Available online at http://scik.org
J. Math. Comput. Sci. 8 (2018), No. 5, 579-583
https://doi.org/10.28919/jmcs/3791
ISSN: 1927-5307

NOTE ON SOFT FRACTIONAL IDEAL OF RING

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Abstract. In this note we introduce soft fractional ideal of soft rings. Then, we study fractional ideal by applying few basic soft operations.

Keywords: soft fractional ideal; soft rings; soft ring of fractions.

2010 AMS Subject Classification: 08A72, 13A15, 03E72, 13C12.

1. Introduction and Preliminaries

Theory of probability, theory of fuzzy sets [13], theory of intuitionistic fuzzy sets [4], theory of vague sets [7], theory of interval mathematics [8], and theory of rough sets [11] which were considered best mathematical tools for dealing with uncertainties. In [10], Molodtsov showed that to fix uncertainties soft set theory works more efficient than any other tool. In [2] authors discussed soft groups, soft subgroups. In [1] soft rings, soft ideals of soft rings have been introduced, furthermore the authors also introduced idealistic soft rings. For basic terminologies

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Received July 1, 2018

of soft set one may consult [10] and, for soft rings and soft ideals we refer [1]. In the begining we recall few useful definitions and terminologies.

Let R be an integral domain, and K be its field of fractions. R-submodule I of K such that there exists a non-zero $r \in R$ such that $rI \subseteq R$ is said to be a fractional ideal of R. Every integral ideal is a fractional ideal of ring R. This type of ideal has its own importance while study Dedekind domains, valuation, domains etc.

Following [10, definition 2.1] pair (F, E) is called a soft set (over U) if and only if F is a mapping on E into the set of subsets of the set U. Assume that (F, A) and (H, B) are two soft sets over a common universe U . We say that (F, A) is a soft subset of (H, B), if it satisfies: (1) $A \subset B$ and (2) $F(x)$ and $H(x)$ are identical approximations for all $x \in A$[10]. In [1, definition 3.1] authors introduced soft rings i.e., Let (F,A) be a non-null soft set over a ring R. Then (F,A) is called a soft ring over R if $F(x)$ is a subring of R for all $x \in A$. Further in [1, definition 4.1] introduce soft ideal of a soft ring i.e., Let (F, A) is a soft ring over R. A non-null soft set (γ, I) over R is called soft ideal of (F, A), if it satisfies: (1) $I \subset A$ and (2) $\gamma(x)$ is an ideal of $F(x)$ for all $x \in Supp(\gamma, I)$. Throughout this paper E is a set of parameters, $P(R)$ is the power set of R , $\mathbb{Z}$ is the ring of integer numbers.

Definition 1. Let (F, A) and (G, B) be two soft sets over a common universe U, we say that (F, A) is a soft subset of (G, B), if it satisfies: (1) $A \subset B$ and (2) $F(x)$ and $G(x)$ are identical approximations for all $x \in A$ [9, definition 2.3]. We write it $(F, A) \tilde{\subset} (G, B)$

Definition 2. Let (F, A) and (G, B) be two soft sets over a common universe U. The intersection of (F, A) and (G, B) is defined as the soft set (H,C) satisfying the following conditions:

(i) $C = A \cap B$

(ii) For all $x \in C$, $H(x) = F(x)$ or $G(x)$ (while the two sets are the same).

In this case we write $F(A) \tilde{\cap} G(B)$[9, definition 2.12].

Definition 3. Let (F, A) and (G, B) be two soft sets over a common universe U. The bi-intersection of (F, A) and (G, B) is defined as the soft set (H,C) satisfying the following conditions:

(i) $C = A \cap B$

(ii) For all $x \in C$, $H(x) = F(x) \cap G(x)$

In this case we write $H(C) = F(A) \tilde{\sqcap} G(B)$ [9].

Definition 4. Let (F, A) and (G, B) be two soft sets over a common universe U. The union of (F, A) and (G, B) is defined as the soft set (H,C) satisfying the following conditions:

(1) $C = A \cup B$

(2) For all $x \in C$,

$$H(x) = \begin{cases} F(x), & \text{if } x \in A - B \\ G(x), & \text{if } x \in B - A \\ F(x) \cup G(x), & \text{if } x \in A \cap B. \end{cases}$$

In this case we write $H(x) = F(A) \tilde{\cup} G(B)$ [9, definition 2.11].

Definition 5. If (F, A) and (G, B) be two soft sets over a common universe U. Then "(F, A) AND (G, B)" denoted by $F(A) \tilde{\wedge} G(B)$ is defined as $F(A) \tilde{\wedge} G(B) = (H, C)$, where $C = A \times B$ and $H(x, y) = F(x) \cap G(y)$ for all $(x, y) \in C$ [9, definition 2.9].

Definition 6. If (F, A) and (G, B) be two soft sets over a common universe U. Then "(F, A) OR (G, B)" denoted by $F(A) \tilde{\vee} G(B)$ is defined as $F(A) \tilde{\vee} G(B) = (H, C)$, where $C = A \times B$ and $H(x, y) = F(x) \cap G(y)$ for all $(x, y) \in C$ [9, definition 2.10].

