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CHAOS SYNCHRONIZATION BETWEEN TWO IDENTICAL RESTRICTED THREE BODY PROBLEM VIA ACTIVE CONTROL AND ADAPTIVE CONTROL METHOD

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Abstract: This article presents the chaos synchronization problem of the restricted three body problem (RTBP) when the massive primary is supposed to be oblate spheroid and the smaller one is a uniform circular ring. The feedback controller for the stability of the closed-loop system are designed using the active control and adaptive control strategy. It is shown that the two methods have excellent performance, with the active control marginally outperforming the adaptive control in terms of transient analysis. Simulation results satisfy the theoretical findings. For validation of results by numerical simulations, the Mathematica 10 is used when the primaries are Saturn and Jupiter.

Keywords: chaos; synchronization; active control; adaptive control; restricted three body problem.

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1. INTRODUCTION

Pecora and Carroll [1] gave idea of synchronization of chaotic systems using the concept of

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master and slave system and they demonstrated that chaotic synchronization could be achieved by driving or replacing one of the variables of a chaotic system with a variable of another similar chaotic device. Many methods for chaos synchronization of various chaotic systems have been developed, such as non linear feedback control [2], OGY approach [3], sliding mode control [4], anti synchronization method [5], adaptive synchronization [6], active control [7] and so on. The active control methods for synchronizing the chaotic systems has been applied to many practical systems such as spatiotemporal dynamical systems (Codreanu [8]), the Rikitake two-disc dynamo-a geographical systems (Vincent 9]), Complex dynamos (Mahmoud [10]) and Hyper-chaotic and time delay systems (Israr Ahmad et al. [11]) etc. Shihua Chen and Jinhu [12] proposed a new adaptive control method for adaptive synchronization of two uncertain chaotic systems, using a speci_c uncertain uni_ed chaotic model.

Many mathematicians have investigated the circular restricted three body problem such as the Euler [13], Hill [14], Poincare [15], Lagrange [16], Deprit [17], Hadjidemetriou [18], Bhatnagar [19,20], Sharma et al. [21,22], Sahoo and Ishwar [23] and many others. These studies focus on the analytical, qualitative and numerical studies of the problem. A detailed analysis of this problem is illustrated in the work of American mathematician Szebehely [24]. Khan and Shahzad [25] investigated the synchronization behavior of the two identical circular restricted three body problem inuenced by radiation evolving from di_erent initial conditions via the active control Arif [26] studied the complete synchronization, anti-synchronization and hybrid synchronization in the planar restricted three p0roblem by taking into consideration the small primary is ellipsoid and bigger primary an oblate spheroid via active control technique.

Being motivated by the above discussion, in this article, the equation of motion of the restricted three body problem when the massive primary is supposed to be oblate spheroid and the smaller one is a uniform circular ring in a dimensionless rotating, co-ordinate system is formulated. we have also designed the controller for the stability of the closed-loop system by using the active control and adaptive control strategy It has been observed that the system is chaotic for some values of parameter. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time. The paper is organized as follows. In section 2 we derive the equations of motion of the system. Section 3 deals with the complete synchronization of the problem via active control and section 4 via adaptive control. Finally, we conclude the paper in section 5.

2. Equation of motion

The equation of motion for the restricted three body problem when the massive primary is supposed to be oblate spheroid and the smaller one is a uniform circular ring in a dimensionless rotating, co-ordinate system are as follows,

$$\ddot{x} - 2\,\omega\,\dot{y} = U_x \qquad (1)$$

$$\ddot{y} + 2\,\omega\,\dot{x} = U_y \qquad (2)$$

Where

$U_x = \frac{\partial U}{\partial x}$ and $U_y = \frac{\partial U}{\partial y}$

$$U = \frac{\omega^2}{2}\,(x^2 + y^2) + \left\{\frac{1-\mu}{r_1} + \frac{I}{2\,r_1^3}\right\} + \frac{2\,\mu}{\pi\,(a^2-b^2)}\,[(b + r_2)\,E(\theta, k_b) + (b - r_2)\,K(\theta, k_b) - (a + r_2)\,E(\theta, k_a) - (a - r_2)\,K(\theta, k_a)] \qquad (3)$$

$\omega =$ mean motion of the primaries, $\mu =$ mass ratio, $a, b =$ outer and inner radii of the ring respectively. $I = \mu(R_e^2 - R_p^2)/5$, R_e , R_p equatorial and polar radius of oblate spheroid respectively.

$\mathrm{K}(\theta, k_{a,b}) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-k_{a,b}^2\,sin^2\theta}}$ Elliptic integral of first kind,

$\mathrm{E}(\theta, k_{a,b}) = \int_0^{\frac{\pi}{2}} \sqrt{1 - k_{a,b}^2 sin^2\theta}\,d\theta$ Elliptic integral of second kind, $k_a = \frac{\sqrt{4a r_2}}{(a+r_2)}$, $k_b = \frac{\sqrt{4b r_2}}{(b+r_2)}$.

$r_1^2 = (x - \mu)^2 + y^2$, $r_2^2 = (x + 1 - \mu)^2 + y^2$.

