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MODELLING THE DYNAMICS OF JIGGERS INFESTATION: INSIGHTS FROM A THEORETICAL MODEL

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Abstract. Tungiasis is a parasitic skin disease caused by jiggers, also known as sand fleas. The disease predominantly affects impoverished populations living in Sub-Saharan Africa, the Caribbean and South America. Mathematical models have been used for decades to inform public health policies and have been useful for the evaluation of control strategies and interventions. In this paper we consider a deterministic model with four compartments that represent the dynamics of the human population and a stage-structured model for the flea. The model equilibria are computed and stability analyses carried out based on the reproduction number R_0 . Sensitivity analysis is carried out on the model parameters and it is observed that the epidemic is driven by infested humans and the survival of the flea through its developmental stages. The model points to a focused control of the flea through larvicides and treatment of infested humans. The paper is concluded by discussing the public health implications of the mathematical results.

Keywords: jiggers; stability; reproduction number; sensitivity analysis; simulations.

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1. INTRODUCTION

Tungapenetrans also known as chigger, jigger, chigoe or sand flea, is a flea mainly found in East and West Africa, the Caribbean, especially in Trinidad, Central and South America and India [10]. As of 2009, Tungiasis was present in 88 countries, with varying degrees of incidence. In 2010 alone, Busoga region in Uganda registered 20 deaths and 20,000 severe cases related to jiggers infestation [16]. According to a study carried out in Muranga South District in Kenya on children between the ages of 5 and 12, the prevalence of Tungiasis was at 57 percent suggesting that it is highly endemic in rural central Kenya [14]. The prevalence of jigger infestation further is found to be, 45.2 percent in Nigerian community in Lagos state [33], 51.1 percent in North Eastern Brazil [25], 75 percent in the village of Rural Haiti [17] and 65.9 percent in Murang'a North District, Kenya [32].

Jiggers are small pin-head-sized chigoe fleas found in sandy terrains of warm, dry climates. It hides in the crevices and cracks found on the floors, walls of dwellings and items like furniture and it feeds on warm blooded hosts including man, cats, dogs, rats, pigs, cattle and sheep. The female flea feeds by burrowing into the skin of its host. The abdomen becomes enormously enlarged between the second and third segments so that the flea forms a round sac with the shape and size of a pea. The impregnated female Tunga embeds itself in the skin under the toe-nails and fingernails of humans, where the resultant sores may fill with pus and become infected. After two weeks, over 100 eggs are released through the exposed skin opening and fall to the ground. The flea then dies and is slowly sloughed by the host's skin. The eggs hatch in the dust within 3-4 days. In 21-28 days, they go through their larval and pupal stages and become adults. The complete life cycle of a Tungapenetrans lasts for about a month [1].

In Kenya, all the eight provinces, now the forty seven counties, have reported cases of jigger infestation, with a few isolated cases in the capital, Nairobi. The effects of jigger infestation are profound. School children often drop out of school, and HIV/AIDS spreads among the infested communities through sharing of pins and other jiggers removing equipment. Heavy infestations may lead to severe inflammation, ulceration, and fibrosis. Lymphangitis, gangrene, sepsis, the loss of toenails, auto amputation of the digits, and even death may occur. The risk of secondary

infections is high with Tetanus being the most common secondary infection associated with jiggers. Jiggers infestation is linked with poverty and lack of proper hygiene [1].

The social implications of jiggers infestation on communities has also been investigated and specifically its effects on development in Kenya, see [27] and knowledge, attitude and practices on jigger infestation among house hold members , see [3]. The effects on academic achievement in public primary schools [26] and the epidemiology of tunga penetrans infestation in selected areas in Kahuru constituency, Murang'a County [14], have also been carried out. The control of tungiasis in impoverished communities [6], and the seasonal variation of tungiasis [11] have also been investigated. Other studies have been done in rural Cameroon, see [8], Lagos state, Nigeria [33] and globally [20]. From a health point of view, a number of studies have also been done, see for instance [9, 30]. Research on similar diseases in areas where tungiasis is common has been done in recent years. These include vector-borne diseases distribution [19, 23, 4], onchocerciasis [18] and the impact of mass administration of ivermectin in the treatment of onchocerciasis [7].

