Available online at http://scik.org
J. Math. Comput. Sci. 9 (2019), No. 3, 340-353
https://doi.org/10.28919/jmcs/4002
ISSN: 1927-5307

ON SOME CLASSES OF CONCIRCULAR CURVATURE TENSOR ON LORENTZIAN PARA-SASAKIAN MANIFOLDS

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Abstract. The present paper deals with the study of different classes of concircular curvature tensor on Lorentzian para-Sasakian manifold admitting a quarter-symmetric metric connection.

Keywords: Lorentzian para-Sasakian manifolds; quarter-symmetric metric connection; concircular curvature tensor; $\eta-$Einstein manifold.

2010 AMS Subject Classification: 53C15, 53C25.

1. Introduction

K. Matsumoto [8] introduced the concept of Lorentzian para- Sasakian manifolds in 1989. Late, the same concept was independently introduced by I. Mihai and R. Rosca [10]. The Lorentzian para-Sasakian manifolds have also been studied by K. Matsumoto and I. Mihai [9], U. C. De and A. A. Shaikh [11] and several others such as ([12], [14], [15]). K. Matsumoto and I. Mihai obtained some interesting results for conformally recurrent and conformally symmetric $P-$Sasakian manifold in [1]. In 1924, the notion of semi-symmetric connection on a differentiable manifold was firstly introduced by Friedmann and Schouten [18]. A linear connection $\bar{\nabla}$

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Received January 18, 2019

on a differentiable manifold M is said to be a semi-symmetric connection if the torsion tensor T of the connection satisfies

$$T(U,V) = \eta(V)U - \eta(U)V,$$

where η is a 1-form and ξ is a vector field defined by $\eta(U) = g(U,\xi)$, for all vector fields U on $\Gamma(TM)$, $\Gamma(TM)$ is the set of all differentiable vector fields on M. A. Barman ([2], [3]) studied para-Sasakian manifold admitting semi-symmetric metric and non metric connection. On the other hand, in 1975, Golab [6] intoduced and studied quarter-symmetric connection in differentiable manifolds along with affine connections.

A liner connection $\bar{\nabla}$ on an n-dimensional Riemannian manifold (M,g) is called a quarter-symmetric connection [6] if its torsion tensor T satisfies

$$T(U,V) = \eta(V)\phi U - \eta(U)\phi V, \tag{1.1}$$

where ϕ is a (1,1) tensor field.

The quarter-symmetric connection generalizes the notion of the semi-symmetric connection because if we assume $\phi U = U$ in the above equation, the quarter-symmetric connection reduces to the semi-symmetric connection [18].

Moreover, if a quarter-symmetric connection $\bar{\nabla}$ satisfies the condition

$$(\bar{\nabla}_U g)(V,W) = 0, \tag{1.2}$$

for all U,V,W on $\Gamma(TM)$, then $\bar{\nabla}$ is said to be a quarter-symmetric metric connection.

Venkatesha and C.S. Bagewadi [19] obtain some interesting results on concircular ϕ-recurrent Lorentzian para-Sasakian manifolds which generalize the concept of locally concircular ϕ-symmetric Lorentzian para-Sasakian manifolds. If curvature tensor R of Riemannian manifold M satisfies $\nabla R = 0$, then M is called locally symmetric. Later, many geometers have considered semi-symmetric spaces as a generalization of locally symmetric spaces. A Riemannian manifold M is said to be semi-symmetric if its curvature tensor R satisfies $R(U,V).R = 0$, where $R(U,V)$ acts on R as a derivation and also it is called Ricci-semisymmetric manifold if the relation $R(U,V).S = 0$ holds, where $R(U,V)$ the curvature operator.

A transformation which transforms every geodesic circle of a Riemannian manifold M into a geodesic circle, is known as concircular transformation ([7], [16]), where geodesic circle means a curve in M whose first curvature is constant and second curvature is identically zero. A concircular transformation is always a conformal transformation [7]. Thus the geometry of concircular transformations is a generalization of inversive geometry in the sense that the change of metric is more general than that induced by a circle preserving diffeomorphism (see also [5]). An invariant of a concircular transformation is the concircular curvature tensor C, which is defined by ([16], [17])

$$C(U,V)W = R(U,V)W - \frac{r}{n(n-1)}[g(V,W)U - g(U,W)V]. \tag{1.3}$$

Using (1.3), we obtain

$$g(C(U,V)W,Z) = g(R(U,V)W,Z) - \frac{r}{n(n-1)}[g(V,W)g(U,Z) - g(U,W)g(V,Z)], \tag{1.4}$$

where $U,V,W,Z \in \Gamma(TM)$ and r is the scalar curvature on Lorentzian para-Sasakian manifolds. Riemannian manifolds with vanishing concircular curvature tensor are of constant curvature. Thus the concircular curvature tensor is a measure of the failure of a Riemannian manifold to be of constant curvature.

