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STABILIZATION OF THE MEMRISTOR-BASED HYPERCHAOTIC COMPLEX LÜ SYSTEM IN A FINITE TIME

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Abstract. In this paper, we deal with the finite-time stabilization problem of a memristor-based hyperchaotic complex Lü system. Based on the finite-time stability theory, two control strategies are presented to achieve stabilization of the memristor-based hyperchaotic complex Lü system in a finite time. Two numerical simulations have been conducted, the simulation results demonstrate the validity and feasibility of the theoretical analysis.

Keywords: finite-time; memristor; stabilization; hyperchaotic system; complex Lü system.

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1. Introduction

Hyperchaos [1] is generally characterized as a chaotic attractor with more than one positive Lyapunov exponent and has richer dynamical behaviors than chaos. Compared to chaotic

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systems, hyperchaotic systems have a greater randomness and higher complexity and unpredictability, so they are more suitable and effective for secure communication and digital cryptography. Over the past three decades, hyperchaotic systems have been investigated extensively [2-5].

Memristors are considered to be the fourth fundamental circuit element with the characteristics of nonlinearity, non-volatility, nanoscale, and low power consumption. Increasing attentions are focused on the memristors for their potential applications in programmable logic, signal processing, neural networks, and so on [6]. Moreover, as a novel element, the circuit based on the memristor shares many interesting phenomenon. Recently, the study of memristors, memristor-based circuits and neural networks have become a key research front in mathematics, computer science and engineering [7-10].

The complex systems appear in physics and engineering fields. Since Fowler et al. [11] generalized the real Lorenz model to a complex Lorenz model, which can be used to describe and simulate the physics of a detuned laser and the thermal convection of liquid flows [12-13], many hyperchaotic complex-variable systems have been reported, such as the hyperchaotic complex Chen system [14], hyperchaotic complex Lorenz system [15], memristor-based hyperchaotic complex Lü System [16], etc.

Recently, the theory and methods on stabilization of chaotic or hyperchaotic systems have received a great deal of attention among scientists, and have also been extensively studied due to their potential applications in secure communication, modeling brain activity, chemical reactions, ecological systems, etc. [17-19]. Many approaches on chaos or hyperchaos stabilization have been proposed, such as active control [20], passive control [21], adaptive control [22], backstepping [23], sliding mode control[24] methods.

The existing methods stabilize chaotic or hyperchaotic systems asymptotically, i.e. the system trajectories converge to zero with infinite settling time. However, from the practical engineering point of view, it is more crucial to stabilize chaotic or hyperchaotic systems in a finite time. Therefore, it is important to consider the problem of finite-time stabilization of chaotic or hyperchaotic systems. Finite-time control is a very useful technique to achieve faster convergence speed in control systems. In addition, the finite-time control technique has demonstrated better robustness and disturbance rejection properties [25].

Motivated by the above discussion, in this paper, we propose controllers to stabilize memristor-based hyperchaotic complex Lü system. Based on the finite-time stability theorem, two control strategies are constructed to stabilize the memristor-based hyperchaotic complex Lü system in a finite time. Numerical simulation results verify the effectiveness of the proposed controllers.

2. Preliminary definition and lemmas and system description

Definition 1 [25]. Consider the nonlinear dynamical system modeled by

$$\dot{x} = f(x), \tag{1}$$

where the state variable $x \in R^n$. If there exists a constant $T > 0(T > 0$ may depend on the initial state $x(0))$, such that

$$\lim_{t \to T} \parallel x(t) \parallel = 0, \tag{2}$$

and $\parallel x(t) \parallel \equiv 0$, if $t \geq T$, then system (1) is finite-time stable.

Lemma 1 [25]. Suppose there exists a continuous function $V : \mathscr{D} \to \mathbb{R}$ such that the following conditions hold:

(i) V is positive definite.

(ii) There exist real numbers $c > 0$ and $\alpha \in (0,1)$ and an open neighborhood $\mathscr{V} \subseteq \mathscr{D}$ of the origin such that

$$\dot{V}(x) + c(V(x))^{\alpha} \leq 0, \; x \in \mathscr{V} \setminus \{0\}. \tag{3}$$

Then the origin is a finite-time stable equilibrium of system (1), and the settling time, depending on the initial state $x(0) = x_0$, satisfies

$$T(x_0) \leq \frac{V^{1-\alpha}(x_0)}{c(1-\alpha)}. \tag{4}$$

In addition, if $\mathscr{D} = \mathbb{R}^n$ and $V(x)$ is also radially unbounded (i.e. $V(x) \to +\infty$ as $\|x\| \to +\infty$), then the origin is a globally finite-time stable equilibrium of system (1).