The only integral of motion available for the system of equations 1and 2 is the Jacobi constant

$$\dot{x}^2 + \dot{y}^2 = 2\mathrm{U} - \mathrm{C} \qquad (4)$$

Once a set of initial conditions is given, the Jacobi constant, through equation 4, defines the forbidden region and allowed regions of motion bounded by the Hill's surfaces.

Let S be that energy surface,

i.e.,

$$S(\mu, C) = \{(x, y, \dot{x}, \dot{y})|\ C(x, y, \dot{x}, \dot{y}) = constant\} \tag{5}$$

The projection of this surface onto position space is called a Hill's region

$$S(\mu, C) = \left\{(x, y)|\ \mathrm{U}(x, y) \geq \frac{C}{2}\right\}. \tag{6}$$

The boundary of $S(\mu, C)$ is the zero velocity curve. The *negligible* mass can move only within this region in the (x, y)-plane. There are basic two configurations for the Hill's region for a given value of μ.

Figure 1

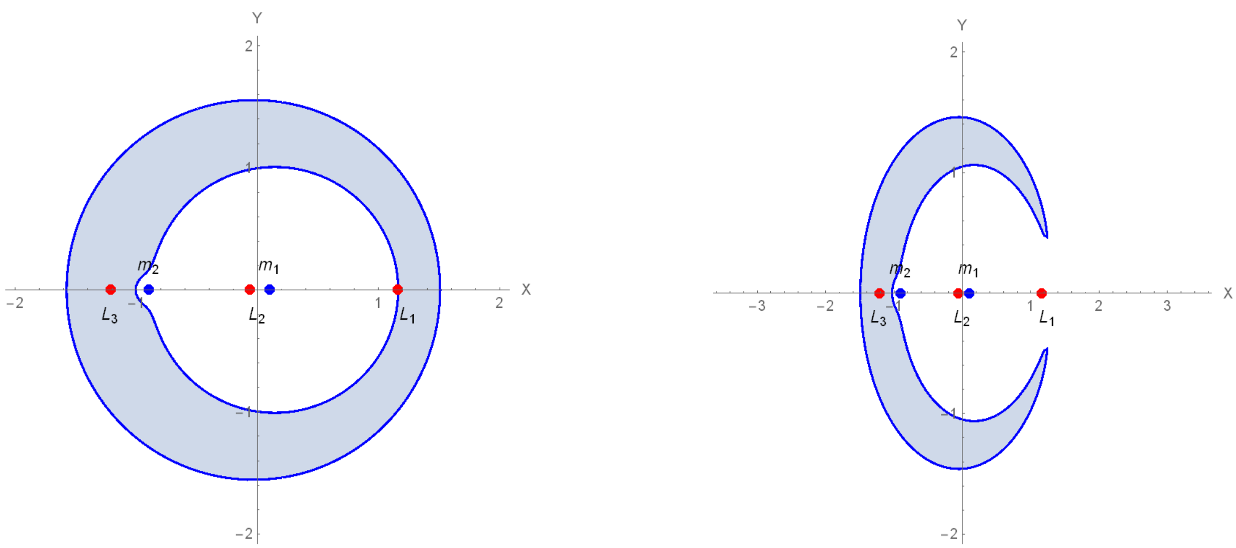


Fig (1a) $C < C_{1,}\ \mu = .1$

Fig (1b) $C_{3,} < C < C_{1,}\ \mu = .1$

The values of C which separate these two cases will be denoted $C_i\ i = 1{,}3$ which are the values corresponding to the equilibrium points L_1 and L_3. These values can be easily calculated for small μ and their graphs are shown in Figure 1. For case 2, the Jacobi constant lies between C_1 and C_3 which are the Jacobi constants of the libration points L_1 and L_3 respectively. In this case, the Hill's region contains a neck around both L_1 and L_3 and the negligible mass can transit from the interior region to the exterior region and vice versa.

3. Synchronization via Active Control

Let

$x = x_1, \quad \dot{x} = x_2, \quad y = x_3, \quad \dot{y} = x_4$

Then the equation (1) and (2) can be written as:

$$\dot{x}_1 = x_2 \tag{7}$$

$$\dot{x}_2 = 2\omega x_4 + \omega^2 x_1 + A_1 \tag{8}$$

$$\dot{x}_3 = x_4 \tag{9}$$

$$\dot{x}_4 = -2\omega x_2 + \omega^2 x_3 + A_2 \tag{10}$$

Where

$$A_1 = \frac{-(1-\mu)(x_1-\mu)}{\mathrm{r}_1^3} - \frac{3\,\mathrm{I}\,(x_1-\mu)}{2\mathrm{r}_1^5}$$

$$- \frac{2\,\mu\,(x_1+1-\mu)}{\pi(a^2-b^2)\,r_2}\Bigg[(\mathrm{E}(\theta,k_b)-\mathrm{K}(\theta,k_b))\left(1+\frac{4b(b-r_2)}{k_b(b+\mathrm{r}_2^2)}\right)$$

$$+ \frac{4b(b-r_2)^2}{(b+r_2)^3}\left(\frac{\left(\mathrm{E}(\theta,k_b)-(1-{k_b}^2)\mathrm{K}(\theta,k_b)\right)}{k_b(1-{k_b}^2)}\right)$$