While a lot of research has been done on jiggers, most of it has focused on social and community awareness. Very little has been done on the mathematical modeling of jigger infestation and interventions in humans which might be key to the fight against the infestation. The most recent ones have been on the dynamics of tungiasis transmission in endemic areas [13] and in zoonotic areas [12]. In the later, the authors formulated a deterministic model of tungiasis which involved the interactions between humans, animals and the sand fleas in the environment. We argue that the two models do not capture the interactions of the humans and the environment adequately as they left out the source of the larvae, i.e the eggs. The authors assumed that that the flea dynamics are such that infested humans and animals contribute the flea larvae into the environment and not the eggs and also that the adult fleas directly develop from severely infested animals, see [12].

In this paper, we consider a model for jiggers infestation with the aim of determining threshold conditions for the persistence of the infestation and parameters that influence the dynamics of Tungiasis. We present a model that comprehensively represents the dynamics of the flea

population's developmental stages. We strictly consider the flea as the sole driver of infestation. The pregnant female flea burrows in a human host for the purposes of laying eggs and our model considers a complete cycle of the flea population. We determine the steady states and their stabilities. Sensitivity analysis and numerical simulations are carried out with the aim of determining the potential impact of some parameters on the *Tungapenetrans* dynamics.

This work is arranged as follows. In Section 2 we perform model formulation and in Section 2.1 we carry out analysis of the model by establishing important thresholds such as the reproduction number R_0 and different equilibria of the model. We then demonstrate the stability of equilibria and carry out some bifurcation analysis. In section 3, we carry out numerical simulations which include, sensitivity analysis, parameter variation and contour plots which are used to assess the influence of some important parameters in the jiggers infestation. Section 3.3 concludes the paper.

2. PRELIMINARIES

MODEL FORMULATION

We propose a deterministic model where the human population is categorized into four compartments at any time $t > 0$, comprising of susceptible humans $S(t)$, infested humans $I(t)$, chronically infested humans, $C(t)$ and recovered humans $R(t)$. The recovered humans are assumed to recover as a result of treatment. The total size of the human population is thus given by

$$N(t) = S(t) + I(t) + C(t) + R(t).$$

The flea cycle is categorized into three compartments, at any $t > 0$, of eggs E , coming as a result of the of adult flea from, those of the infested in compartment I and the chronically infested in compartment C , the larvae and pupa, combined into one stage, L and the adult flea F .

The human population is recruited at the rate Π which is assumed to be generated through births and as the female flea burrows into the susceptible human skin, a susceptible individual moves to the infested compartment I . All recruited individuals are assumed to be susceptible. The generation of new infestations is modeled by the expression βSF where β is the effective

infestation rate of the susceptible humans by the adult female fleas. The expression βSF , is basically the force of infestation. However, some infested humans may become chronically infested and join compartment C at a rate α . By chronically infested we mean individuals with severe infestations that may require thorough treatment. The chronically infested can be treated and recover at a rate γ_2 . Once an individual is infested, recovery is possible through treatment at a rate γ_1 . This could be made possible by removing the fleas from their cavity using sterile instruments followed by thorough cleaning and covering the remaining crater with topical antibiotic to prevent secondary infestation. This could be more difficult if the infestation is engorged [29]. Also a two component dimethicone, administered directly to affected area reduces all embedded sand fleas by 80 to 95 percent [22]. Recovery from infestation does not provide protection from further infestation. Individuals in the recovered class can move back to the susceptible class at a rate ω . Individuals in each class die naturally at a rate μ .

Fleas' eggs are released by individuals infested, that is, those in classes I and C at rates τ_1 and τ_2 respectively. The parameter τ_1 is assumed to be smaller than τ_2 since the chronically infested humans generate more eggs than the infested humans in I . We assume that from each infested individual N_i and N_c eggs are released from those in I and C respectively. The laid eggs develop into a combined pupa and larvae stage at a rate ρ and the eggs die naturally at a rate v_e . The larvae develop into adult fleas at a rate δ and die naturally at a rate v_l . We assume that a proportion ε of the larvae develop into adult female fleas. The development of the adult fleas is modeled by a saturation function $\frac{\varepsilon\delta L}{1+L}$ where $\varepsilon\delta$ is the maximum number of female adult fleas that will eventually be involved in the transmission of jiggers to the susceptible population. However the adult female fleas die naturally at a rate v_f . The diagram for the model describing the dynamics of jiggers infestation is shown in Figure 1.