Lemma 2 [26]. For any real number $\alpha_i,\ i = 1,2,...,k$ and $0 < r < 1$, the following inequality holds

$$(|\alpha_1| + |\alpha_2| + \cdots + |\alpha_k|)^r \leq |\alpha_1|^r + |\alpha_2|^r + \cdots + |\alpha_k|^r. \tag{5}$$

In [16], a memristor-based hyperchaotic complex Lü system was proposed by introducing complex variables to its real counterpart [10]. The system takes the following form

$$\begin{cases} \dot{x} = \alpha(y - x), \\ \dot{y} = -xz + \beta y - \rho W(w)x, \\ \dot{z} = (\bar{x}y + x\bar{y})/2 - \gamma z, \\ \dot{w} = (x + \bar{x})/2. \end{cases} \tag{6}$$

where α, β, ρ and γ are positive parameters. $W(w) = a + 3bw^2$ denotes the memductance function of a flux-controlled memristor, which is characterized by a smooth continuous cubic nonlinearity. $x = v_1 + iv_2$ and $y = v_3 + iv_4$ are complex variables, $i = \sqrt{-1}$; $v_i (i = 1,2,3,4)$, $z = v_5$ and $w = v_6$ are real variables. Dots represent derivatives with respect to time, $\bar{x}$ and $\bar{y}$ are conjugates of x and y, respectively. overbar represents complex conjugate variables.

Separating the real and imaginary parts of system (6) yields the following equivalent system

$$\begin{cases} \dot{v}_1 = \alpha(v_3 - v_1), \\ \dot{v}_2 = \alpha(v_4 - v_2), \\ \dot{v}_3 = -v_1 v_5 + \beta v_3 - \rho(a + 3bv_6^2)v_1, \\ \dot{v}_4 = -v_2 v_5 + \beta v_4 - \rho(a + 3bv_6^2)v_2, \\ \dot{v}_5 = v_1 v_3 + v_2 v_4 - \gamma v_5, \\ \dot{v}_6 = v_1. \end{cases} \tag{7}$$

The study in [16] indicates that the system (7) has three line sets of equilibrium points and can generate abundant behaviors, such as periodic operations, transient phenomena, hyperchaotic

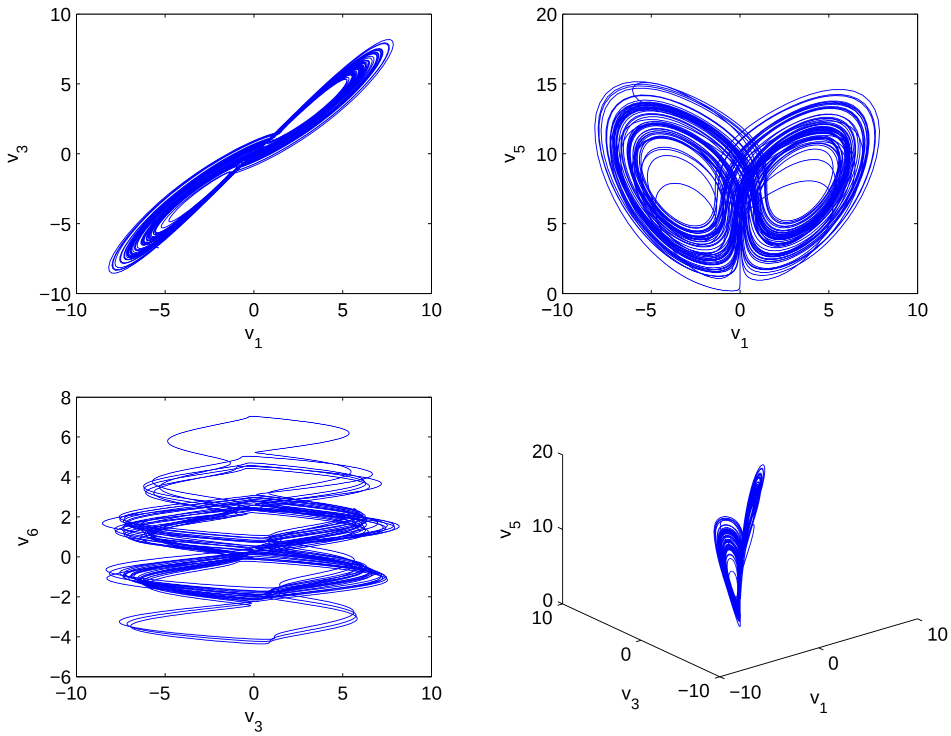

FIGURE 1. Hyperchaos of system (7).

and chaotic attractors with different shapes. Fig.1 shows the hyperchaotic dynamics of system (7), please refer to [16] for more properties and dynamical behaviors.