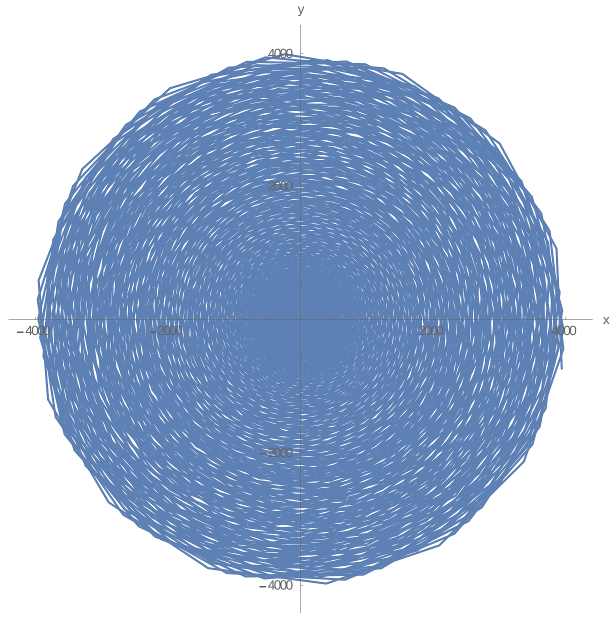
$$+ (\mathrm{E}(\theta,k_a)-\mathrm{K}(\theta,k_a))\left(1+\frac{4a(a-r_2)}{k_a(a+\mathrm{r}_2^2)}\right)$$

$$+ \frac{4a(a-r_2)^2}{(a+r_2)^3}\left(\frac{\left(\mathrm{E}(\theta,k_a)-(1-{k_b}^2)\mathrm{K}(\theta,k_a)\right)}{k_a(1-{k_a}^2)}\right)\Bigg]$$

$$A_2 = \frac{-(1-\mu)x_3}{\mathrm{r}_1^3} - \frac{3\mathrm{I}\,x_3}{2\mathrm{r}_1^5} - \frac{2\,(\mu)\,x_3}{\pi(a^2-b^2)\,r_2}\Bigg[\left(\mathrm{E}(\theta,k_b)-\mathrm{K}(\theta,k_b)\right)\left(1+\frac{4b(b-r_2)}{k_b(b+\mathrm{r}_2^2)}\right) + \frac{4b(b-r_2)^2}{(b+r_2)^3}\left(\frac{\left(\mathrm{E}(\theta,k_b)-(1-{k_b}^2)\mathrm{K}(\theta,k_b)\right)}{k_b(1-{k_b}^2)}\right) + (\mathrm{E}(\theta,k_a)-\mathrm{K}(\theta,k_a))\left(1+\frac{4a(a-r_2)}{k_a(a+\mathrm{r}_2^2)}\right) + \frac{4a(a-r_2)^2}{(a+r_2)^3}\left(\frac{\left(\mathrm{E}(\theta,k_a)-(1-{k_b}^2)\mathrm{K}(\theta,k_a)\right)}{k_a(1-{k_a}^2)}\right)\Bigg]$$

$$r_1^2 = (x_1-\mu)^2 + {x_3}^2, \qquad r_2^2 = (x_1+1-\mu)^2 + {x_3}^2.$$

The state orbits of this master system are shown in Figure (2) and this figure shows that the system is chaotic.



Fig(2)

Corresponding to master system ((7), … (10)), the identical slave system is defined as:

$$\dot{y}_1 = y_2 + u_1(t) \tag{11}$$

$$\dot{y}_2 = 2\omega y_4 + \omega^2 y_1 + B_1 + u_2(t) \tag{12}$$

$$\dot{y}_3 = y_4 + u_3(t) \tag{13}$$

$$\dot{y}_4 = -2\omega y_2 + \omega^2 y_3 + B_2 + u_4(t) \tag{14}$$

Where

$$B_1 = \frac{-(1-\mu)(y_1-\mu)}{\mathrm{r}_{11}^3} - \frac{3\,\mathrm{I}\,(y_1-\mu)}{2\mathrm{r}_{11}^5}$$

$$- \frac{2\,(\mu)\,(y_1+1-\mu)}{\pi(a^2-b^2)\,r_{21}}\Bigg[(\mathrm{E}(\theta,k_b)-\mathrm{K}(\theta,k_b))\left(1+\frac{4b(b-r_{21})}{k_b(b+\mathrm{r}_{21}^2)}\right)$$

$$+\frac{4b(b-r_{21})^2}{(b+r_{21})^3}\left(\frac{(\mathrm{E}(\theta,k_b)-(1-{k_b}^2)\mathrm{K}(\theta,k_b))}{k_b(1-{k_b}^2)}\right)$$

$$+(\mathrm{E}(\theta,k_a)-\mathrm{K}(\theta,k_a))\left(1+\frac{4a(a-r_{21})}{k_a(a+\mathrm{r}_{21}^2)}\right)$$

$$+\frac{4a(a-r_{21})^2}{(a+r_{21})^3}\left(\frac{(\mathrm{E}(\theta,k_a)-(1-{k_b}^2)\mathrm{K}(\theta,k_a))}{k_a(1-{k_a}^2)}\right)\Bigg]$$