The model description, assumptions and Figure 1 result in the following system of non-linear ordinary differential equations:

TABLE 1. Symbols and descriptions of state variables considered in the model

Variables	Description
$S(t)$	Susceptible humans,
$I(t)$	Infested humans,
$C(t)$	Chronically infested humans,
$R(t)$	Recovered humans,
$F(t)$	Adult flea,
$L(t)$	Pupa + larvae stage,
$E(t)$	Eggs from adult fleas in humans

$$(1) \quad \left. \begin{aligned} \frac{dS}{dt} &= \Pi + \omega R - \beta FS - \mu S, \\ \frac{dI}{dt} &= \beta FS - (\mu + \alpha + \gamma_1)I, \\ \frac{dC}{dt} &= \alpha I - (\mu + \gamma_2)C, \\ \frac{dR}{dt} &= \gamma_1 I + \gamma_2 C - (\mu + \omega)R, \\ \frac{dE}{dt} &= N_i \tau_1 I + N_c \tau_2 C - (v_e + \rho)E, \\ \frac{dL}{dt} &= \rho E - \left(v_l + \frac{\varepsilon \delta}{1+L} \right) L, \\ \frac{dF}{dt} &= \frac{\varepsilon \delta L}{1+L} - v_f F. \end{aligned} \right\}$$

The initial conditions of the system (1) are given by: $S(0) = S_0 > 0$, $I(0) = I_0 \geq 0$, $C(0) = C_0 \geq 0$, $R(0) = R_0 \geq 0$, $E(0) = E_0 \geq 0$, $L(0) = L_0 \geq 0$ and $F(0) = F_0 > 0$.

2.1. Invariant region. Given that N represents the total human population so that $N(t) = S(t) + I(t) + C(t) + R(t)$. On substituting the derivatives in model system (1) and simplifying, we have

$$\frac{dN}{dt} \leq \Pi - \mu N.$$

TABLE 2. Symbols and definitions of parameters used in the model

Parameter	Description
Π	Recruitment rate of susceptibles
β	Rate at which the susceptible humans become infested with the flea
γ_1	Rate at which infested humans recover after treatment
α	Rate at which infested humans become chronically infested
μ	Natural death rate for humans
δ	Rate at which larvae develop into adult fleas
ε	proportion of larvae that develop into adult female fleas
ν_f	Natural death rate of the adult female flea
ν_l	Natural death rate of the larvae
ν_e	Natural death rate of the eggs
ρ	Rate at which the eggs develop into the combined pupa and larvae stage
τ_1	Rate of egg production from infested humans by adult female fleas
τ_2	Rate of egg production from chronically infested humans by adult female fleas
γ_1	Rate at which chronically infested humans recover after treatment
ω	Rate at which the recovered humans become susceptible to jigger infestation
N_i	Number of eggs are released per adult flea from infested humans
N_c	Number of eggs are released per adult flea from chronically infested humans

Using integrating factor, we obtain

$$N(t) \leq \frac{\Pi}{\mu} + \left(N_0 - \frac{\Pi}{\mu}\right) e^{-\mu t}.$$

Thus as t approaches infinity the right hand side of the inequality becomes $\frac{\Pi}{\mu}$. Thus we conclude that $N(t) \leq \max \left\{ N(0), \frac{\Pi}{\mu} \right\}$ for all time $t > 0$.

From the flea cycle we consider the egg, larvae and flea populations separately. For the egg we have

$$\frac{dE}{dt} \leq N_i \tau_1 \frac{\Pi}{\mu} + N_c \tau_2 \frac{\Pi}{\mu} - (\nu_e + \rho)E \leq \Lambda - (\nu_e + \rho)E.$$