3. Main results

In this section, we investigate the finite-time stabilization schemes for the memristor-based hyperchaotic complex Lü system, and present the main results of this paper.

Consider the following controlled system

$$
\begin{cases}
\dot{v}_1 = \alpha(v_3 - v_1) + \mu_1, \\
\dot{v}_2 = \alpha(v_4 - v_2) + \mu_2, \\
\dot{v}_3 = -v_1 v_5 + \beta v_3 - \rho(a + 3bv_6^2)v_1 + \mu_3, \\
\dot{v}_4 = -v_2 v_5 + \beta v_4 - \rho(a + 3bv_6^2)v_2 + \mu_4, \\
\dot{v}_5 = v_1 v_3 + v_2 v_4 - \gamma v_5 + \mu_5, \\
\dot{v}_6 = v_1 + \mu_6.
\end{cases} \tag{8}
$$

where $\mu_i (i = 1, 2, ..., 6)$ are controllers to be determined.

In the following, we will present two strategies to achieve stabilization of system (7) in a finite time.

Strategy 1:

Theorem 1. If the controllers are designed as

$$\begin{cases} \mu_1 = -\alpha v_3 - v_1^k, \\ \mu_2 = -\alpha v_4 - v_2^k, \\ \mu_3 = -\beta v_3 + \rho(a+3bv_6^2)v_1 - v_3^k, \\ \mu_4 = -\beta v_4 + \rho(a+3bv_6^2)v_2 - v_4^k, \\ \mu_5 = -v_5^k, \\ \mu_6 = -v_1 - v_6^k. \end{cases} \tag{9}$$

where $k = q/p$ is a proper rational number, p and q are positive odd integers and $p > q$. The memristor-based hyperchaotic complex Lü system will achieve finite-time stabilization, i.e., the system (8)will be asymptotically stabilized at the equilibrium $O(0,0,0,0,0,0)$ in a finite time.

Proof. Choose a positive definite function in the form of

$$V = \frac{1}{2}(v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2 + v_6^2). \tag{10}$$

Take the time derivative of V, one obtains

(11)

$$\begin{aligned} \dot{V} =& v_1\dot{v}_1 + v_2\dot{v}_2 + v_3\dot{v}_3 + v_4\dot{v}_4 + v_5\dot{v}_5 + v_6\dot{v}_6 \\ =& v_1(\alpha(v_3 - v_1) + \mu_1) + v_2(\alpha(v_4 - v_2) + \mu_2) + v_3(-v_1v_5 + \beta v_3 - \rho(a+3bv_6^2)v_1 + \mu_3) \\ & + v_4(-v_2v_5 + \beta v_4 - \rho(a+3bv_6^2)v_2 + \mu_4) + v_5(v_1v_3 + v_2v_4 - \gamma v_5 + \mu_5) + v_6(v_1 + \mu_6). \end{aligned}$$

Introducing the controllers (9) into the above equation, we have

$$
\begin{aligned}
\dot{V} =& v_1(\alpha(v_3-v_1)-\alpha v_3-v_1^k)+v_2(\alpha(v_4-v_2)-\alpha v_4-v_2^k)\\
&+v_3(-v_1v_5+\beta v_3-\rho(a+3bv_6^2)v_1-\beta v_3+\rho(a+3bv_6^2)v_1-v_3^k)\\
&+v_4(-v_2v_5+\beta v_4-\rho(a+3bv_6^2)v_2-\beta v_4+\rho(a+3bv_6^2)v_2-v_4^k)\\
&+v_5(v_1v_3+v_2v_4-\gamma v_5-v_5^k)+v_6(v_1-v_1-v_6^k)\\
=&-\alpha v_1^2-v_1^{k+1}-\alpha v_2^2-v_2^{k+1}-v_3^{k+1}-v_4^{k+1}-\gamma v_5^2-v_5^{k+1}-v_6^{k+1}\\
\leq&-v_1^{k+1}-v_2^{k+1}-v_3^{k+1}-v_4^{k+1}-v_5^{k+1}-v_6^{k+1}\\
\leq&-(\frac{1}{2})^{-\frac{k+1}{2}}[(\frac{1}{2}v_1^2)^{\frac{k+1}{2}}+(\frac{1}{2}v_2^2)^{\frac{k+1}{2}}+(\frac{1}{2}v_3^2)^{\frac{k+1}{2}}+(\frac{1}{2}v_4^2)^{\frac{k+1}{2}}+(\frac{1}{2}v_5^2)^{\frac{k+1}{2}}+(\frac{1}{2}v_6^2)^{\frac{k+1}{2}}].
\end{aligned}\tag{12}
$$