In this paper, we study a type of quarter-symmetric metric connection on Lorentzian para-Sasakian manifolds. The paper is organized as follows: After introduction section two is equipped with some prerequisites of a Lorentzian para-Sasakian manifold. In section three, curvature tensor and Ricci tensor of Lorentzian para-Sasakian manifold with respect to the quarter-symmetric metric connection are given. Section four is devoted to study ξ-concircularly flat Lorentzian para-Sasakian manifold with respect to the quarter-symmetric metric connection. Quasi-concircularly flat and ϕ-concircularly flat Lorentzian para-Sasakian manifolds with respect to the quarter-symmetric metric connection have been studied in section five and six respectively. In next section, we investigate Ricci-semisymmetric manifolds with respect to the quarter-symmetric metric connection of a Lorentzian para-Sasakian manifold.

2. PRELIMINARIES

An n-dimensional differentiable manifold M is said to be a Lorentzian almost para-contact manifold, if it admits an almost para-contact structure (ϕ,ξ,η,g) consisting of a $(1,1)$ tensor field ϕ, vector field ξ, 1-form η and a Lorentzian metric g satisfying

$$\phi(\xi)=0,\;\; \eta\circ\phi=0,\;\; \eta(\xi)=-1,\;\; g(U,\xi)=\eta(U), \tag{2.1}$$

$$\phi^2U=U+\eta(U)\xi, \tag{2.2}$$

$$g(\phi U,\phi V)=g(U,V)+\eta(U)\eta(V), \tag{2.3}$$

$$(\nabla_U\eta)V=g(U,\phi V)=(\nabla_V\eta)U, \tag{2.4}$$

for any vector fields U, V on M. Such a manifold M is termed as Lorentzian para-contact manifold and the structure (ϕ,ξ,η,g) a Lorentzian para-contact structure [8].

If moreover (ϕ,ξ,η,g) satisfies the conditions

$$d\eta=0,\;\; \nabla_U\xi=\phi U, \tag{2.5}$$

$$(\nabla_U\phi)V=g(U,V)\xi+\eta(V)U+2\eta(U)\eta(V)\xi, \tag{2.6}$$

for U,V tangent to M, then M is called a Lorentzian para-Sasakian manifold or briefly LP-Sasakian manifold, where ∇ denotes the covariant differentiation with respect to Lorentzian metric g.

Moreover, the curvature tensor R, the Ricci tensor S and the Ricci operator Q in a Lorentzian para-Sasakian manifold M with respect to the Levi-Civita connection ∇ satisfies the following relations [13]

$$\eta(R(U,V)W)=g(V,W)\eta(U)-g(U,W)\eta(V), \tag{2.7}$$

$$R(\xi,U)V=g(U,V)\xi-\eta(V)U, \tag{2.8}$$

$$R(\xi,U)\xi=-R(U,\xi)\xi=U+\eta(U)\xi, \tag{2.9}$$

$$R(U,V)\xi = \eta(V)U - \eta(U)V, \tag{2.10}$$

$$S(U,\xi) = (n-1)\eta(U),\ Q\xi = (n-1)\xi \tag{2.11}$$

$$S(\phi U,\phi V) = S(U,V) + (n-1)\eta(U)\eta(V), \tag{2.12}$$

for all vector fields $U,V,W \in \Gamma(TM)$.

Definition 2.1. A Lorentzian para-Sasakian manifold M is said to be an $\eta-$Einstein manifold [13] if its Ricci tensor S of the Levi-Civita connection is of the form

$$S(U,V) = ag(U,V) + b\eta(U)\eta(V) \ \ for\ all\ U,V \in \Gamma(TM) \tag{2.13}$$

where a and b are smooth functions on the manifold M.

