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MORE RESULTS ON ECCENTRIC COLORING IN GRAPHS

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Abstract. The eccentricity $e(u)$ of a vertex u is the maximum distance from u in G. A vertex v is an eccentric vertex of u if the distance from u to v is equal to $e(u)$. An eccentric coloring of a graph $G = (V,E)$ is a function $color : V \longrightarrow N$ such that

(i) for all $u, v \in V$, $(color(u) = color(v)) \Longrightarrow d(u,v) > color(u)$,

(ii) for all $v \in V$, color $(v) \leq e(v)$.

The eccentric chromatic number $\chi_e \in N$ for a graph G is the least number of colors for which it is possible to eccentrically color G by $colors : V \longrightarrow \{1, 2, ..., \chi_e\}$. In this paper, we have proved that a cycle with a chord between vertices at any distance up to the radius of the cycle is eccentric colorable thereby making the results of [5] particular cases. Also, here we have extended results on eccentric coloring of Lexicographic product graphs proved earlier and found a sharp upper bound and shown its attainability.

Keywords: eccentricity of a vertex; eccentric vertex; eccentric coloring; eccentric chromatic number; lexicographic product.

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1. Introduction

Unless mentioned otherwise for terminology and notation the reader may refer Buckley and Harary [2] and Chartrand and Lensiak [3], new ones will be introduced as and when found necessary. In this paper we consider simple undirected graphs without multiple edges and self loops. The order p is the number of vertices in G and size q is the number of edges in G.
The distance $d(u,v)$ between u and v is the length of a shortest path joining u and v. If there exists no path between u and v then we define $d(u,v)=\infty$. The eccentricity $e(u)$ of u is the distance to a vertex farthest from u. If $d(u,v)=e(u)(v\neq u)$, we say that v is an eccentric vertex of u. The radius $rad(G)$ is the minimum eccentricity of the vertices, where as the diameter $diam(G)$ is the maximum eccentricity. A vertex v is a central vertex if $e(v)=rad(G)$, and the center $C(G)$ is the set of all central vertices. A graph G is self-centered if $rad(G)=diam(G)$.
The join of two graphs G_1 and G_2, defined by Zykov [8], is denoted G_1+G_2 and consists of $V_1\cup V_2$ and all edges joining V_1 with V_2.
Sloper [7] introduced the concept of eccentric coloring of a graph and studied the eccentric coloring of trees. which is a generalization of the broadcast coloring studied by many [1], [4] to name a few.
In [5] Itagi Huilgol et al. have established several bounds on the radius and the diameter of an eccentric colorable graph and have also found eccentric coloring number explicitly for cycles with chords between two vertices at distance two and three. In this paper, we prove that a cycle with a chord between vertices at any distance up to the radius of the cycle is eccentric colorable thereby making the results of [5] particular cases. In [5] Itagi Huilgol et al. have considered the eccentric coloring of Lexicographic product of cycles with $\overline{K_2}$. Here, we extend this result to the Lexicographic product of $C_p[\overline{K_3}]$ and show that it is not possible to eccentrically color $C_p[\overline{K_n}]$ for higher values of n.

2. Some basic results

In this section we prove some results about eccentric coloring.

Lemma 1. *For a complete graph K_p, the eccentric chromatic number remains unaltered after an edge deletion, that is, $\chi_e(K_p-e^{'})=\chi_e(K_p)$, for all $e^{'}\in E(K_p), p\geq 4$.*

Proof. Given a complete graph $K_p, p \geq 4$, we know that $\chi_e(K_p) = p$, by [5]. Now any edge $e^{'}$, when deleted does not change its eccentric coloring as the other adjacencies are kept intact and hence $\chi_e(K_p - e^{'}) = p$. Hence the result.

□

Lemma 2. *A self-centered, regular graph is not eccentrically colorable if its diameter is less than or equal to its regularity.*

Proof. Let G be a self-centered graph of radius d and regularity k. If a vertex v is colored with color say 1, then its k neighbors require k different colors. So to eccentrically color the vertices of G we require at least $k+1$ colors. If $d \leq k$, then the number of permissible colors is d. Hence, it is not possible to eccentrically color G. □

Next we consider Mycieleski graph of a graph G and prove that it is not eccentrically colorable. For ready reference we give the definition of the Mycieleski of a graph here.

