1}{12}$	$-\frac{13}{48}$	$-\frac{1}{4}$	$-\frac{17}{48}$	$-\frac{1}{5}$	$-\frac{1}{120}$	$-\frac{5}{24}$
$\Delta_{50,4,3}(2z)$	$-\frac{29}{60}$	$-\frac{17}{12}$	$-\frac{13}{6}$	0	$-\frac{11}{6}$	$-\frac{23}{20}$	$-\frac{7}{60}$	$-\frac{7}{6}$
$\Delta_{50,4,4}(z)$	$\frac{2}{63}$	$\frac{5}{63}$	$\frac{1}{18}$	$\frac{1}{14}$	$\frac{55}{252}$	$\frac{19}{126}$	0	$\frac{25}{126}$
$\Delta_{50,4,4}(2z)$	$\frac{22}{63}$	$\frac{10}{9}$	$\frac{64}{63}$	0	$\frac{80}{63}$	$\frac{58}{63}$	$\frac{4}{105}$	$\frac{10}{9}$
$\Delta_{50,4,5}(z)$	$-\frac{17}{360}$	$\frac{1}{36}$	$\frac{31}{144}$	$\frac{1}{4}$	$\frac{47}{144}$	$\frac{47}{180}$	$-\frac{1}{40}$	$\frac{23}{72}$
$\Delta_{50,4,5}(2z)$	$\frac{1}{180}$	$\frac{41}{36}$	$\frac{11}{9}$	0	$\frac{13}{9}$	$\frac{259}{180}$	$\frac{1}{12}$	$\frac{25}{18}$
$\Delta_{100,4,1}(z)$	$\frac{1}{90}$	$\frac{1}{3}$	$\frac{7}{18}$	0	$\frac{5}{12}$	$\frac{17}{45}$	$-\frac{1}{45}$	$\frac{1}{3}$
$\Delta_{100,4,2}(z)$	$-\frac{1}{18}$	$-\frac{2}{9}$	$-\frac{7}{18}$	0	$-\frac{5}{18}$	$-\frac{2}{9}$	0	$-\frac{2}{9}$
$\Delta_{100,4,3}(z)$	$-\frac{1}{9}$	$-\frac{5}{18}$	$-\frac{55}{144}$	0	$-\frac{25}{72}$	$-\frac{17}{72}$	0	$-\frac{5}{18}$
$\Delta_{100,4,4}(z)$	$\frac{1}{20}$ $-\frac{4}{285}\sqrt{19}$	$\frac{1}{4}$ $-\frac{1}{228}\sqrt{19}$	$\frac{13}{48}$	0	$\frac{5}{16}$ $-\frac{5}{912}\sqrt{19}$	$\frac{4}{285}\sqrt{19}$ $+\frac{29}{120}$	$-\frac{1}{285}\sqrt{19}$ $\frac{19}{1140}$	$\frac{1}{4}$ $-\frac{1}{228}\sqrt{19}$
$\Delta_{100,4,5}(z)$	$\frac{1}{20}$ $\frac{4}{285}\sqrt{19}$	$\frac{1}{4}$ $\frac{1}{228}\sqrt{19}$	$\frac{13}{48}$	0	$\frac{5}{16}$ $+\frac{5}{912}\sqrt{19}$	$-\frac{2}{285}2\sqrt{19}$ $+\frac{29}{120}$	$\frac{1}{285}\sqrt{19}$ $\frac{19}{1140}$	$\frac{1}{4}$ $+\frac{1}{228}\sqrt{19}$