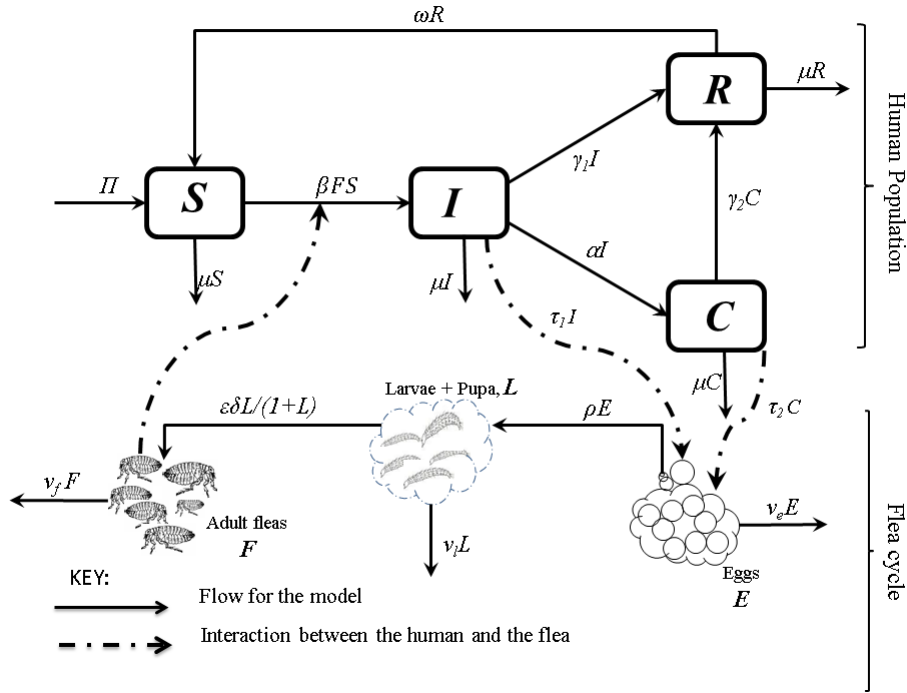


FIGURE 1. The model diagram describing the dynamics of jigger infestation

Through integration, we obtain

$$E(t) = \frac{\Lambda}{v_e + \rho} + \left(E_0 - \frac{\Lambda}{v_e + \rho} \right) e^{-(v_e + \rho)t}.$$

As time t goes to infinity the right hand side of the equation approaches $\frac{\Lambda}{v_e + \rho}$ where $\Lambda = \frac{\Pi}{\mu} (N_i \tau_1 + N_c \tau_2)$.

We see that

$$E(t) \leq \max \left(E(0), \frac{\Lambda}{v_e + \rho} \right) \text{ for all time } t > 0.$$

For the larvae we have

$$\frac{dL}{dt} \leq \left(\frac{\Lambda}{v_e + \rho} \right) \rho - \left(v_e + \frac{\varepsilon \delta}{1 + L} \right) L,$$

but since $\frac{\varepsilon \delta}{1 + L} < \varepsilon \delta$ the inequality reduces to

$$\frac{dL}{dt} \leq \left(\frac{\Lambda}{v_e + \rho} \right) \rho - (v_e + \varepsilon \delta) L.$$

We thus obtain

$$L(t) \leq \frac{\rho \Lambda}{v_e + \varepsilon \delta} - \left(L_0 - \frac{\rho \Lambda}{v_e + \varepsilon \delta} \right) e^{-(v_e + \varepsilon \delta)t}.$$

As t tends to infinity we obtain $L(t) \leq \frac{\rho\Lambda}{v_e + \epsilon\delta}$. Thus

$$L(t) \leq \max\left(L(0), \frac{\rho\Lambda}{v_e + \epsilon\delta}\right).$$

Finally, for the flea we have

$$\frac{dF}{dt} \leq \frac{\epsilon\delta K_L}{1 + K_L} - v_f F.$$

Through integration we obtain

$$F(t) \leq \frac{\Gamma}{v_f} + \left(F_0 - \frac{\Gamma}{v_f}\right) e^{-v_f t}$$

where $\Gamma = \frac{\epsilon\delta K_L}{1 + K_L}$ and as t approaches infinity we get $F \leq \frac{\Gamma}{v_f}$. Since $\frac{\epsilon\delta}{1 + K_L} < \epsilon\delta$, the inequality reduces to $F \leq \frac{\epsilon\delta}{v_f}$. Clearly we have