Mycielski graph: Let G be a graph with p vertices. Let these p vertices be labeled as $\{u_1, u_2, \ldots, u_p\}$. The Mycielski graph $\mu(G)$ contains G itself as a subgraph, together with $p+1$ additional vertices: a vertex v_i corresponding to each vertex u_i of G, and an extra vertex w. Each vertex v_i is connected by an edge to w, so that these vertices form a subgraph in the form of a star $K_{1,p}$. In addition, for each edge u_iu_j of G, the Mycielski graph includes two edges, v_iu_j and u_iv_j. Thus, if G has p vertices and q edges, then $\mu(G)$ has $2p+1$ vertices and $3q+p$ edges. In [6] Itagi Huilgol et al. have proved the following result.

Theorem 1. [6] *Let G be any connected graph with $V(G) = \{u_1, u_2, \ldots, u_p\}$. Then the Mycieleski of G, $\mu(G)$, has its distance degree sequence as follows:*

$dds_{(\mu(G))}(u_i) = (d_0, 2d_1, 2d_2+2, 2d_3+d_4+d_5+d_6+\ldots+d_{ecc_G(u_i)}, d_4+d_5+d_6+\ldots+d_{ecc_G(u_i)}))$;

$dds_{\mu(G)}(v_i) = (d_0, d_1+1, p+d_2, p-1-d_1-d_2)$;

$dds_{\mu(G)}(w) = (d_0, p, p)$.

From the above result we can prove the following result.

Lemma 3. *Mycieleski of a graph G is not eccentrically colorable.*

Proof. From the above result of [6] we know that $\mu(G)$ has diameter 4 and radius 2.
We can see that if $ecc_G(u_i) \geq 4$, then $ecc_{\mu(G)}(u_i) = 4$ and if $ecc_G(u_i) \leq 3$, then $ecc_{\mu(G)}(u_i) = ecc_G(u_i)$.
Also by the structure of $\mu(G)$, it is clear that $ecc_{\mu(G)}(v_i) = 3$ and $ecc_{\mu(G)}(w) = ecc_G(w) = 2$.
Now if we try to eccentrically color $\mu(G)$, for the vertex w, only possible colors are 1 and 2. Since, w and $v_i'^s$ induce a star, $K_{1,p}$, for optimality we assign w with color 2 and all the vertices $v_i'^s$ color 1. Hence the only possible colors to be assigned to the vertices $u_i'^s$ are 3 and 4. With these two colors it is impossible to color all the p vertices $u_i'^s$. Hence, $\mu(G)$ is not eccentrically colorable. □

3. Eccentric coloring of cycle with chord

Here we consider the eccentric coloring of cycle with chord. In [5] Itagi Huilgol et al. have proved that a cycles are eccentrically colorable with eccentric chromatic number 3 or 4. Also they have proved that a cycle with a chord between vertices at distance two or three (in cycle) is eccentrically colorable. Here we prove that a cycle with a chord between vertices at any distance up to the radius of the cycle is eccentric colorable thereby making the results of [5] particular cases.

Lemma 4. *A cycle $C_p, p \geq 9$ with a chord between two vertices at distance four from each other is eccentric colorable with $\chi_e(C_p + e) = 5$.*

Proof. Let cycle $C_p, p \geq 9$ be labeled as $v_1, v_2, v_3, ..., v_p$. Let e be a chord between v_{p-2} and v_2. Hence denote by $C_p + e$, the graph obtained by adding the chord e to C_p. Eccentric coloring of $C_p + e$ is given based on the order p as follows:

Case (i): Let $p = 4m + 5, m \geq 1$.
Assign colors $1, 2, 1, 3$, respectively, $m-$ times to the vertices of $C_p + e$ sequentially in the order starting from v_1, that is, $v_1, v_2, ..., v_{p-5}$. Now the remaining five more vertices namely, $v_{p-4}, v_{p-3}, v_{p-2}, v_{p-1}, v_p$ are colored in the order $1, 4, 1, 3, 5$, respectively. Hence, $C_p + e$ is eccentrically colorable with $\chi_e(C_p + e) = 5$.

Case (ii): Let $p = 4m+6, m \geq 1$.

Assign colors $1,2,1,3$, respectively, $m-$ times to the vertices $v_1, v_2, ..., v_{p-6}$ of $C_p + e$ sequentially in the order starting from v_1. The remaining six vertices namely,
$v_{p-5}, v_{p-4}, v_{p-3}, v_{p-2}, v_{p-1}, v_p$ are colored in the order $1,2,1,4,1,5$ respectively. Hence, $C_p + e$ is eccentrically colorable with $\chi_e(C_p + e) = 5$.

Case (iii): Let $p = 4m+7, m \geq 1$.