3. Curvature Tensor of Lorentzian Para-Sasakian Manifold with Respect to the Quarter-Symmetric Metric Connection

A relation between the quarter-symmetric metric connection $\bar{\nabla}$ and the Levi-Civita connection ∇ in an n-dimensional Lorentzian para-Sasakian manifold M is given by [15]

$$\bar{\nabla}_U V = \nabla_U V + \eta(V)\phi U - g(\phi U,V)\xi. \tag{3.1}$$

The curvature tensor $\bar{R}$ of a Lorentzian para-Sasakian manifold M with respect to the quarter-symmetric metric connection $\bar{\nabla}$ is defined by

$$\bar{R}(U,V)W = \bar{\nabla}_U\bar{\nabla}_V W - \bar{\nabla}_V\bar{\nabla}_U W - \bar{\nabla}_{[U,V]}W. \tag{3.2}$$

From the equations $(2.1)-(2.6)$, (3.1) and (3.2), we obtain

$$\begin{aligned} \bar{R}(U,V)W &= R(U,V)W + [g(\phi U,W)\phi V - g(\phi V,W)\phi U] \\ &\quad + [g(V,W)\eta(U) - g(U,W)\eta(V)]\xi \\ &\quad + \eta(W)[\eta(V)U - \eta(U)V]. \end{aligned} \tag{3.3}$$

where $U,V,W \in \Gamma(TM)$ and $R(U,V)W = \nabla_U \nabla_V W - \nabla_V \nabla_U W - \nabla_{[U,V]} W$ is the Riemannian curvature tensor with respect to the Levi-Civita connection ∇.

The Ricci tensor $\bar{S}$ and the Scalar curvature $\bar{r}$ in a Lorentzian para-Sasakian manifold M with respect to the quarter-symmetric metric connection $\bar{\nabla}$ are defined by

$$\bar{S}(V,W) = \sum_{i=1}^{n} \varepsilon_i g(\bar{R}(e_i,V)W,e_i), \tag{3.4}$$

$$\bar{r} = \sum_{i=1}^{n} \varepsilon_i \bar{S}(e_i,e_i) \tag{3.5}$$

where $\{e_1,e_2,....,e_{n-1},e_n = \xi\}$ be a local orthonormal basis of vector fields in M and $\varepsilon_i = g(e_i,e_i)$.

Now contracting U in (3.3), we get

$$\bar{S}(V,W) = S(V,W) + (n-1)\eta(V)\eta(W) - (trace\phi)g(\phi V,W). \tag{3.6}$$

Again contracting V and W in (3.6), we get

$$\bar{r} = r - (n-1) - (trace\phi)^2. \tag{3.7}$$

From equation (3.3) and (3.6), we have

$$\bar{R}(U,V)\xi = \bar{R}(\xi,U)V = 0, \tag{3.8}$$

$$\bar{S}(V,\xi) = 0, \tag{3.9}$$

$$\bar{S}(\phi U,\phi V) = \bar{S}(U,V). \tag{3.10}$$

4. ξ-Concircularly Flat Lorentzian Para-Sasakian Manifold with Respect to the Quarter-Symmetric Metric Connection

Definition 4.1. Concircular curvature tensor $\bar{C}$ of Lorentzian para-Sasakian manifold M with respect to the quarter-symmetric metric connection is defined by

$$\bar{C}(U,V)W = \bar{R}(U,V)W - \frac{\bar{r}}{n(n-1)}[g(V,W)U - g(U,W)V] \tag{4.1}$$

for all $U,V,W \in \Gamma(TM)$ where $\bar{R}$ is the curvature tensor and $\bar{r}$ is the scalar curvature of M with respect to the quarter-symmetric metric connection $\bar{\nabla}$

Definition 4.2. A Lorentzian para-Sasakian manifold is said to be $\xi-$concircularly flat [4] with respect to the quarter-symmetric metric connection $\bar{\nabla}$ if

$$\bar{C}(U,V)\xi = 0 \tag{4.2}$$

for all $U, V \in \Gamma(TM)$.

Putting $W = \xi$ in (4.1) and using (3.8) and (4.2), we have

$$\bar{r}[\eta(V)U - \eta(U)V] = 0. \tag{4.3}$$

Putting $U = \xi$ in (4.3) and using (2.1), we have

$$\bar{r}[V + \eta(V)\xi] = 0. \tag{4.4}$$

Taking inner product of (4.4) with W and replacing V by QV, we have

$$\bar{r}[g(QV,W) + \eta(QV)\eta(W)] = 0. \tag{4.5}$$

Using $S(V,W) = g(QV,W)$ and equations (2.11) and (3.7) in (4.5), we have

$$[r-(n-1)-(trace\phi)^2][S(V,W)+(n-1)\eta(V)\eta(W)] = 0 \tag{4.6}$$

Equation (4.6) implies that either $r = (n-1)+(trace\phi)^2$ or $S(V,W) = -(n-1)\eta(V)\eta(W)$. Thus we can state the following:

Theorem 4.3. *If a Lorentzian para-Sasakian manifold M admitting a quarter-symmetric metric connection is $\xi-$concircularly flat with respect to the quarter-symmetric metric connection, then either scalar curvature of M is $(n-1)+(trace\phi)^2$ or the manifold M is a special type of $\eta-$Einstein manifold with respect to the Levi-Civita connection.*

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5. Quasi-Concircularly Flat Lorentzian Para-Sasakian Manifold with Respect to the Quarter-Symmetric Metric Connection

Definition 5.1. A Lorentzian para-Sasakian manifold M is said to be quasi$-$concircularly flat with respect to the quarter-symmetric metric connection if

$$g(\bar{C}(\phi U,V)W,\phi Z)=0 \tag{5.1}$$

where U, V, W, $Z\in\Gamma(TM)$.

From equation (4.1), we have

$$\begin{aligned} g(\bar{C}(\phi U,V)W,\phi Z) &= g(\bar{R}(\phi U,V)W,\phi Z)-\frac{\bar{r}}{n(n-1)}[g(V,W)g(\phi U,\phi Z) \\ &\quad -g(\phi U,W)g(V,\phi Z)]. \end{aligned} \tag{5.2}$$

Using (5.1) in (5.2), we have

$$g(\bar{R}(\phi U,V)W,\phi Z)=\frac{\bar{r}}{n(n-1)}[g(V,W)g(\phi U,\phi Z)-g(\phi U,W)g(V,\phi Z)]. \tag{5.3}$$

Let $\{e_1,e_2,....,e_{n-1},\xi\}$ be a local orthonormal basis of vector fields in M, then $\{\phi e_1,\phi e_2,....,\phi e_{n-1},\xi\}$ is also a local orthonormal basis. Putting $U=Z=e_i$ in (5.3) and summing over $i=1$ to $n-1$, we obtain

$$\sum_{i=1}^{n-1} g(\bar{R}(\phi e_i,V)W,\phi e_i)=\frac{\bar{r}}{n(n-1)}\sum_{i=1}^{n-1}[g(V,W)g(\phi e_i,\phi e_i)-g(\phi e_i,W)g(V,\phi e_i)], \tag{5.4}$$

On LP-Sasakian manifold it can be verify that

$$\sum_{i=1}^{n-1} g(\bar{R}(\phi e_i,V)W,\phi e_i)=\bar{S}(V,W), \tag{5.5}$$

$$\sum_{i=1}^{n-1} g(\phi e_i,\phi e_i)=n-1, \tag{5.6}$$

$$\sum_{i=1}^{n-1} g(\phi e_i,W)g(V,\phi e_i)=g(V,W)+\eta(V)\eta(W). \tag{5.7}$$

So by virtue of (5.5), (5.6) and (5.7), the equation (5.4) takes the form

$$\bar{S}(V,W)=\left[\frac{\bar{r}(n-2)}{n(n-1)}\right]g(V,W)-\left[\frac{\bar{r}}{n(n-1)}\right]\eta(V)\eta(W).$$

or

$$\bar{S}(V,W) = ag(V,W) + b\eta(V)\eta(W),$$

where $a = \left[\frac{\bar{r}(n-2)}{n(n-1)}\right]$ and $b = -\left[\frac{\bar{r}}{n(n-1)}\right]$.

From which it follows that the manifold is an $\eta-$Einstein manifold with respect to the quarter-symmetric metric connection.

Hence we can state the following theorem:

Theorem 5.2. *If a Lorentzian para-Sasakian manifold admitting a quarter-symmetric metric connection is quasi-concircularly flat, then the manifold with respect to the quarter-symmetric metric connection is an $\eta-$Einstein manifold.*

6. ϕ-Concircularly Flat Lorentzian Para-Sasakian Manifold with Respect to the Quarter-Symmetric Metric Connection

Definition 6.1. A Lorentzian para-Sasakian manifold is said to be $\phi-$concircularly flat with respect to the quarter-symmetric metric connection if

$$g(\bar{C}(\phi U,\phi V)\phi W,\phi Z) = 0, \tag{6.1}$$

where U, V, W, $Z \in \Gamma(TM)$.