$$F(t) \leq \max\left(F(0), \frac{\epsilon\delta}{v_f}\right)$$

We have, through the above derivations, shown the existence of a bounded positive invariant region for our model system (1). We denote this region by $\Omega \in \mathbb{R}_+^7$, where

$$\Omega = \left\{ (S, I, C, R, E, L, F) \in \mathbb{R}_+^7 : N(t) \leq \max\left(N(0), \frac{\Pi}{\mu}\right), E(t) \leq \max\left(E(0), \frac{\Lambda}{v_e + \rho}\right), \right. \\ \left. L(t) \leq \max\left(L(0), \frac{\rho\Lambda}{v_e + \epsilon\delta}\right), F(t) \leq \max\left(F(0), \frac{\epsilon\delta}{v_f}\right) \right\}$$

Moreover, any solution of our model system (1) which commences in Ω at any time $t \geq 0$ will always remain confined in that region. We therefore deduce that the region Ω is positively invariant and attracting with respect to the dynamics of jigger infestation model. Our dynamics of jigger infestation model are therefore well posed mathematically and biologically meaningful.

MODEL ANALYSIS

2.2. Basic reproduction number. In the absence of any infestation, we have the jigger free steady state (JFS) of the system (1), given by $\mathcal{E}^0 = (S^0, I^0, C^0, R^0, E^0, L^0, F^0)$. At \mathcal{E}^0 , the state variables $I(t), C(t), R(t), E(t), L(t)$ and $F(t)$ are equal to zero, hence we obtain, $S^0 = \frac{\Pi}{\mu}$. The JFS point (\mathcal{E}^0) for the system (1) is thus given by

$$\mathcal{E}^0 = \left(\frac{\Pi}{\mu}, 0, 0, 0, 0, 0, 0\right).$$

Following the description in [24], the reproduction number R_0 is defined as the average number of the secondary cases arising from an average primary jiggers infestation case in an entirely

non-infested population.

Following [24], the transmission matrix \mathcal{F} and the matrix of transitions \mathcal{V} at \mathcal{E}^0 , are

$$\mathcal{F} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\beta\Pi}{\mu} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \mathcal{V} = \begin{pmatrix} \mu + \gamma_1 + \alpha & 0 & 0 & 0 & 0 \\ -\alpha & \mu + \gamma_2 & 0 & 0 & 0 \\ -N_i\tau_1 & -N_c\tau_2 & \rho + v_e & 0 & 0 \\ 0 & 0 & -\rho & \delta\varepsilon + v_l & 0 \\ 0 & 0 & 0 & -\delta\varepsilon & v_f \end{pmatrix}.$$

The basic reproduction number R_0 , is thus the spectral radius of $\mathcal{F}\mathcal{V}^{-1}$, and is given by

$$(2) \quad R_0 = R_0^I + R_0^{CI}$$

where

$$R_0^{CI} = \left(\frac{\Pi}{\mu}\right) \left(\frac{\beta}{\mu + \gamma_2}\right) \left(\frac{\alpha}{\mu + \gamma_1 + \alpha}\right) \left(\frac{\rho}{\rho + v_e}\right) \left(\frac{\delta\varepsilon}{\delta\varepsilon + v_l}\right) \left(\frac{\tau_2 N_c}{v_f}\right) \quad \text{and}$$

$$R_0^I = \left(\frac{\Pi}{\mu}\right) \left(\frac{\beta}{\mu + \gamma_1 + \alpha}\right) \left(\frac{\rho}{\rho + v_e}\right) \left(\frac{\delta\varepsilon}{\delta\varepsilon + v_l}\right) \left(\frac{\tau_1 N_i}{v_f}\right).$$

From the basic reproduction number R_0 , it can be clearly seen that; $\frac{1}{\mu + \gamma_2}$ is the duration of stay in class C , $\frac{1}{\mu + \gamma_1 + \alpha}$ is the duration of stay in class I , $\frac{\alpha}{\mu + \gamma_1 + \alpha}$ is the fraction of individual that move from compartment I to compartment C , $\frac{\rho}{\rho + v_e}$ is the fraction of eggs that become pupa/larvae, $\frac{\delta\varepsilon}{\delta\varepsilon + v_l}$ is the fraction of larvae/pupa that become adult fleas and $\frac{1}{v_f}$ is the survival of the fleas.