Assign colors $1,2,1,3$, respectively, $(m+1)-$ times to the vertices $v_1, v_2, ..., v_{p-3}$ of $C_p + e$ sequentially in the order starting from v_1. The remaining three vertices namely, v_{p-2}, v_{p-1}, v_p are colored in the order $1,4,5$ respectively. Hence, $C_p + e$ is eccentrically colorable with $\chi_e(C_p + e) = 5$.

Case (iv): Let $p = 4m+8, m \geq 1$.

Assign colors $1,2,1,3$, respectively, $(m+1)-$ times to the vertices $v_1, v_2, ..., v_{p-4}$ of $C_p + e$ sequentially in the order starting from v_1. The remaining four vertices namely, $v_{p-3}, v_{p-2}, v_{p-1}, v_p$ are colored in the order $1,4,1,5$ respectively. Hence, $C_p + e$ is eccentrically colorable with $\chi_e(C_p + e) = 5$. Referring to all the above cases we see that a cycle with a chord at distance four from each other is eccentric colorable. □

Illustration: Let us consider the graphs as shown in Figure 1. In this example we consider $m = 2$. We have also shown the eccentric coloring of the vertices.

Lemma 5. *A cycle $C_p, p \geq 9$ with a chord between two vertices at distance five from each other is eccentric colorable with $\chi_e(C_p + e) = 5$.*

Proof. Proof is written on similar lines to the previous one. Let cycle $C_p, p \geq 9$ be labeled as $v_1, v_2, v_3, ..., v_p$. Let e be a chord between v_{p-3} and v_2. Hence denote by $C_p + e$, the graph obtained by adding the chord e to C_p. Eccentric coloring of $C_p + e$ is given based on the order p as follows:

Case (i): Let $p = 4m+5, m \geq 1$.

Assign colors $1,2,1,3$, respectively, $m-$ times to the vertices of $C_p + e$ sequentially in the

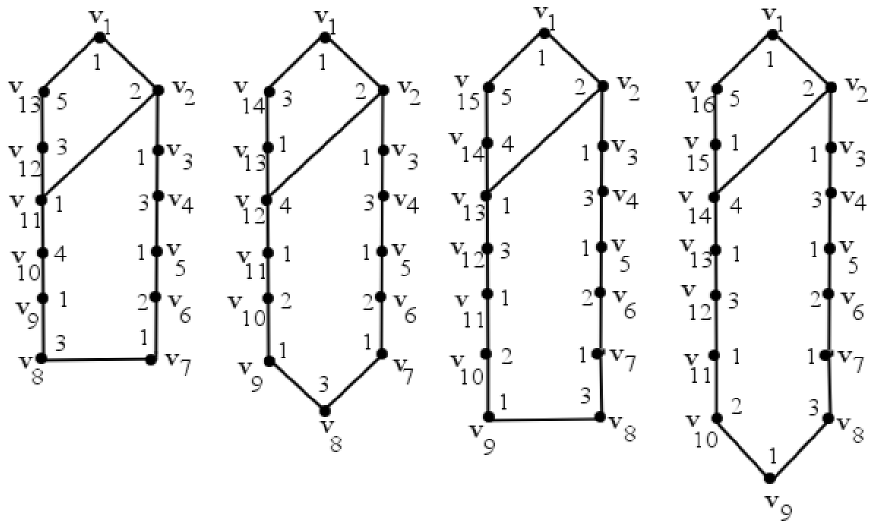

FIGURE 1. Chord at distance 4

order starting from v_1, that is, $v_1, v_2, ..., v_{p-5}$. Now the remaining five more vertices namely, $v_{p-4}, v_{p-3}, v_{p-2}, v_{p-1}, v_p$ are colored in the order $1,4,1,3,5$, respectively. Hence, $C_p + e$ is eccentrically colorable with $\chi_e(C_p + e) = 5$.

Case (ii): Let $p = 4m + 6, m \geq 1$.
Assign colors $1,2,1,3$, respectively, $m-$ times to the vertices $v_1, v_2, ..., v_{p-6}$ of $C_p + e$ sequentially in the order starting from v_1. The remaining six vertices namely,
$v_{p-5}, v_{p-4}, v_{p-3}, v_{p-2}, v_{p-1}, v_p$ are colored in the order $1,2,1,4,1,5$ respectively. Hence, $C_p + e$ is eccentrically colorable with $\chi_e(C_p + e) = 5$.