According to Lemma 2, we have

$$
\begin{aligned}
\dot{V}\leq&-(\frac{1}{2})^{-\frac{k+1}{2}}(\frac{1}{2}v_1^2+\frac{1}{2}v_2^2+\frac{1}{2}v_3^2+\frac{1}{2}v_4^2+\frac{1}{2}v_5^2+\frac{1}{2}v_6^2)^{\frac{k+1}{2}}\\
=&-(\frac{1}{2})^{-\frac{k+1}{2}}(V)^{\frac{k+1}{2}}.
\end{aligned}\tag{13}
$$

Then in light of Lemma 1, the controlled system (8) is finite-time stable. This implies there exists a $T>0$ such that $v_i\equiv 0(i=1,2,...,5)$ if $t\geq T$.

Strategy 2:

Theorem 2. If the controllers are designed as

$$
\begin{cases}
\mu_1=-\alpha v_3-v_1^k,\\
\mu_2=-\alpha v_4-v_2^k,\\
\mu_3=-\beta v_3-v_3^k,\\
\mu_4=-\beta v_4-v_4^k,\\
\mu_5=-v_5^k,\\
\mu_6=-v_6^k.
\end{cases}\tag{14}
$$

where $k=q/p$ is a proper rational number, p and q are positive odd integers and $p>q$. The memristor-based hyperchaotic complex Lü system will achieve finite-time stabilization, i.e., the system (8)will be asymptotically stabilized at the equilibrium $O(0,0,0,0,0,0)$ in a finite time.

Proof. The design procedure consists of two steps.

Step 1. Introducing the controllers μ_1 and μ_2 into the first two parts of system (8) yields

$$\begin{cases} \dot{v}_1 = -\alpha v_1 - v_1^k, \\ \dot{v}_2 = -\alpha v_2 - v_2^k. \end{cases} \tag{15}$$

Construct the following positive definite function

$$V_1 = \frac{1}{2}(v_1^2 + v_2^2). \tag{16}$$

The derivative of V_1 along the trajectory of Eq.(15) is

$$\begin{aligned} \dot{V}_1 &= v_1\dot{v}_1 + v_2\dot{v}_2 \\ &= v_1(-\alpha v_1 - v_1^k) + v_2(-\alpha v_2 - v_2^k) \\ &\leq -v_1^{k+1} - v_2^{k+1} \\ &= -(\frac{1}{2})^{-\frac{k+1}{2}}[(\frac{1}{2}v_1^2)^{\frac{k+1}{2}} + (\frac{1}{2}v_2^2)^{\frac{k+1}{2}}] \\ &\leq -(\frac{1}{2})^{-\frac{k+1}{2}}(\frac{1}{2}v_1^2 + \frac{1}{2}v_2^2)^{\frac{k+1}{2}} \\ &= -(\frac{1}{2})^{-\frac{k+1}{2}}V_1^{\frac{k+1}{2}}. \end{aligned} \tag{17}$$

In light of Lemma 1, system (15) is finite-time stable. That means there is a $T_1 > 0$ such that $v_1 \equiv 0$ and $v_2 \equiv 0$, for any $t \geq T_1$.