$$B_2 = \frac{-(1-\mu)y_3}{r_{11}^3} - \frac{3\,\mathrm{I}\,y_3}{2r_{11}^5} - \frac{2\,(\mu)\,y_3}{\pi(a^2-b^2)\,r_{21}}\Bigg[(\mathrm{E}(\theta,k_b) - \mathrm{K}(\theta,k_b))\left(1+\frac{4b(b-r_{21})}{k_b(b+r_{21}^2)}\right) + \frac{4b(b-r_{21})^2}{(b+r_{21})^3}\left(\frac{(\mathrm{E}(\theta,k_b)-(1-{k_b}^2)\mathrm{K}(\theta,k_b))}{k_b(1-{k_b}^2)}\right) + (\mathrm{E}(\theta,k_a) - \mathrm{K}(\theta,k_a))\left(1+\frac{4a(a-r_{21})}{k_a(a+r_{21}^2)}\right) + \frac{4a(a-r_{21})^2}{(a+r_{21})^3}\left(\frac{(\mathrm{E}(\theta,k_a)-(1-{k_b}^2)\mathrm{K}(\theta,k_a))}{k_a(1-{k_a}^2)}\right)\Bigg]$$

$$r_{11}^2 = (y_1-\mu)^2 + {y_3}^2, \quad r_{21}^2 = (y_1+1-\mu)^2 + {y_3}^2.$$

where $u_i(t)$; i =1 ,2,3,4 are control functions to be determined.

Let $e_i = y_i - x_i$; i = 1, 2, 3, 4 be the synchronization errors. From ((7)….(10)) and ((11)….(14)), we obtain the error dynamics as follows:

$$\dot{e_1} = e_2 + u_1(t) \tag{15}$$

$$\dot{e_2} = 2\omega e_4 + \omega^2 e_1 + B_1 - A_1 + u_2(t) \tag{16}$$

$$\dot{e_3} = e_4 + u_3(t) \tag{17}$$

$$\dot{e_4} = -2\omega e_2 + \omega^2 e_3 + B_2 - A_2 + u_4(t) \tag{18}$$

Let us redefine the control functions so that the terms in (15) to (18) which cannot be expressed as linear terms in e_i's are eliminated :

$$u_1(t) = v_1(t)$$

$$u_2(t) = -B_1 + A_1 + v_2(t)$$

$$u_3(t) = v_3(t)$$

$$u_4(t) = -B_2 + A_2 + v_4(t)$$

The new error system can be expressed as**:**

$$\left.\begin{aligned} \dot{e_1} &= e_2 + v_1(t) \\ \dot{e_2} &= 2\omega e_4 + \omega^2 e_1 + v_2(t) \\ \dot{e_3} &= e_4 + v_3(t) \\ \dot{e_4} &= -2\omega e_2 + \omega^2 e_3 + v_4(t) \end{aligned}\right\} \tag{19}$$

The above error system to be controlled is a linear system with a control input $v_i(t)$ ($i = 1, \ldots 4$) as function of the error states e_i ($i = 1, \ldots 4$). As long as these feedbacks stabilize the system e_i ($i = 1, \ldots 4$) converge to zero as time t tends to infinity. This implies that master and the slave system are synchronized with active control .We choose.

$$\begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \\ v_4(t) \end{bmatrix} = A \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \tag{20}$$

Here A is a 4×4 coefficient matrix to be determined. As per Lyapunov stability theory and Routh-Hurwitz criterion, in order to make the closed loop system (20) stable, proper choice of elements of A has to be made so that the system (20) must have all eigen values with negative real parts. Choosing

$$A = \begin{bmatrix} -1 & -1 & 0 & 0 \\ -\omega^2 & -1 & 0 & -2\omega \\ 0 & 0 & -1 & -1 \\ 0 & 2\omega & -\omega^2 & -1 \end{bmatrix} \tag{21}$$

and, defining a matrix B as

$$\begin{bmatrix} \dot{e_1} \\ \dot{e_2} \\ \dot{e_3} \\ \dot{e_4} \end{bmatrix} = B \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \tag{22}$$

Where B is

$$B = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{23}$$

Clearly, B has eigen values with negative real parts. This implies $\lim_{t\to\infty}|e_i| = 0;\ i = 1, 2, 3, 4$ and hence, complete synchronization is achieved between the master and slave systems.

4. Synchronization via Adaptive Control

In this section we design an adaptive controller for the slave system (11)...(14). Lyapunov stability theory state that when controller satisfies the assumption with V (e) = $\frac{1}{2}\, e^t e$ a positive definite function and the first derivative of this function $\dot{V}$< 0, the chaos synchronization of two identical systems (master and slave) for different initial conditions is achieved. Construct a Lyapunov function as:

$\dot{V} = \frac{1}{2}({e_1}^2 + {e_2}^2 + {e_3}^2 + {e_4}^2).$

Then its derivative along the error system (15) to (18) is

$\dot{V} = e_1(e_2 + u_1) + e_2\{2\omega e_4 + \omega^2 e_1 + B_1 - A_1 + u_2\} + e_3(e_4 + u_3)$

$+e_4\{-2\omega e_2 + \omega^2 e_3 + B_2 - A_2 + u_4\}$.