Case (iii): Let $p = 4m + 7, m \geq 1$.
Assign colors $1,2,1,3$, respectively, $m-$ times to the vertices $v_1, v_2, ..., v_{p-7}$ of $C_p + e$ sequentially in the order starting from v_1. The remaining seven vertices namely,
$v_{p-6}, v_{p-5}, v_{p-4}, v_{p-3}, v_{p-2}, v_{p-1}, v_p$ are colored in the order $1,2,1,4,1,2,5$ respectively. Hence, $C_p + e$ is eccentrically colorable with $\chi_e(C_p + e) = 5$.

Case (iv): Let $p = 4m+8, m \geq 1$.

Assign colors $1,2,1,3$, respectively, $(m+1)-$ times to the vertices $v_1, v_2, ..., v_{p-4}$ of $C_p + e$ sequentially in the order starting from v_1. The remaining four vertices namely, $v_{p-3}, v_{p-2}, v_{p-1}, v_p$ are colored in the order $1,4,1,5$ respectively. Hence, $C_p + e$ is eccentrically colorable with $\chi_e(C_p + e) = 5$.

Referring to all the above cases we see that a cycle with a chord at distance five from each other is eccentric colorable. □

Illustration: Let us consider the graphs as shown in Figure 2. In this example we consider $m = 2$. We have also shown the eccentric coloring of the vertices.

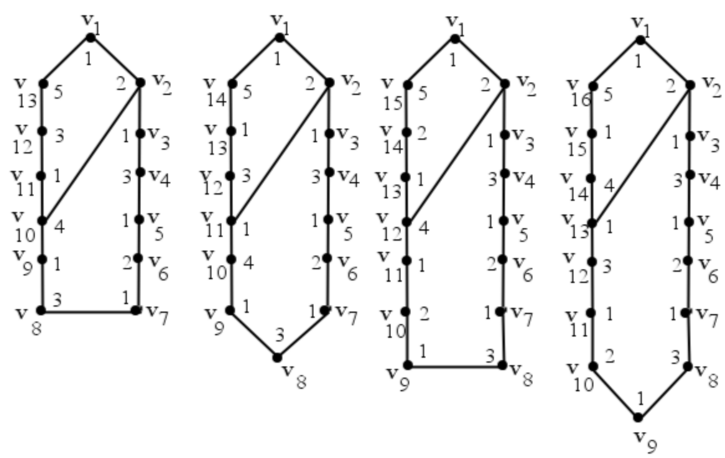

FIGURE 2. Chord at distance 5

Remark 1. *A cycle $C_p, p \geq 9$ with a chord e between two vertices at distance $i, 1 \leq i \leq \lfloor \frac{p}{2} \rfloor$, from each other is eccentric colorable, as the eccentric coloring is achieved based on the order p similar to the two previous lemmas. Since the chord divides the cycle into two smaller cycles, whose lengths are $i+1$ and $p-(i+1)+2 = p-i+1$. Depending on the value of i and the four cases of order of C_p, that is, p, we can show the eccentric colorability of $C_p + e$.*

4. ECCENTRIC COLORING OF LEXICOGRAPHIC PRODUCTS

In this section we consider the eccentric coloring of lexicographic product of class of graphs. First, we recollect the definition of lexicographic product as follows:

Given two graphs G and H, the lexicographic product $G[H]$ has its vertex set

$\{(g,h): g \in V(G), h \in V(H)\}$ and two vertices $(g,h),(g^{'},h^{'})$ are adjacent if and only if either $[g,g^{'}]$ is an edge of G or $g=g^{'}$ and $[h,h^{'}]$ is edge of H.

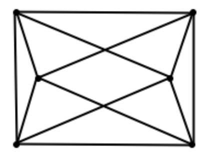

FIGURE 3. Lexicographic product $C_4[\overline{K_2}]$

As mentioned earlier we give an extension of the result proved in [5], in the following theorem. The proof runs similarly, except for the additional case.

Theorem 6. *For any even integer $p \geq 16, C_p[\overline{K_3}]$ is eccentric colorable.*

Proof. Let a cycle C_p be labeled as $v_1, v_2, v_3, ...v_p$, where $p(\geq 16)$ is an even integer and $C_p[\overline{K_3}]$ be the lexicographic product of C_p and $\overline{K_3}$. Let S_i denote the set of vertices of $\overline{K_3}$ replaced in place of v_i.

Case(i): $p = 8n+8$, $n \geq 1$.