	A_{25}	A_{26}	A_{27}	A_{28}	A_{29}	A_{30}
$\Delta_{25,4,3}(z)$	$-\frac{1}{3}$	$\frac{11}{48}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{60}$	$\frac{7}{24}$
$\Delta_{25,4,3}(2z)$	$\frac{7}{6}$	$-\frac{3}{2}$	$-\frac{13}{2}$	$-\frac{7}{6}$	$-\frac{61}{60}$	$-\frac{25}{12}$
$\Delta_{25,4,3}(4z)$	0	$\frac{26}{3}$	$\frac{80}{3}$	0	$\frac{20}{3}$	8
$\Delta_{50,4,1}(z)$	$-\frac{1}{3}$	$\frac{1}{24}$	$-\frac{7}{12}$	$-\frac{1}{3}$	$-\frac{1}{24}$	$-\frac{5}{24}$
$\Delta_{50,4,1}(2z)$	$\frac{5}{3}$	$\frac{17}{12}$	$\frac{17}{3}$	$\frac{5}{3}$	$\frac{5}{6}$	$\frac{5}{3}$
$\Delta_{50,4,2}(z)$	$-\frac{19}{63}$	$-\frac{1}{84}$	$-\frac{37}{63}$	$-\frac{19}{63}$	$-\frac{1}{42}$	$-\frac{5}{63}$
$\Delta_{50,4,2}(2z)$	$\frac{80}{63}$	$\frac{19}{21}$	$\frac{248}{63}$	$\frac{80}{63}$	$\frac{4}{7}$	$\frac{55}{63}$
$\Delta_{50,4,3}(z)$	0	$-\frac{7}{24}$	$-\frac{5}{12}$	0	$-\frac{3}{8}$	$-\frac{5}{24}$
$\Delta_{50,4,3}(2z)$	-3	$-\frac{19}{12}$	$-\frac{25}{3}$	-3	$-\frac{3}{2}$	$-\frac{5}{3}$
$\Delta_{50,4,4}(z)$	$\frac{13}{63}$	$\frac{55}{252}$	$\frac{25}{63}$	$-\frac{13}{63}$	$\frac{29}{126}$	$\frac{5}{63}$
$\Delta_{50,4,4}(2z)$	$\frac{32}{63}$	$\frac{65}{63}$	$\frac{200}{63}$	$-\frac{32}{63}$	$\frac{68}{63}$	$\frac{55}{63}$
$\Delta_{50,4,5}(z)$	$\frac{1}{18}$	$\frac{2}{9}$	$\frac{13}{18}$	$-\frac{1}{18}$	$\frac{29}{360}$	$\frac{11}{72}$
$\Delta_{50,4,5}(2z)$	$-\frac{5}{9}$	$\frac{37}{36}$	$\frac{25}{9}$	$\frac{5}{9}$	$\frac{5}{18}$	$\frac{19}{18}$
$\Delta_{100,4,1}(z)$	$\frac{5}{18}$	$\frac{1}{3}$	$\frac{10}{9}$	$\frac{5}{18}$	$\frac{1}{6}$	$\frac{25}{72}$
$\Delta_{100,4,2}(z)$	$\frac{1}{18}$	$-\frac{5}{18}$	$-\frac{10}{9}$	$-\frac{1}{18}$	$-\frac{5}{18}$	$-\frac{25}{72}$
$\Delta_{100,4,3}(z)$	$-\frac{5}{36}$	$-\frac{35}{144}$	$-\frac{35}{36}$	$\frac{5}{36}$	$-\frac{25}{72}$	$-\frac{5}{18}$
$\Delta_{100,4,4}(z)$	$\frac{5}{12} -\frac{5}{114}\sqrt{19}$	$\frac{1}{912}\sqrt{19} +\frac{1}{4}$	$\frac{5}{4} -\frac{5}{57}\sqrt{19}$	$\frac{5}{12} -\frac{5}{114}\sqrt{19}$	$\frac{5}{24} -\frac{5}{228}\sqrt{19}$	$\frac{5}{24}$
$\Delta_{100,4,5}(z)$	$\frac{5}{12} +\frac{5}{114}\sqrt{19}$	$-\frac{1}{1824}2\sqrt{19} +\frac{1}{4}$	$\frac{5}{4} +\frac{5}{57}\sqrt{19}$	$\frac{5}{12} \; \frac{5}{114}\sqrt{19}$	$\frac{5}{24} \; \frac{5}{228}\sqrt{19}$	$\frac{5}{24}$