From Theorem 2 in [24], we have the following result:

Theorem 2.1. *The JFE of the system of equations (1) is locally asymptotically stable when $R_0 < 1$ and unstable otherwise.*

2.3. The jigger persistent equilibrium. Solving the equations in (1) at steady states by equating the left hand side to zero we obtain the following equation in terms of L , after some tedious algebraic manipulations:

$$(3) \quad (A_2L^2 + A_1L + A_0)L = 0,$$

Note that the case $L = 0$, corresponds to the jigger-free equilibrium treated earlier. Thus, the existence and number of endemic equilibria is determined by the positive roots of the following polynomial

$$(4) \quad h(L) = A_2L^2 + A_1L + A_0 = 0.$$

We note that

$$A_0 \begin{cases} < 0 & \text{if } R_0 > 1, \\ > 0 & \text{if } R_0 < 1, \end{cases}$$

where A_2, A_1, A_0 and K_1 are worked out as shown below.

We let $Q_1 = \mu + \alpha + \gamma_1$, $Q_2 = \mu + \gamma_2$, $Q_3 = \mu + \omega$, $Q_4 = v_e + \rho$, $Q_5 = v_l + \varepsilon\delta$, $\tau_3 = N_i\tau_1$ and $\tau_4 = N_c\tau_2$, so that

$$(5) \quad \left. \begin{aligned} C &= \psi_1 I, R = \psi_2 I, E = \psi_3 I, I = \frac{L(\delta\varepsilon + v_l + Lv_l)}{(1+L)\rho\psi_3}, \\ F &= \frac{L\delta\varepsilon}{(1+L)v_f}, S = \frac{v_f(L\omega(\delta\varepsilon + (1+L)v_l)\psi_2 + (1+L)\Pi\rho\psi_3)}{\rho(L\beta\delta\varepsilon + (1+L)\mu v_f)\psi_3} \end{aligned} \right\}$$

where $\psi_1 = \frac{\alpha}{Q_2}$, $\psi_2 = \frac{\gamma_1 + \gamma_2\psi_1}{Q_3}$ and $\psi_3 = \frac{\tau_3 + \tau_4\psi_1}{Q_4}$.

Substituting the variables in (5), into the second equation of system (1), we obtain

$$L[(\delta\varepsilon + (1+L)v_l)(Q_1(L\beta\delta\varepsilon + (1+L)\mu v_f) - L\beta\delta\varepsilon\omega\psi_2) - (1+L)\beta\delta\varepsilon\Pi\rho\psi_3] = 0,$$

from which we obtain $L = 0$ corresponding to the jiggers free equilibrium and

$$h(L) = (\delta\varepsilon + (1+L)v_l)(Q_1(L\beta\delta\varepsilon + (1+L)\mu v_f) - L\beta\delta\varepsilon\omega\psi_2) - (1+L)\beta\delta\varepsilon\Pi\rho\psi_3 = 0.$$

The roots of the polynomial $h(L) = 0$ give the endemic equilibrium which is in our case the jigger persistent equilibrium

$$A_2 = \beta\delta\varepsilon v_l Q_1 \left(1 - \frac{\omega\psi_2}{Q_1} \right) + \mu v_f v_l Q_1 > 0,$$

$$\begin{aligned} A_1 &= \mu v_f Q_1 Q_5 (1 - \mathcal{R}_0) + \mu v_f v_l Q_1 + \beta\delta\varepsilon Q_1 Q_5 \left(1 - \frac{\omega\psi_2}{Q_1} \right), \\ &= (K_1 - \mathcal{R}_0) \end{aligned}$$

$$A_0 = \mu v_f Q_1 Q_5 (1 - \mathcal{R}_0),$$

where $K_1 = \mu v_f Q_1 Q_5 + \mu v_f v_l Q_1 + \beta \delta \varepsilon Q_1 Q_5 \left(1 - \frac{\omega \psi_2}{Q_1}\right) > 0$.