In this case, let $V(C_p[\overline{K_3}]) = A \cup B \cup C \cup D$ be the partition of $V(C_p[\overline{K_3}])$, where

$A = \{\cup S_{2k-1}/1 \leq k \leq p/2\}$,

$B = \{\cup S_{4k-2}/1 \leq k \leq p/4\}$,

$C = \{\cup S_{8k-2}/1 \leq k \leq p/8\}$,

$D = \{\cup S_{8k}/1 \leq k \leq p/8\}$.

The eccentric coloring of $C_p[\overline{K_3}]$ is given as follows:

The three vertices in each $S_{2k-1}, 1 \leq k \leq p/2$, are colored with color 1. Among the three vertices in each $S_{4k-2}, 1 \leq k \leq p/4$ in B, one vertex is colored with color 2 and the other two vertices are colored with color 3 and color 4. Among three vertices in each $S_{8k-2}, 1 \leq k \leq p/8$ in C, one vertex is colored with color 4 and the other two vertices are colored with color 5 and color 6. Among the three vertices in each $S_{8k}, 1 \leq k \leq p/8$ in D, one vertex is colored with color 5 and the other two vertices are colored with color 6 and color 7.

Hence, in this case the eccentric coloring number is 7.

Case(ii): $p = 8n+10, n \geq 1$.

In this case, let $V(C_p[\overline{K_3}]) = A \cup B \cup C \cup D \cup E$ be the partition of $V(C_p[\overline{K_3}])$, where

$A = \{\cup S_{2k-1}/1 \leq k \leq p/2\}$,

$B = \{\cup S_{4k-2}/1 \leq k \leq p-2/4\}$,

$C = \{\cup S_{8k-4}/1 \leq k \leq p-2/8\}$,

$D = \{\cup S_{8k}/1 \leq k \leq p-2/8\}$,

$E = \{S_p\}$.

The eccentric coloring of $C_p[\overline{K_3}]$ is given as follows:

The three vertices in each $S_{2k-1}, 1 \leq k \leq p/2$, are colored with color 1. Among the three vertices in each $S_{4k-2}, 1 \leq k \leq p-2/4$ in B, one vertex is colored with color 2 and the other two vertices are colored with color 3 and color 4. Among three vertices in each $S_{8k-4}/1 \leq k \leq p-2/8$ in C, one vertex is colored with color 4 and the other two vertices are colored with color 5 and color 6. Among the three vertices in each $S_{8k}, 1 \leq k \leq p-2/8$ in D, one vertex is colored with color 5 and the other two vertices are colored with color 6 and color 7. Among the three vertices in E, one vertex is colored with color 7 and the other two are colored with color 8 and color 9.

Hence, in this case the eccentric coloring number is 9.

Case(iii): $p = 8n+12, n \geq 1$.

In this case, let $V(C_p[\overline{K_3}]) = A \cup B \cup C \cup D \cup E$ be the partition of $V(C_p[\overline{K_3}])$, where

$A = \{\cup S_{2k-1}/1 \leq k \leq p/2\}$,

$B = \{\cup S_{4k-2}/1 \leq k \leq p/4\}$,

$C = \{\cup S_{8k-4}/1 \leq k \leq p-4/8\}$,

$D = \{\cup S_{8k}/1 \leq k \leq p-4/8\}$,

$E = \{S_p\}$.

The eccentric coloring of $C_p[\overline{K_3}]$ is given as follows:

The three vertices in each $S_{2k-1}, 1 \leq k \leq p/2$, are colored with color 1. Among the three vertices in each $S_{4k-2}, 1 \leq k \leq p/4$ in B, one vertex is colored with color 2 and the other two vertices are colored with color 3 and color 4. Among three vertices in each $S_{8k-4}/1 \leq k \leq p-4/8$ in C, one vertex is colored with color 4 and the other two vertices are colored with color 5 and color 6. Among the three vertices in each $S_{8k}, 1 \leq k \leq p-4/8$ in D, one vertex is colored with color 5 and the other two vertices are colored with color 6 and color 7. Among the three vertices in E, one vertex is colored with color 7 and the other two vertices are colored with color 8 and color 9.

Hence, in this case the eccentric coloring number is 9.

Case(iv): $p = 8n+14, n \geq 1$.

In this case, let $V(C_p[\overline{K_3}]) = A \cup B \cup C \cup D \cup E$ be the partition of $V(C_p[\overline{K_3}])$, where

$A = \{\cup S_{2k-1}/1 \leq k \leq p/2\}$,

$B = \{\cup S_{4k-2}/1 \leq k \leq p-2/4\}$,

$C = \{\cup S_{8k-4}/1 \leq k \leq p+2/8\}$,

$D = \{\cup S_{8k}/1 \leq k \leq p-6/8\}$,

$E = \{S_p\}$.