Definition 7. Let (F, A) be a soft set. The support of (F,A) i.e., $Supp(F,A) = \{x \in A |\ F(x) \neq \emptyset\}$. A soft set (F, A) is said to be non-null if its support is not equal to empty set [6].

Definition 8. Let (F,A) be a non-null soft set over a ring R. Then (F,A) is called a soft ring over R if $F(x)$ is a subring of R for all $x \in A$ [1, definition 3.1].

Definition 9. Let (F, A) is a soft ring over R, a non-null soft set (γ, I) over R is called soft ideal of (F, A), and is denoted by $(\gamma, I) \tilde{\lhd} (F, A)$ if it satisfies:

(1) $I \subset A$

(2) $\gamma(x)$ is an ideal of $F(x)$ for all $x \in Supp(\gamma, I)$ [1, definition 4.1].

Definition 10. Let (F,A) and (G, B) be non-null soft sets over a ring R. Then (G, B) is called a soft subring of (F, A) if it satisfy the following

(1) $A \subset B$

(2) $G(x)$ is a subring of $F(x)$, for all $x \in Supp(G,B)$ [1, definition 4.1].

Definition 11. Let (F,A) be a non-null soft sets over a ring R. Then (F,A) is called an idealistic soft ring over R, if $F(x)$ is an ideal of R for all $x \in Supp(F,A)$ [1, definition 5.1].

Definition 12. Let M be a left R-module, A be any nonempty set $F : A \to P(M)$ refers to a set-valued function and the pair (F, A) is a soft set over M. Then, (F, A) is said to be a soft module over M if and only if $F(x) < M$ for all $x \in A$ [12].

2. Soft fractional ideal of rings

Fuzzy fractionary ideal has been introduced and discussed in the literature(see[5]). Different types of soft ideals have been also introduced in the literature. Soft substructures of rings, fields and modules have been discussed in the literature [3]. Soft module and submodules have been introduced in the literature [12]. In this section we introduce and discuss about soft fractional ideals of soft rings. Throughout by R we mean an integral domain and K be its field of fraction. We begin with the definition.

Definition 13. Let μ be a soft set over the field K and $\mu_\alpha = \{x \in K : \mu(x) \supseteq \alpha\}$ be a level set for every $\alpha \in P(K)$.

We let χ_A the characteristic function for a subset A of a ring $R \subseteq K$. Let χ_A^α be a soft subset of K such that $\chi_A^\alpha(x) = U$, if $x \in R$, and $\chi_A^\alpha(x) = \alpha$ if $x \in K - R$, where $\alpha \in P(K)$.

A soft subset μ is said to be a soft ideal of a ring R if $\mu(x-y) \supseteq \mu(x) \cap \mu(y)$ and $\mu(xy) \supseteq \mu(x) \cup \mu(y)$. A soft subset of R is said to be an ideal iff $\mu(0) \supseteq \mu(x)$ for every $x \in R$ and μ_α is an ideal for every $\alpha \in P(K)$.

Definition 14. Let R be a ring contained in a field K, and (β, K) be a soft subset over the field K. Then β is said to be soft R-submodule of K if:

(i) $\beta(x-y) \supseteq \beta(x) \cap \beta(y)$

(ii) $\beta(rx) \supseteq \beta(x)$

(iii) $\beta(0) = R$, for every $x, y \in K$, $r \in R$.

For $d \in K$ and $\alpha \in P(K)$, we let d_α denote the soft subset of K, defined by: for every $x \in K$, $d_\alpha(x) = \alpha$ if $x = d$ and $d_\alpha(x) = 0$, otherwise. We call $d_\alpha(x)$ a soft singleton.

Definition 15. A soft R-submodule of K is called a fractionary soft ideal of R ifthere exists $d \in R$; $d \neq 0$, such that $d_R \circ \beta \subseteq \chi_R^\alpha$ for some $\alpha \in K - R$.

Theorem 1. *Let α, β be fractional soft ideals of R. Then $\alpha + \beta$ and $\alpha \circ \beta$ are fractional soft ideals of R.*

Proof. Since α, β are fractional soft ideals of R there exist $0 \neq d, d' \in R$ such that $d_R \circ \alpha \subseteq \chi_R^\alpha$, $d'_R \circ \beta \subseteq \chi_R^\beta$ for some $\alpha, \beta \in R$. Thus $(d'd)_R \circ \alpha = d'_R \circ d \circ \alpha \subseteq d'_R \circ \chi_R^\beta$. Similarly, $(dd')_R \subseteq \chi_R^\alpha$. Hence $(d'd)_R \circ (\alpha + \beta) = (d'd)_R \circ \alpha + (d'd)_R \circ \beta \subseteq \chi_R^\alpha + \chi_R^\beta$. And $(d'd)_R \circ (\alpha \circ \beta) \subseteq \chi_R^\alpha \circ \chi_R^\beta$. Hence, $\alpha + \beta$ and $\alpha \circ \beta$ are fractional soft ideals of R. □

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