	A_{31}	A_{32}	A_{33}	A_{34}	A_{35}	A_{36}
$\Delta_{25,4,3}(z)$	$\frac{17}{48}$	$-\frac{1}{10}$	$\frac{1}{240}$	$\frac{1}{6}$	$-\frac{3}{5}$	$\frac{23}{48}$
$\Delta_{25,4,3}(2z)$	$-\frac{127}{48}$	$\frac{9}{20}$	$-\frac{1}{15}$	$-\frac{23}{30}$	$\frac{4}{5}$	$-\frac{17}{6}$
$\Delta_{25,4,3}(4z)$	10	$-\frac{14}{15}$	$\frac{1}{3}$	$\frac{4}{15}$	$-\frac{8}{15}$	$\frac{25}{3}$
$\Delta_{50,4,1}(z)$	$-\frac{25}{96}$	$\frac{1}{12}$	$\frac{1}{48}$	$\frac{1}{10}$	$\frac{4}{15}$	$-\frac{11}{48}$
$\Delta_{50,4,1}(2z)$	$\frac{25}{12}$	$-\frac{1}{6}$	$\frac{1}{12}$	$-\frac{2}{5}$	$\frac{2}{3}$	$\frac{13}{12}$
$\Delta_{50,4,2}(z)$	$-\frac{2}{21}$	$\frac{11}{252}$	$\frac{1}{36}$	$-\frac{4}{315}$	$\frac{32}{105}$	$-\frac{8}{63}$
$\Delta_{50,4,2}(2z)$	$\frac{15}{14}$	$-\frac{1}{63}$	$\frac{1}{63}$	$-\frac{4}{315}$	$-\frac{8}{21}$	$\frac{40}{63}$
$\Delta_{50,4,3}(z)$	$-\frac{25}{96}$	$\frac{1}{12}$	$-\frac{1}{48}$	$\frac{1}{6}$	$\frac{8}{15}$	$-\frac{1}{48}$
$\Delta_{50,4,3}(2z)$	$-\frac{25}{12}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{8}{15}$	$\frac{34}{15}$	$-\frac{23}{12}$
$\Delta_{50,4,4}(z)$	$\frac{2}{21}$	$-\frac{11}{252}$	$\frac{5}{252}$	$\frac{8}{63}$	$-\frac{8}{63}$	$\frac{1}{21}$
$\Delta_{50,4,4}(2z)$	$\frac{15}{14}$	$-\frac{1}{63}$	$\frac{1}{63}$	$\frac{4}{63}$	$-\frac{88}{63}$	$\frac{4}{3}$
$\Delta_{50,4,5}(z)$	$\frac{17}{96}$	$-\frac{19}{360}$	$-\frac{13}{720}$	$-\frac{1}{18}$	$-\frac{8}{45}$	$-\frac{1}{48}$
$\Delta_{50,4,5}(2z)$	$\frac{5}{4}$	$-\frac{7}{45}$	$-\frac{7}{36}$	$-\frac{8}{45}$	$-\frac{34}{45}$	$\frac{5}{12}$
$\Delta_{100,4,1}(z)$	$\frac{5}{12}$	$\frac{5}{72}$	$\frac{1}{72}$	$\frac{1}{45}$	$\frac{4}{15}$	$\frac{2}{9}$
$\Delta_{100,4,2}(z)$	$-\frac{5}{12}$	$-\frac{5}{72}$	$-\frac{1}{72}$	$\frac{1}{9}$	$-\frac{4}{9}$	$-\frac{1}{3}$
$\Delta_{100,4,3}(z)$	$-\frac{35}{96}$	$-\frac{1}{18}$	$\frac{1}{72}$	$\frac{5}{36}$	$-\frac{2}{9}$	$-\frac{5}{12}$
$\Delta_{100,4,4}(z)$	$\frac{25}{96}$	$\frac{1}{24}$	$\frac{1}{228}2\sqrt{19}$	$\frac{1}{60}$ $-\frac{11}{570}\sqrt{19}$	$\frac{1}{5}$ $-\frac{1}{285}\sqrt{19}$	$\frac{1}{4}$ $-\frac{7}{228}\sqrt{19}$
$\Delta_{100,4,5}(z)$	$\frac{25}{96}$	$\frac{1}{24}$	$-\frac{1}{228}2\sqrt{19}$	$\frac{1}{60}$ $+\frac{11}{570}\sqrt{19}$	$\frac{1}{5}$ $+\frac{1}{285}\sqrt{19}$	$\frac{1}{4}$ $+\frac{7}{228}\sqrt{19}$

3. Main results

Theorem 3. *The set*

$$\{E_4, E_4(2z), E_4(4z), E_4(5z), E_4(10z), E_4(20z), E_4(25z), E_4(50z), E_4(100z),$$
$$E_4^{\psi,\psi^3}, E_4^{\psi,\psi^3}(2z), E_4^{\psi,\psi^3}(4z), E_4^{\psi^2,\psi^2}, E_4^{\psi^2,\psi^2}(2z), E_4^{\psi^2,\psi^2}(4z), E_4^{\psi^3,\psi}, E_4^{\psi^3,\psi}(2z), E_4^{\psi^3,\psi}(4z)\}$$

is a basis of Eisenstein subspace of $M_4(\Gamma_0(100))$ *and, the set* $\{A_1, A_2, \ldots, A_{36}\}$ *is a basis of* $S_4(\Gamma_0(100))$.