The eccentric coloring of $C_p[\overline{K_3}]$ is given as follows:

The three vertices in each $S_{2k-1}, 1 \leq k \leq p/2$, are colored with color 1. Among the three vertices in each $S_{4k-2}, 1 \leq k \leq p-2/4$ in B, one vertex is colored with color 2 and the other two vertices are colored with color 3 and color 4. Among three vertices in each $S_{8k-4}/1 \leq k \leq p+2/8$ in C, one vertex is colored with color 4 and the other two vertices are colored with color 5 and color 6. Among the three vertices in each $S_{8k}, 1 \leq k \leq p-6/8$ in D, one vertex is colored with color 5 and the other two vertices are colored with color 6 and color 7. Among the three vertices in E, one vertex is colored with color 7 and the other two vertices are colored with color 8 and color 9.

Hence, in this case the eccentric coloring number is 9.

$\square$

Illustration: For example, let us consider $C_{16}[\overline{K_3}]$, $C_{18}[\overline{K_3}]$, $C_{20}[\overline{K_3}]$,$C_{22}[\overline{K_3}]$ as shown in Figure 4 and Figure 5.

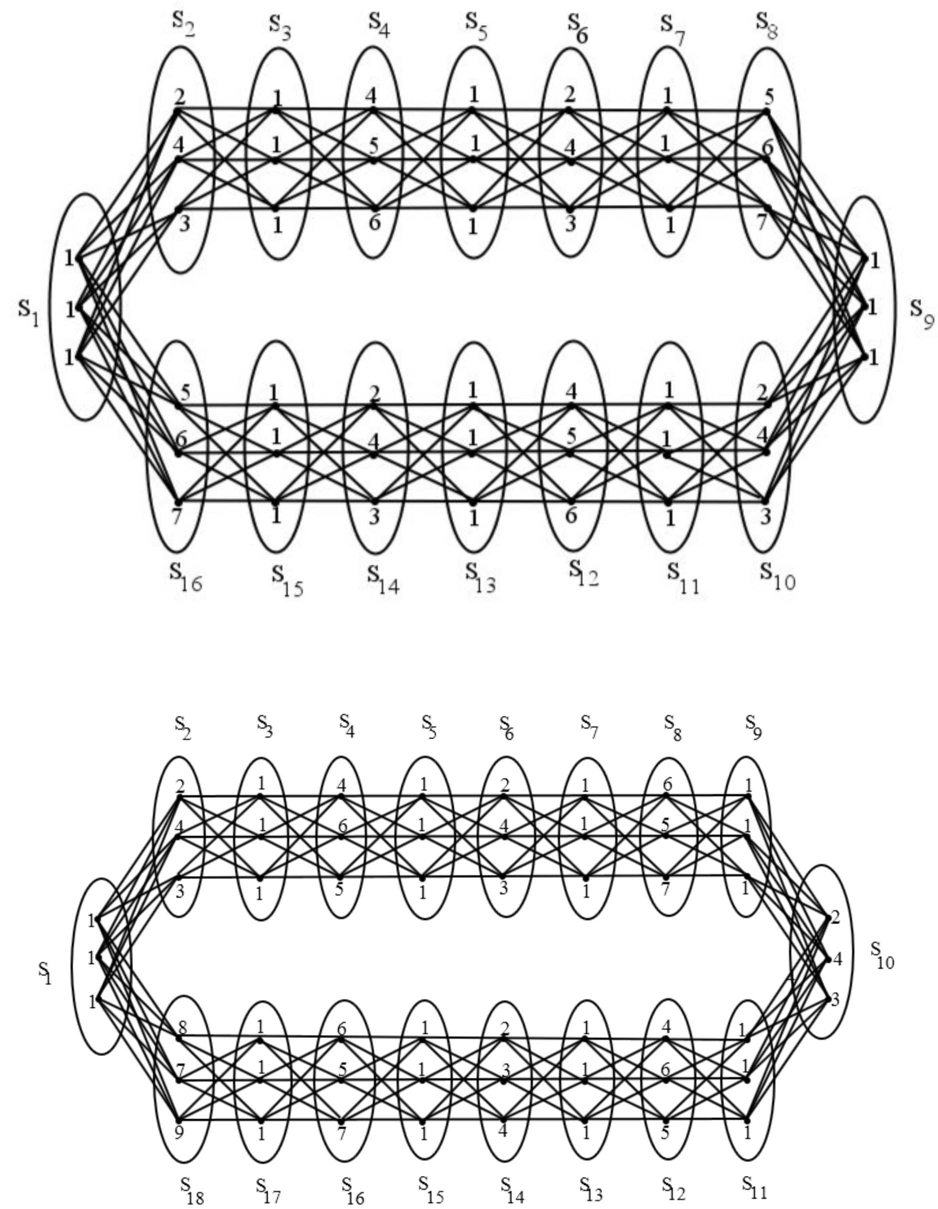

FIGURE 4. Lexicographic product $C_{16}[\overline{K_3}], C_{18}[\overline{K_3}]$

For $p = 8n+8 = 16$, $V(C_p[\overline{K_3}]) = A \cup B \cup C \cup D$ be the partition of $V(C_p[\overline{K_3}])$, where

$A = \{\cup S_{2k-1}/1 \leq k \leq p/2\} = \{S_1, S_3, S_5, S_7, S_9, S_{11}, S_{13}, S_{15}\}$,