Hence, if we choose the controller u as follows,

$u_1 = -e_1 - e_2$

$u_2 = -\omega^2 e_1 - B_1 + A_1 - e_2$

$u_3 = -e_3 - e_4$

$u_4 = -\omega^2 e_3 - B_2 + A_2 - e_4$

Then

$$\dot{V} = -{e_1}^2 - {e_2}^2 - {e_3}^2 - {e_4}^2 < 0.$$

Hence the error state

$$\lim_{t\to\infty} \|e(t)\| = 0.$$

which gives asymptotic stability of the system. This means that the controlled chaotic systems (master and slave) are synchronized for deferent initial conditions.

NUMERICAL SIMULATION

Let us consider an example of Jupiter-Saturn system in the restricted three body problem in which the primary m_2 is taken as the Saturn and primary m_1 as the Jupiter and small body as a space- craft. From the astrophysical data we have

Mass of the Saturn m_2 = 5:683 $\times$ 10^{26} kg

Mass of the Jupiter m_1 = 1:89712 $\times$ 10^{27} kg

The distance between the Jupiter and Saturn = 646,270,000 Km.

In dimensionless system we have m_1+m_2 = 1 unit and the distance between the Jupiter-Saturn is 1 unit.

The initial conditions of the master system and the slave system are set to be (.967, 2.29, 0.00, 5.0) and (-6.5, .29, 3.0, 0,0), respectively. We have simulated the system under consideration using mathematica 10. Results for uncontrolled system are given in figures 3,6,9,12 and that of controlled system Via active control are shown in figures 4,7,10 and 13 and via Adaptive control are shown in figures 5,8,11,14 respectively. These figures shows that the state $[x_1(t), x_2(t), x_3(t),\ x_4(t)]$ of master system [7 to 10] asymptotically synchronize with the state $[y_1(t), y_2(t), y_3(t),\ y_4(t)]$of slave system [11 to 14]. Fig. (15) shows the synchronization error (e) for the two system. We find that at t = 6s, the synchronization was already attained for adaptive control while synchronization was attained at a later time (t = 10s) for active control, the time delay being 4 s. Though it is clear that the adaptive control performs better and is much easier to design.

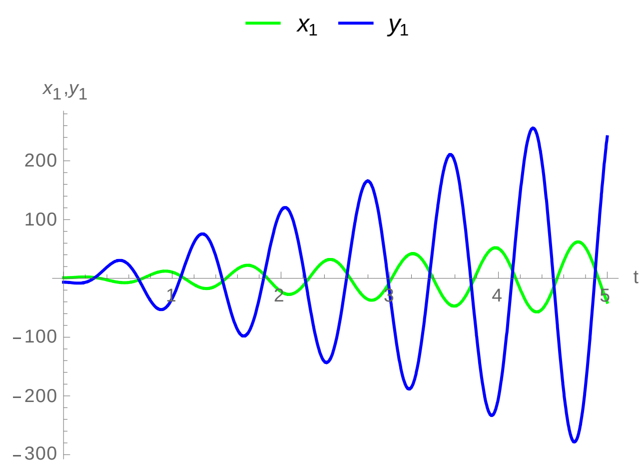


Fig (3): Time series of the Uncontrolled states x_1, y_1.