When $t > T_1$, the last four parts of system (8) become:

$$\begin{cases} \dot{v}_3 = \beta v_3 + \mu_3, \\ \dot{v}_4 = \beta v_4 + \mu_4, \\ \dot{v}_5 = -\gamma v_5 + \mu_5, \\ \dot{v}_6 = \mu_6. \end{cases} \tag{18}$$

Construct the following candidate Lyapunov function for system (18)

$$V_2 = \frac{1}{2}(v_3^2 + v_4^2 + v_5^2 + v_6^2). \tag{19}$$

The derivative of V_2 along the trajectories of Eq.(18) is

$$\begin{aligned} \dot{V}_2 &= v_3\dot{v}_3 + v_4\dot{v}_4 + v_5\dot{v}_5 + v_6\dot{v}_6 \\ &= v_3(\beta v_3 + \mu_3) + v_4(\beta v_4 + \mu_4) + v_5(-\gamma v_5 + \mu_5) + v_6\mu_6. \end{aligned} \tag{20}$$

Introducing the controllers $\mu_3,\ \mu_4,\ \mu_5$ and μ_6 in Eq.(14) into the above equation, yields

$$
\begin{aligned}
\dot{V}_2 =&v_3(-v_3-\beta v_3-v_3^k)+v_4(-v_4-\beta v_4-v_4^k)+v_5(-v_5-v_5^k)+v_6(-v_6-v_6^k)\\
\leq&-v_3^{k+1}-v_4^{k+1}-v_5^{k+1}-v_6^{k+1}\\
=&-(\frac{1}{2})^{-\frac{k+1}{2}}[(\frac{1}{2}v_3^2)^{\frac{k+1}{2}}+(\frac{1}{2}v_4^2)^{\frac{k+1}{2}}+(\frac{1}{2}v_5^2)^{\frac{k+1}{2}}+(\frac{1}{2}v_6^2)^{\frac{k+1}{2}}]\\
\leq&-(\frac{1}{2})^{-\frac{k+1}{2}}(\frac{1}{2}v_3^2+\frac{1}{2}v_4^2+\frac{1}{2}v_5^2+\frac{1}{2}v_6^2)^{\frac{k+1}{2}}\\
=&-(\frac{1}{2})^{-\frac{k+1}{2}}V_2^{\frac{k+1}{2}}.
\end{aligned}
\tag{21}
$$

Then in light of Lemma 1, the states $v_3,\ v_4,\ v_5$ and v_6 will converge to zero at a finite time T_2. Then after T_2, the states of system (8) will stay at zero, i.e., the trajectories of the controlled system (8) converge to zero in a finite time.

5. Numerical simulations

In this section, we present two numerical examples to illustrate the theoretical analysis. In the following numerical simulations, the fourth-order Runge-kutta method is applied with time step size 0.01. The system parameters are selected as $a=4,\ b=0.01, \alpha=36,\ \beta=20, \rho=3$ and $\gamma=3$, so that memristor-based hyperchaotic complex Lü system exhibits hyperchaotic behaviors. The initial conditions for this system are given as $(x(0),y(0),z(0),w(0))=(0.2+0.2i,0.4+0.4i,0.5,0.5)$, i.e. $(v_1(0),v_2(0),v_3(0),v_4(0),v_5(0),v_6(0))=(0.2,0.2,0.4,0.4,0.5,0.5)$.

Example 1. Consider strategy 1 with the controllers (9). We choose $k=3/5$. Fig. 2 shows the result of the numerical simulation. From Fig. 2, we can see that it takes only a very short time to stabilize the controlled system (8) at the equilibrium $O(0,0,0,0,0,0)$.

Example 2. Consider strategy 2 with the controllers (14). We still choose $k=3/5$. Fig.3 shows that the controlled system (8) is stabilize at the equilibrium $O(0,0,0,0,0,0)$. From Figs.2 and 3, we can see the stabilization times of the controlled system (8) are almost the same.