$B = \{\cup S_{4k-2}/1 \leq k \leq p/4\} = \{S_2, S_6, S_{10}, S_{14}\}$,

$C = \{\cup S_{8k-2}/1 \leq k \leq p/8\} = \{S_4, S_{12}\}$,

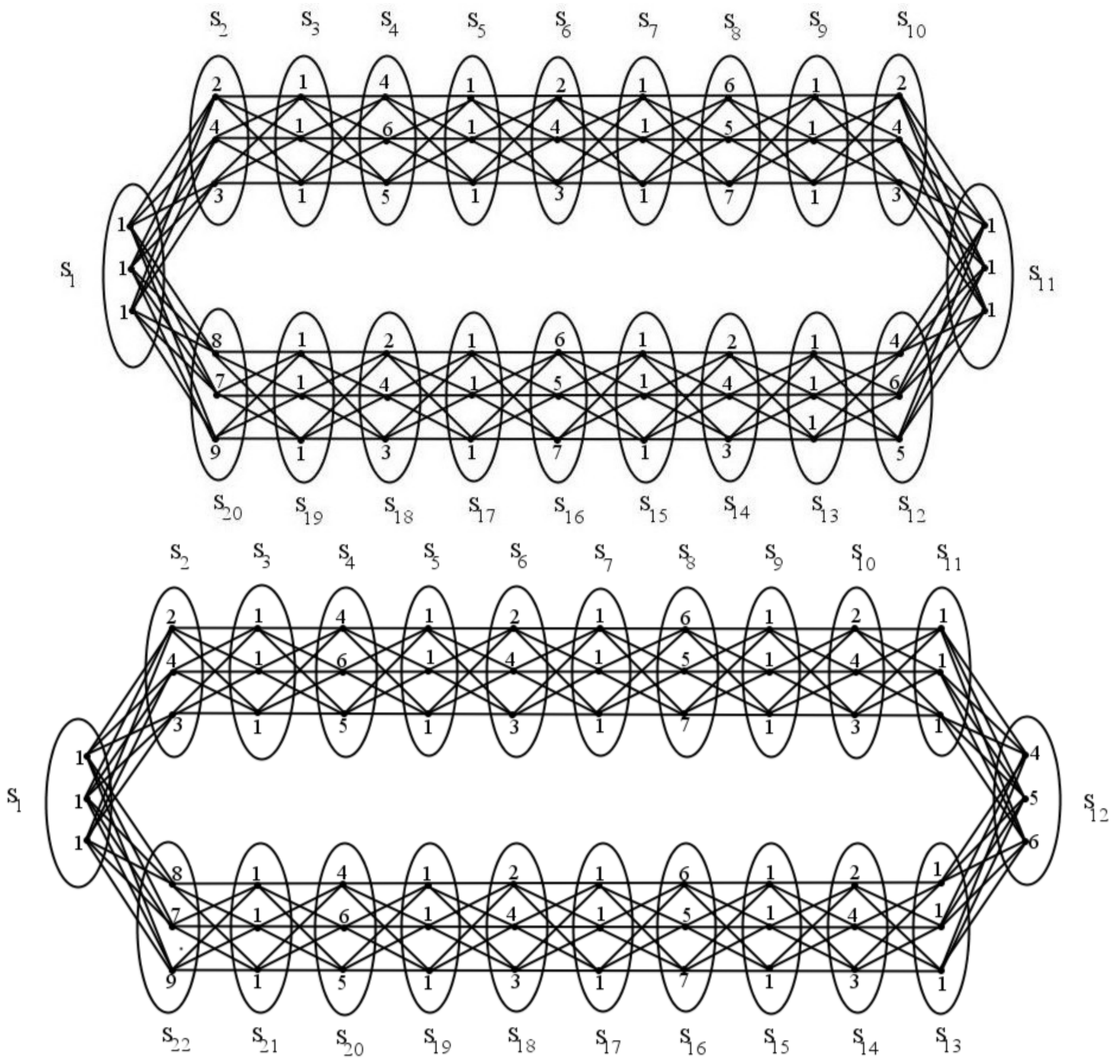

FIGURE 5. Lexicographic product $C_{20}[\overline{K_3}], C_{22}[\overline{K_3}]$

$D = \{\cup S_{8k}/1 \leq k \leq p/8\} = \{S_8, S_{16}\}$.

For $p = 8n+10 = 18, V(C_p[\overline{K_3}]) = A \cup B \cup C \cup D \cup E$ be the partition of $V(C_p[\overline{K_3}])$, where

$A = \{\cup S_{2k-1}/1 \leq k \leq p/2\} = \{S_1, S_3, S_5, S_7, S_9, S_{11}, S_{13}, S_{15}, S_{17}\}$,

$B = \{\cup S_{4k-2}/1 \leq k \leq p-2/4\} = \{S_2, S_6, S_{10}, S_{14}\}$,

$C = \{\cup S_{8k-4}/1 \leq k \leq p-2/8\} = \{S_4, S_{12}\}$,

$D = \{\cup S_{8k}/1 \leq k \leq p-2/8\} = \{S_8, S_{16}\}$,

$E = \{S_p\} = \{S_{18}\}$.

For $p = 8n+12 = 20$, $V(C_p[\overline{K_3}]) = A \cup B \cup C \cup D \cup E$ be the partition of $V(C_p[\overline{K_3}])$, where

$A = \{\cup S_{2k-1}/1 \leq k \leq p/2\} = \{S_1, S_3, S_5, S_7, S_9, S_{11}, S_{13}, S_{15}, S_{17}, S_{19}\}$,

$B = \{\cup S_{4k-2}/1 \leq k \leq p/4\} = \{S_2, S_6, S_{10}, S_{14}, S_{18}\}$,

$C = \{\cup S_{8k-4}/1 \leq k \leq p-4/8\} = \{S_4, S_{12}\}$,

$D = \{\cup S_{8k}/1 \leq k \leq p-4/8\} = \{S_8, S_{16}\}$,

$E = \{S_p\} = \{S_{20}\}$.