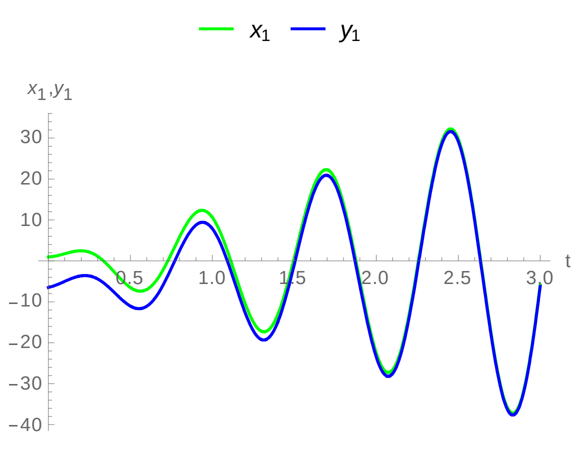


Fig (4): Time series of the controlled states x_1, y_1via Active control

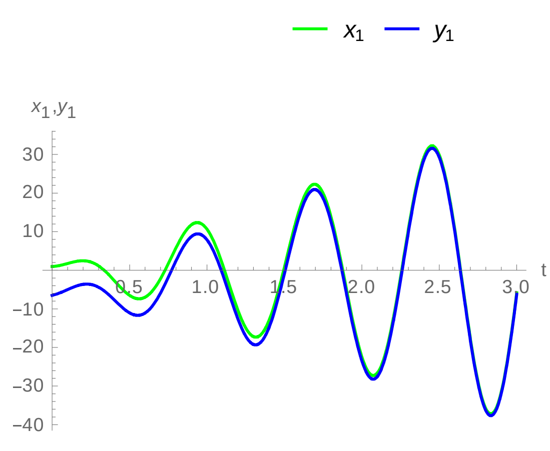


Fig (5): Time series of the controlled states x_1, y_1via Adaptive control

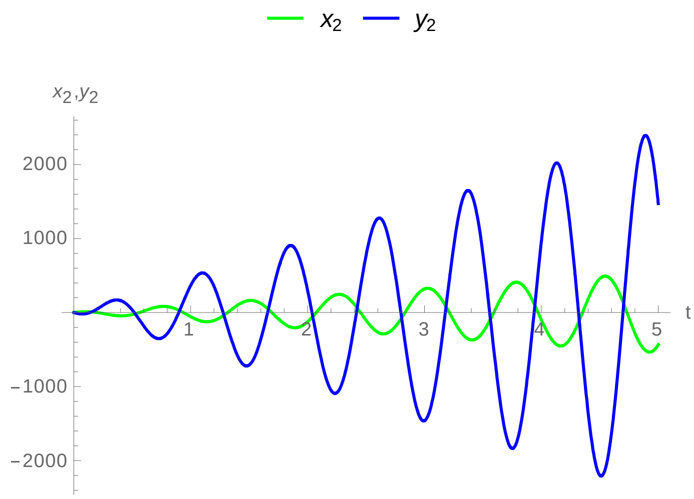


Fig (6) :Time series of the Uncontrolled states x_2, y_2.

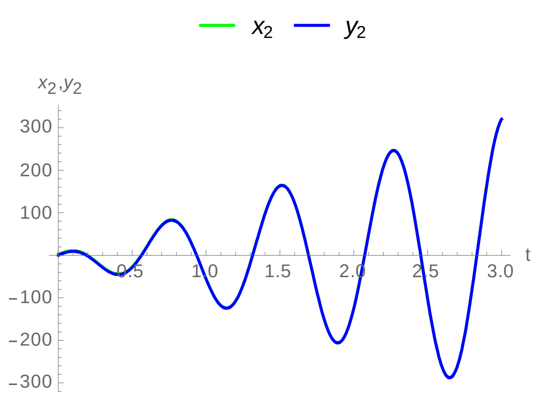


Fig (7) :Time series of the controlled states x_2, y_2 via Active control

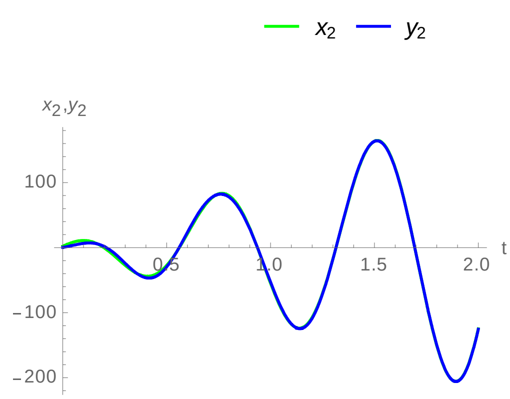


Fig (8) :Time series of the controlled states x_2, y_2 via Adaptive control

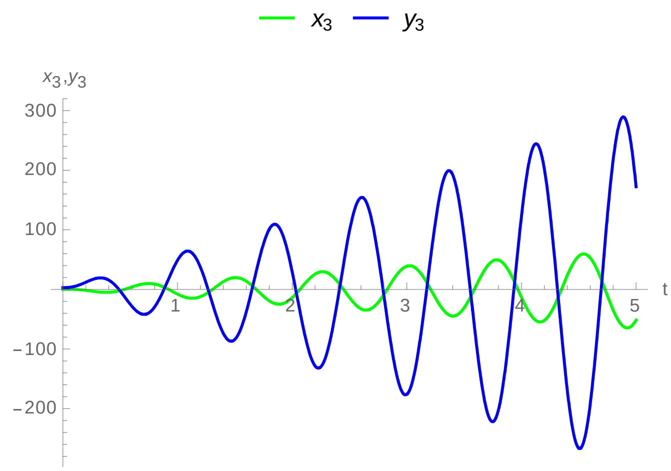
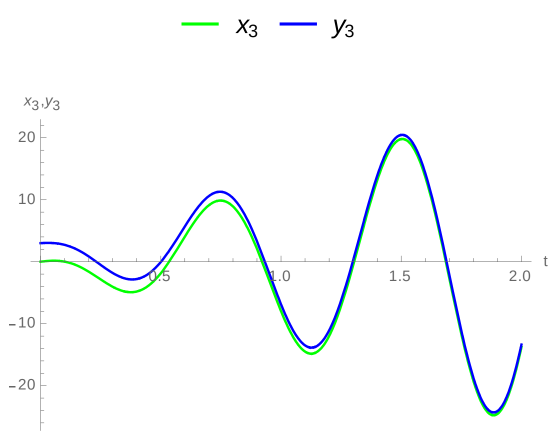


Fig (9) :Time series of the Uncontrolled states x_3, y_3.