From equation (4.1), we have

$$\begin{aligned} g(\bar{C}(\phi U,\phi V)\phi W,\phi Z) &= g(\bar{R}(\phi U,\phi V)\phi W,\phi Z) - \frac{\bar{r}}{n(n-1)}[g(\phi V,\phi W)g(\phi U,\phi Z) \\ &\quad -g(\phi U,\phi W)g(\phi V,\phi Z)]. \end{aligned} \tag{6.2}$$

Using (6.1) in (6.2), we have

$$g(\bar{R}(\phi U,\phi V)\phi W,\phi Z) = \frac{\bar{r}}{n(n-1)}[g(\phi V,\phi W)g(\phi U,\phi Z) - g(\phi U,\phi W)g(\phi V,\phi Z)]. \tag{6.3}$$

Let $\{e_1, e_2, ..., e_{n-1}, \xi\}$ be a local orthonormal basis of vector fields in M, then $\{\phi e_1, \phi e_2, ..., \phi e_{n-1}, \xi\}$ is also a local orthonormal basis. Putting $U = Z = e_i$ in (6.3) and summing over $i=1$ to $n-1$, we obtain

(6.4)
$$\sum_{i=1}^{n-1} g(\bar{R}(\phi e_i,\phi V)\phi W,\phi e_i)=\frac{\bar{r}}{n(n-1)}\sum_{i=1}^{n-1}[g(\phi V,\phi W)g(\phi e_i,\phi e_i)-g(\phi e_i,\phi W)g(\phi V,\phi e_i)],$$

So by virtue of (3.10), (5.5), (5.6) and (5.7), the equation (6.4) takes the form

$$\bar{S}(V,W)=\left[\frac{\bar{r}(n-2)}{n(n-1)}\right]g(V,W)-\left[\frac{\bar{r}(n-2)}{n(n-1)}\right]\eta(V)\eta(W).$$

or

$$\bar{S}(V,W)=ag(V,W)+b\eta(V)\eta(W),$$

where $a=\left[\frac{\bar{r}(n-2)}{n(n-1)}\right]$ and $b=-\left[\frac{\bar{r}(n-2)}{n(n-1)}\right]$.

From which it follows that the manifold is an $\eta-$Einstein manifold with respect to the quarter-symmetric metric connection.

Hence we can state following theorem:

Theorem 6.2. *A ϕ-concircularly flat Lorentzian para-Sasakian manifold with respect to the quarter-symmetric metric connection is an $\eta-$Einstein manifold with respect to the quarter-symmetric metric connection.*

7. Lorentzian Para-Sasakian Manifold Satisfying $\bar{C}\cdot\bar{S}=0$ with Respect to the Quarter-Symmetric Metric Connection

We consider Lorentzian para-Sasakian manifolds with respect to a quarter-symmetric metric connection $\bar{\nabla}$ satisfying the curvature condition $\bar{C}\cdot\bar{S}=0$. Then

$$\left(\bar{C}(U,V)\cdot\bar{S}\right)(W,Z)=0.$$

So,

$$\bar{S}(\bar{C}(U,V)W,Z)+\bar{S}\left(W,\bar{C}(U,V)Z\right)=0. \tag{7.1}$$

Putting $U=\xi$ in (7.1), we get

$$\bar{S}(\bar{C}(\xi,V)W,Z)+\bar{S}\left(W,\bar{C}(\xi,V)Z\right)=0. \tag{7.2}$$

Now from (3.8) and (4.1), we have

$$\bar{C}(\xi,V)W=-\frac{\bar{r}}{n(n-1)}[g(V,W)\xi-\eta(W)V]. \tag{7.3}$$

Using (7.3) in (7.2) and putting $W=\xi$ and using (3.9), we obtain

$$\bar{r}\bar{S}(V,Z)=0.$$

This implies that $\bar{r}=0$.

Hence we can state following:

Theorem 7.1. If Lorentzian para-Sasakian manifolds satisfying $\bar{C}\cdot\bar{S}=0$ with respect to the quarter-symmetric metric connection, then the manifold is scalar flat with respect to the quarter-symmetric metric connection.

Example 1. Example of a *LP*-Sasakian manifold with respect to Quarter-symmetric metric connection.