Proof. $M_4(\Gamma_0(100))$ is 54 dimensional and $S_4(\Gamma_0(100))$ is 36 dimensional, see [6] (Chapter 3,

pg.87 and Chapter 5, pg.197). Since $1 = \psi.\psi^3 = \psi^3.\psi = \psi^2.\psi^2$, the first statement is clear. Second statement follows from lemma 1.

Corollary 4. Θ_Q *can be expressed as linear combinations of the basis elements*

$$\{E_4, E_4(2z), E_4(4z), E_4(5z), E_4(10z), E_4(20z), E_4(25z), E_4(50z), E_4(100z),$$
$$E_4^{\psi,\psi^3}, E_4^{\psi,\psi^3}(2z), E_4^{\psi,\psi^3}(4z), E_4^{\psi^2,\psi^2}, E_4^{\psi^2,\psi^2}(2z), E_4^{\psi^2,\psi^2}(4z), E_4^{\psi^3,\psi}, E_4^{\psi^3,\psi}(2z), E_4^{\psi^3,\psi}(4z)$$
$$, A_1, A_2, \ldots, A_{36}\}$$

for Q in the Table 1. *The formulas are given in Table* 2.

□

Theorem 5. *Each* A_i, $i = 1, .., 36$, *can be expressed as linear combinations of newforms and their rescalings as in Table* 3.

Proof. The space $S_4(\Gamma_0(100))$ is generated by

$$\Delta_{5,4}, \Delta_{5,4}(2z), \Delta_{5,4}(4z), \Delta_{5,4}(5z), \Delta_{5,4}(10z), \Delta_{5,4}(20z),$$
$$\Delta_{10,4}, \Delta_{10,4}(2z), \Delta_{10,4}(5z), \Delta_{10,4}(10z),$$

$$\Delta_{20,4}(z), \Delta_{20,4}(5z),$$

$$\Delta_{25,4,1}(z), \Delta_{25,4,1}(2z), \Delta_{25,4,1}(4z),$$

$$\Delta_{25,4,2}(z), \Delta_{25,4,2}(2z), \Delta_{25,4,2}(4z),$$

$$\Delta_{25,4,3}(z), \Delta_{25,4,3}(2z), \Delta_{25,4,3}(4z),$$

$$\Delta_{50,4,1}(z), \Delta_{50,4,1}(2z),$$

$$\Delta_{50,4,2}(z), \Delta_{50,4,2}(2z),$$

$$\Delta_{50,4,3}(z), \Delta_{50,4,3}(2z),$$

$$\Delta_{50,4,4}(z), \Delta_{50,4,4}(2z),$$

$$\Delta_{50,4,5}(z), \Delta_{50,4,5}(2z),$$

$$\Delta_{100,4,1}(z), \Delta_{100,4,2}(z), \Delta_{100,4,3}(z), \Delta_{100,4,4}(z), \Delta_{100,4,5}(z),$$

where the unique newform in $S_4(\Gamma_0(5))$ is $\Delta_{5,4}$, the unique newform in $S_4(\Gamma_0(10))$ is $\Delta_{10,4}$, the unique newform in $S_4(\Gamma_0(20))$ is $\Delta_{20,4}$, the three unique newforms in $S_4(\Gamma_0(25))$ are $\Delta_{25,4,1}, \Delta_{25,4,2}, \Delta_{25,4,3}$,the five unique newforms in $S_4(\Gamma_0(50))$ are $\Delta_{50,4,1}, \Delta_{50,4,2}, \Delta_{50,4,3}, \Delta_{50,4,4}, \Delta_{50,4,5}$ and, the three unique newforms in $S_4(\Gamma_0(100))$ are $\Delta_{100,4,1}, \Delta_{100,4,2}, \Delta_{100,4,3}$.So the result follows. □

CONCLUSION: As a consequence of Table 2, we immediately get formulae for the representation numbers

$$N(1,5^3,25^4;n), N(1^1,5^2,25^5;n), N(1^1,5^4,25^3;n), N(1^1,5^6,25^1;n), N(1^2,5^0,25^6;n),$$

$$N(1^2,5^2,25^4;n), N(1^2,5^4,25^2;n), N(1^2,5^6,25^0;n), N(1^3,5^0,25^5;n), N(1^3,5^2,25^3;n),$$

$$N(1^3,5^4,25^1;n), N(1^4,5^0,25^4;n), N(1^4,5^2,25^2;n), N(1^4,5^4,25^0;n), N(1^5,5^0,25^3;n),$$

$$N(1^5,5^2,25^1;n), N(1^6,5^0,25^2;n), N(1^6,5^2,25^0;n), N(1^7,5^0,25^1;n), N(1^8,5^0,25^0;n).$$