6. Conclusions

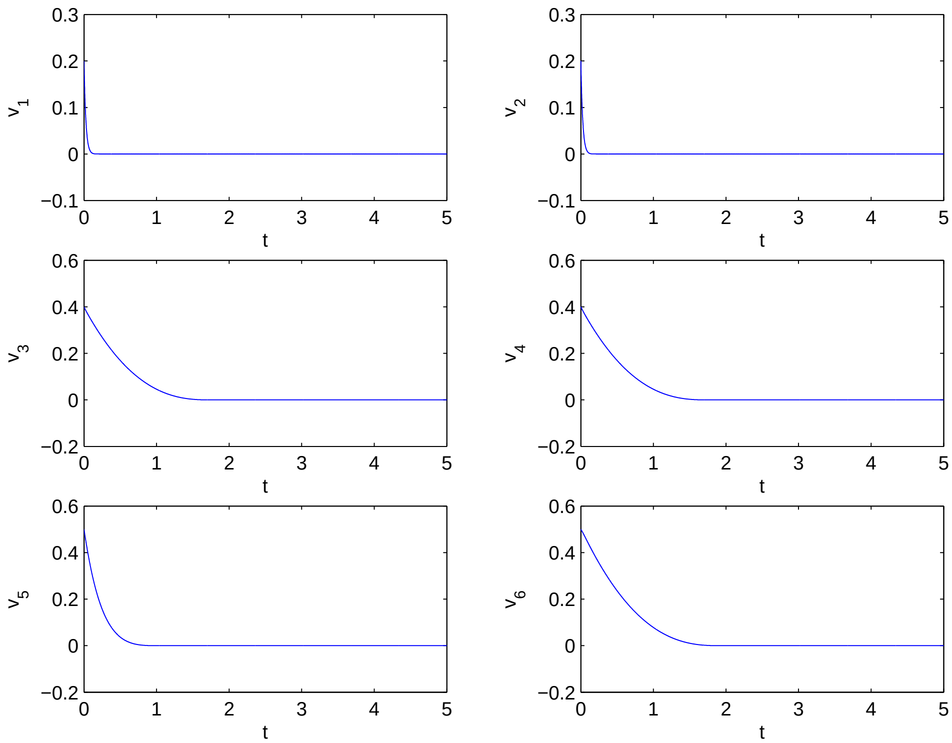

FIGURE 2. The states of the controlled system (8) with controllers (9).

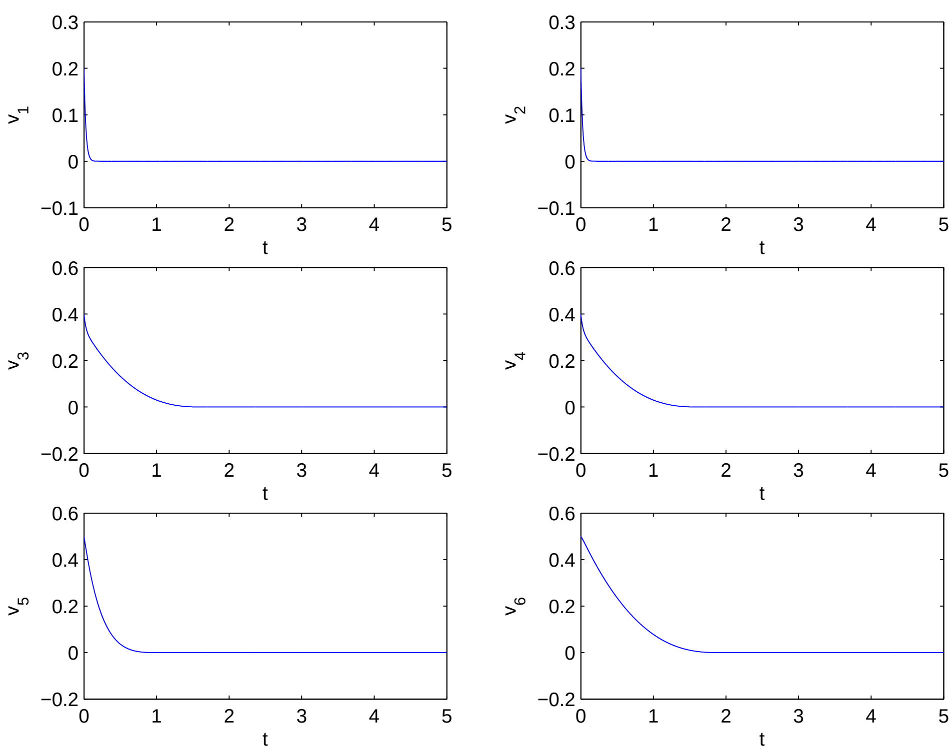

FIGURE 3. The states of the controlled system (8) with controllers (14).

In this paper, the finite-time stability of the memristor-based hyperchaotic complex Lü system has been studied. Based on the finite-time stability theory, two kinds of simple but effective controllers are proposed to stabilize the memristor-based hyperchaotic complex Lü system. From the proof process, we can see that the two methods can be extended effectively to other systems, such as the hyperchaotic complex Lorenz system, hyperchaotic complex Chen system, and some other hyperchaotic complex systems.

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Conflict of Interests

The authors declare that there is no conflict of interests.

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