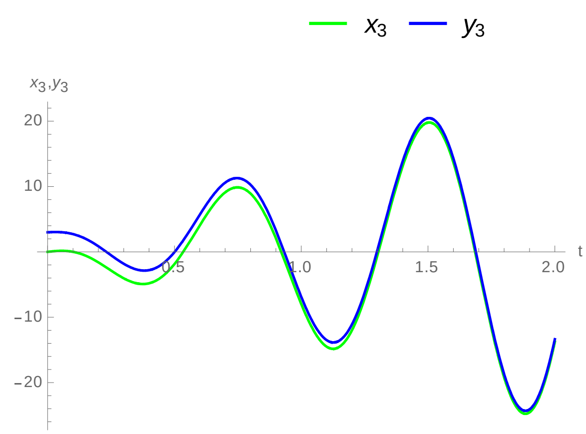


Fig (10) :Time series of the controlled states x_2, y_2via Active control   Fig (11) :Time series of the controlled states x_2, y_2via Adaptive control

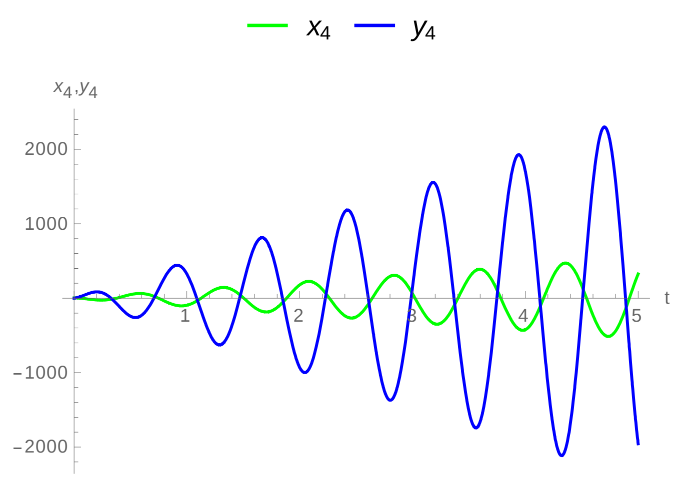


Fig (12) :Time series of the Uncontrolled states x_3, y_3.

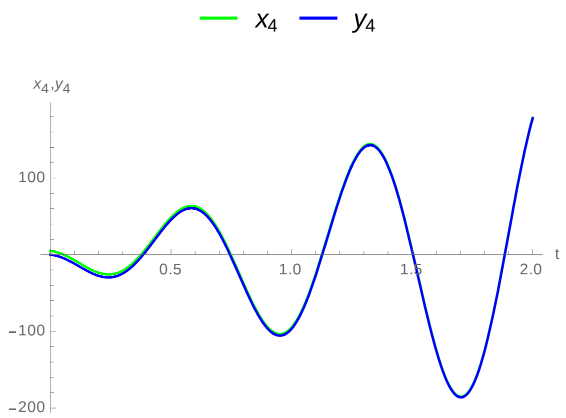


Fig (13) :Time series of the controlled states x_2, y_2via Active control

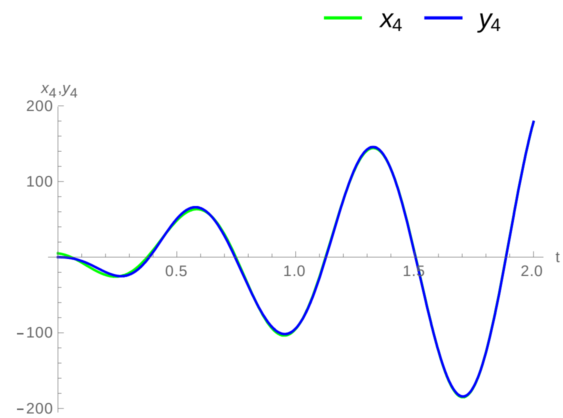


Fig (14) :Time series of the controlled states x_2, y_2via Adaptive control

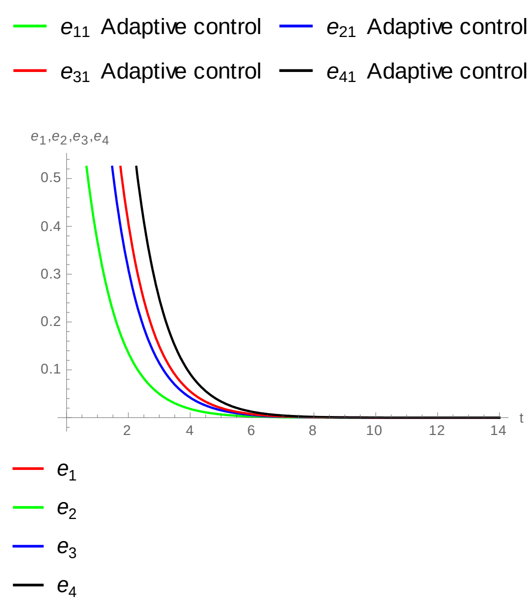


Fig (15a) :Error system via Active control

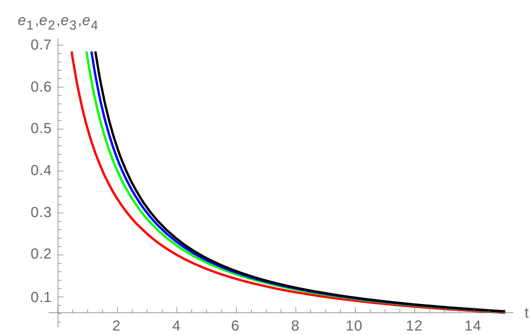


Fig (15 b) :Error system via Adaptive control

5. Conclusions

The equation of motion of the restricted three-body problem when the massive primary is supposed to be oblate spheroid and the smaller one is an uniform circular ring formulated. We have investigated the complete synchronization behavior of the problem via adaptive and active control method. Here two systems (master and slave) are compete synchronized when start with deferent initial conditions. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time. For validation of results by numerical simulations we used the Mathematica 10 when the primaries are Jupiter and Saturn.