Two examples are given explicitly in the following.

$$N(1,5^3,25^4;n) = \tfrac{1}{1170000}240\sigma_3(n) - \tfrac{1}{585000}240\sigma_3\left(\tfrac{n}{2}\right) + \tfrac{1}{73125}240\sigma_3\left(\tfrac{n}{4}\right)$$
$$-\tfrac{7}{65000}240\sigma_3\left(\tfrac{n}{5}\right) + \tfrac{7}{32500}240\sigma_3\left(\tfrac{n}{10}\right) - \tfrac{14}{8125}240\sigma_3\left(\tfrac{n}{20}\right) + \tfrac{125}{1872}240\sigma_3\left(\tfrac{n}{25}\right)$$

$$-\tfrac{125}{936}240\sigma_3\left(\tfrac{n}{50}\right)+\tfrac{125}{117}240\sigma_3\left(\tfrac{n}{100}\right)+\tfrac{1}{4875}\left(\tfrac{5}{n}\right)\sigma_3(n)+\tfrac{2}{4875}\left(\tfrac{5}{n/2}\right)\sigma_3\left(\tfrac{n}{2}\right)$$
$$+\tfrac{16}{4875}\left(\tfrac{5}{n/4}\right)\sigma_3\left(\tfrac{n}{4}\right)+\tfrac{147\,902}{901\,875}a_1(n)+\tfrac{1413\,414\,629}{9018\,750}a_2(n)+\tfrac{5963}{60\,125}a_3(n)$$
$$+\tfrac{217\,240\,406}{4509\,375}a_4(n)-\tfrac{26\,572\,553}{6012\,500}a_5(n)-\tfrac{17\,071\,921}{360\,750}a_6(n)+\tfrac{375\,513\,917}{3607\,500}a_7(n)$$
$$-\tfrac{37\,134\,731}{4509\,375}a_8(n)-\tfrac{808\,546\,769}{9018\,750}a_9(n)+\tfrac{28\,530\,022}{1503\,125}a_{10}(n)-\tfrac{5342}{13\,875}a_{11}(n)$$
$$-\tfrac{563\,549\,807}{18\,037\,500}a_{12}(n)-\tfrac{1240\,426\,503}{6012\,500}a_{13}(n)-\tfrac{148\,859\,039}{6012\,500}a_{14}(n)+\tfrac{132\,762\,329}{9018\,750}a_{15}(n)$$
$$+\tfrac{9151\,007}{360\,750}a_{16}(n)-\tfrac{428\,046\,227}{3607\,500}a_{17}(n)+\tfrac{21\,519\,683}{1803\,750}a_{18}(n)-\tfrac{83\,402\,908}{4509\,375}a_{19}(n)$$
$$+\tfrac{134\,876}{901\,875}a_{20}(n)+\tfrac{163\,867}{138\,750}a_{21}(n)+\tfrac{49\,384\,056}{1503\,125}a_{22}(n)-\tfrac{37\,584}{1503\,125}a_{23}(n)$$
$$-\tfrac{125\,991}{115\,625}a_{24}(n)-\tfrac{1984}{4875}a_{25}(n)+\tfrac{7936}{8125}a_{26}(n)+\tfrac{160\,256}{24\,375}a_{27}(n)-\tfrac{20\,992}{1625}a_{28}(n)$$
$$-\tfrac{81\,664}{4875}a_{29}(n)-\tfrac{5632}{8125}a_{30}(n)+\tfrac{20\,093\,132}{4509\,375}a_{31}(n)+\tfrac{3142\,283}{346\,875}a_{32}(n)$$
$$-\tfrac{13\,569\,693}{1503\,125}a_{33}(n)-\tfrac{51\,167}{1202\,500}a_{34}(n)+\tfrac{637\,753}{120\,250}a_{35}(n),$$

$$N(1^8,5^0,25^0;n)=\tfrac{1}{15}240\sigma_3(n)-\tfrac{2}{15}240\sigma_3\left(\tfrac{n}{2}\right)+\tfrac{16}{15}240\sigma_3\left(\tfrac{n}{4}\right).$$

Conflict of Interests

The authors declare that there is no conflict of interests.

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