For $p = 8n+14 = 22$, $V(C_p[\overline{K_3}]) = A \cup B \cup C \cup D \cup E$ be the partition of $V(C_p[\overline{K_3}])$, where

$A = \{\cup S_{2k-1}/1 \leq k \leq p/2\} = \{S_1, S_3, S_5, S_7, S_9, S_{11}, S_{13}, S_{15}, S_{17}, S_{19}, S_{21}\}$,

$B = \{\cup S_{4k-2}/1 \leq k \leq p-2/4\} = \{S_2, S_6, S_{10}, S_{14}, S_{18}\}$,

$C = \{\cup S_{8k-4}/1 \leq k \leq p+2/8\} = \{S_4, S_{12}, S_{20}\}$,

$D = \{\cup S_{8k}/1 \leq k \leq p-6/8\} = \{S_8, S_{16}\}$,

$E = \{S_p\} = \{S_{22}\}$.

Note: We can see that $\chi_e(C_p[\overline{K_2}]) = \chi_e(C_p[\overline{K_3}])$ since $C_p[\overline{K_2}]$ or $C_p[\overline{K_3}]$ are self-centered graphs of radius $p/2$, since p is even, $\chi_e(C_p[\overline{K_2}])$ or $\chi_e(C_p[\overline{K_3}])$ cannot exceed $p/2-1$. But in eccentric coloring of $C_p[\overline{K_2}]$ and $C_p[\overline{K_3}]$, we have used all possible $p/2-1$ colors. Hence it is not possible to eccentrically color $C_p[\overline{K_n}]$ or any higher value of n.

Theorem 7. *The Lexicographic product $C_p[\overline{K_n}]$ for $p \geq 16$, even and n positive integer is eccentrically colorable if and only if $n \leq 3$.*

Proof. Proof follows from the above Note and Theorem 4.1 of [5]. □

Conflict of Interests

The authors declare that there is no conflict of interests.

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