Conflict of Interests

The authors declare that there is no conflict of interests.

REFERENCES

[1] Pecora L. M.,Carroll T.L., Synchronization in chaotic systems, Rev. Lett. 64 (1990), 821- 824.

[2] Lu L., Zhang C., Guo Z. A. Synchronization between two different chaotic systems with non linear feedback control, Chinese Phys. 16-6 (2007), 1603-1607.

[3] Ott., Grebogi C., Yorke J. A. Controlling Chaos, Phys. Rev. Lett., 64 (1990), 1196-1199.

[4] Haeri M., Emadzadeh A. Synchronizing different chaotic systems using active sliding mode control, Chaos Solitons Fractals, 1 (2003). 119-129.

[5] Park J. H., Chaos synchronization between two different chaotic dynamical systems. Chaos Solitons Fractals, 27-2 (2006), 549-554.

[6] Wang Y., Guan Z. H.,Wang H. O., Feedback and adaptive control for the synchronization of Chen system via a single variable, Phys. Lett. , 312-1 (2003), 30-40.

[7] Bai E.W., Lonngren K.E., Synchronization of two Lorenz systems using active control, Chaos Solitons Fractals, 8 (1997), 51-58.

[8] Codreanu, S., Synchronization of spatiotemporal nonlinear dynamical systems by an active control. Chaos Solitons Fractals, 15 (2003), 507-510.

[9] Vincent, U. E., Synchronization of Rikitake chaotic attractor via active control. Phys. Lett. A, 343 (2005), 133-138.

[10] Mahmoud, G. M., Aly, S. A. and Farghaly, A. On chaos synchronization of a complex two coupled dynamos system. Chaos Solitons Fractals, 33 (2007), 178-187.

[11] Ahmad, I., Saaban, A. B., Ibrahim, A. B., Al-Hadhrami, S., Shahzad, M. and Al-Mahrouqi, S. HA research on adaptive control to stabilize and synchronize a hyperchaotic system with uncertain parameters. An International J. Optim. Control., Theor. Appl. 5(2) (2015), 51-62.

[12] Shihua Chen, Jinhu Leu., Synchronization of an uncertain unified chaotic system via adaptive control. Chaos Solitons Fractals 14 (2002), 643–647.

[13] Euler, L., Un corps etant attire en raison reciproque quarree des distances vers deux points fixes donnes. Mem. Berlin (1760), 228.

[14] Hill, G. W., Reasearches in the lunar theory. Am. J. Math. 1 (1878).

[15] Poincare, H., Sciences et Methode. (1908).

[16] Lagrange, J. Ocuvres 14, vols Gauthier-Villars, Paris. (1772).

[17] Deprit A. and Price J. F., The computation of characteristic exponents in the planar restricted three body problem. Astro. J.,70 (1964), 836-849.

[18] Hadjidemetriou, J. D., Asteroid motion near the 3:1. Celest Mech. Dyn. Astron., 56 (1- 2) (1993), 201-219.

[19] Bhatnagar, K. B., Periodic orbits of collision in the restricted problem of three bodies in a three-dimensional co-ordinate system. Ind. J. Pure Appl. Math., 3 (1) (1969), 101-117.

[20] Bhatnagar, K. B. and Chawla, J. M., A study of the Lagrangian points in the photogravitational restricted three body problem. Ind. J. Pure Appl. Math., 10(11) (1979), 1443-1451.

[21] Sharma, R. K. and Subbarao, collinear equilibria and their characteristic exponents in the restricted three body problem when the primaries are oblate spheroids. Celestial Mechanics, 12 (1975), 189-201.

[22] Sharma, R. K., Taqvi, Z. A. and Bhatnagar, K.B., Existence and stability of the libration point of the restricted three body problem when the bigger primary is a triaxial body and a source of radiation. Indian J. Pure Appl. Math. 32(2) (2001), 255-266.

[23] Sahoo, S. k. and Ishwar, B., Stability of the collinear equilibrium points in generalized the photogravitational elliptical restricted three body problem. Bull Astr. Soc. India, 28, (2000), 579-586.

[24] Szebehely, V., Theory of Orbits., Academic press, New York. (1967).

[25] Khan, A., Shahzad, M., Chaos Synchronization in a Circular Restricted Three Body Problem Under the Effect of Radiation. Chaos Complex Syst. (2012), 59-68.

[26] M. Arif, Complete synchronization antisynchronization and hybrid synchronization of restricted three body problem when bigger primary is oblate spheroid and smaller primary is ellipsoid. Int. J. Adv. Appl. Math. Mech., 4(3) (2017), 32-44.