Taking a $3-$dimensional manifold $M=\{(x,y,v)\in R^3\}$, where (x,y,v) are standard coordinates of R^3. Let e_1,e_2,e_3 are vector fields on M, given by

$$e_1=-e^v\frac{\partial}{\partial x},\qquad e_2=-e^{v-x}\frac{\partial}{\partial y},\qquad e_3=-\frac{\partial}{\partial v}=\xi,$$

Clearly, $\{e_1,e_2,e_3\}$ is linearly independent set of vectors on M. So it forms a basis of $\Gamma(TM)$. The Lorentzian metric g is defined by

$$\begin{aligned} g(e_i,e_j) &= 0,\ \ for\ i\neq j\ and\ 1\leq i,j\ \leq 3 \\ \text{and}\quad g(e_1,e_1) &= g(e_2,e_2)=1,\quad g(e_3,e_3)=-1. \end{aligned}$$

Let η be a $1-$form on M defined as $\eta(U)=g(U,e_3)=g(U,\xi)$, for all $U\in\Gamma(TM)$, and let ϕ be a $(1,1)$ tensor field on M defined as

$$\phi(e_1) = -e_1, \ \ \phi(e_2) = -e_2, \quad \phi(e_3) = 0.$$

By applying linearity of ϕ and g, we have

$$\eta(e_3) = -1, \quad \phi^2(U) = U + \eta(U)\xi,$$

and

$$g(\phi U, \phi V) = g(U,V) + \eta(U)\eta(V) \quad \text{for all} \quad U,V \in \Gamma(TM).$$

Let ∇ be a Levi-Civita connection with respect to the Riemannian metric g, we have

$$[e_1, e_2] = -e^v e_2, \quad [e_2, e_3] = -e_2, \quad [e_1, e_3] = -e_1,$$

The Riemannian connection ∇ of the metric g is given by

$$\begin{aligned} 2g(\nabla_U V, W) &= Ug(V,W) + Vg(W,U) - Wg(U,V) \\ &\quad -g(U,[V,W]) - g(V,[U,W]) + g(W,[U,V]), \end{aligned}$$

which is known as Koszul's formula, we can easily calculate

$$\begin{aligned} \nabla_{e_1} e_1 &= e_3, & \nabla_{e_1} e_2 &= 0, & \nabla_{e_1} e_3 &= -e_2, \\ \nabla_{e_2} e_1 &= -e^v e_2, & \nabla_{e_2} e_2 &= -e_3 - e^v e_1, & \nabla_{e_2} e_3 &= -e_2, \\ \nabla_{e_3} e_1 &= 0, & \nabla_{e_3} e_2 &= 0, & \nabla_{e_3} e_3 &= 0, \end{aligned}$$

From the above it follows that the manifold satisfies $\nabla_U \xi = \phi U$, for $\xi = e_3$ and $(\nabla_U \phi)V = g(U,V)\xi + \eta(V)U + 2\eta(U)\eta(V)\xi$. Hence the manifold is $LP-$Sasakian manifold.

Using (3.1), we have

$$\begin{aligned}
\overline{\nabla}_{e_1}e_1 &= 0, \qquad \overline{\nabla}_{e_1}e_2 = 0, \qquad \overline{\nabla}_{e_1}e_3 = 0,\\
\overline{\nabla}_{e_2}e_1 &= -e^v e_2, \qquad \overline{\nabla}_{e_2}e_2 = -e^v e_1, \qquad \overline{\nabla}_{e_2}e_3 = 0,\\
\overline{\nabla}_{e_3}e_1 &= 0, \qquad \overline{\nabla}_{e_3}e_2 = 0, \qquad \overline{\nabla}_{e_3}e_3 = 0,
\end{aligned}$$

Using (1.1), the torsion tensor $\overline{T}$, with respect to quarter symmetric metric connection $\overline{\nabla}$ as follows :

$$\begin{aligned}
\overline{T}(e_i,e_i) &= 0, \quad \forall i = 1,2,3,\\
\overline{T}(e_1,e_2) &= 0, \quad \overline{T}(e_1,e_3) = e_3, \quad \overline{T}(e_2,e_3) = e_2,
\end{aligned}$$

Also,

$$(\overline{\nabla}_{e_1}g)(e_2,e_3) = 0, \quad (\overline{\nabla}_{e_2}g)(e_3,e_1) = 0, \quad (\overline{\nabla}_{e_3}g)(e_1,e_2) = 0,$$

Thus M is a Lorentzian para-Sasakian manifold admitting quarter-symmetric metric connection $\overline{\nabla}$.

Conflict of Interests

The authors declare that there is no conflict of interests.

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