The quadratic equation in (4) can be analyzed to investigate the existence of multiple equilibria when the reproduction number is less than unity. If the parameter, v_f , that accounts for more jigger infestations in humans in model system (1) is excluded that is, $v_f = 0$, (4) reduces to a linear equation

$$A_2' L + A_1' = 0,$$

where $A_2' = \beta \delta \varepsilon v_l Q_1 \left(1 - \frac{\omega \psi_2}{Q_1}\right)$ and $A_1' = \beta \delta \varepsilon Q_1 Q_5 \left(1 - \frac{\omega \psi_2}{Q_1}\right)$. So the model (1) will have a unique solution $L = -\frac{A_1'}{A_2'}$, which is non negative if and only if $R_0 > 1$. Hence if $v_f = 0$ model system (1) has a unique endemic equilibrium whenever $R_0 > 1$ and thus equilibrium approaches zero as R_0 tends to one ($R_0 \rightarrow 1+$) because $A_1' \rightarrow 0$ and there will be no positive endemic equilibria if $R_0 < 1$.

For the case $v_f \neq 0$, if $R_0 = 1$, then $A_0 = 0$ and there is a unique nonzero solution of equation (4), $L = -\frac{(R_0 - K_1)}{A_2}$ which is positive if and only if $(K_1 - R_0) < 0$ since $A_2 > 0$. Depending on the signs of $A_1 (K_1 - R_0)$ and A_0 , we may have a unique positive root, two or no positive roots. In fact $h(L)$ is quadratic function that is concave up with $h(0) > 0$ if $R_0 < 1$. If $h(0) > 0$ then the function $h(L)$ has at most two positive roots. Also, if $h(0) < 0$, then $R_0 > 1$. In this case a geometrical consideration of $h(L)$ shows that $h(L)$ has a unique positive root. We thus have the following results on the existence of the equilibria of the system (1).

Theorem 2.2.

- (i) *A unique endemic equilibrium point exists if $R_0 > 1$,*
- (ii) *A positive endemic equilibrium point exists if $(K_1 - R_0) < 0$ and*

$$\Delta = ((K_1 - R_0))^2 - 4A_2A_0 = 0 \text{ or } R_0 = 1,$$
- (iii) *Two endemic equilibria exists if $(K_1 - R_0) < 0$ and $R_0 < 1$,*
- (iv) *No endemic equilibrium exists otherwise.*

The result in Theorem 2.2 (iii), suggests that the model system (1) exhibits backward bifurcation for $R_0 < 1$ and case (i) of Theorem 2.3 demonstrates that the model has a unique endemic

equilibrium when $R_0 > 1$. So, case (iii) shows the possibility of backward bifurcation in which a locally asymptotically stable jiggers free equilibrium point coexists with a locally asymptotically stable endemic equilibrium point when $R_0 < 1$. In this case an endemic equilibrium point is reached instead of the jigger free equilibrium point even when the reproduction number is less than unity depending on how many infestations occur in the population at some critical value of R_0 , denoted by R_0^c . Here, R_0^c is the positive root of $\Delta = 0$ when solved for R_0 .

It is important to note that R_0^c is critical threshold because no endemic exists when $R_0 < R_0^c$. To successfully clear the jiggers infestation, the reproduction number should be brought below R_0^c . The condition $R_0 < 1$ is not sufficient for the elimination of jiggers infestation. The direction of bifurcation $R_0 = 1$ of the endemic equilibrium is proved by the direct use of the Center Manifold Theory (CMT) as described in [5]. The theorem is stated as follows;

Theorem 2.3. *Considering the following general system of ordinary differential equations with parameter ϑ*

$$(6) \quad \frac{dx}{dt} = f(x, \theta), \quad f : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}, \quad f \in \mathbb{C}^2(\mathbb{R}^2 \times \mathbb{R}),$$

where 0 is the equilibrium of the model system (1) such that $f(0, \theta) = 0$ for all θ with the assumption that;

Let f_k be the k^{th} component and

$$(7) \quad \begin{aligned} a &= \sum_{k,i,j=1}^n v_k w_i w_j \frac{\partial^2 f_k}{\partial x_i \partial x_j}(0,0) \\ b &= \sum_{k,i=1}^n v_k w_i \frac{\partial^2 f_k}{\partial x_i \partial \theta}(0,0) \end{aligned}$$

Thus the local dynamics of (6) around 0 are totally governed by a and b .

- (1) $a > 0, b > 0$. When $\theta < 0$ with $|\theta| \leq 1, 0$ is locally asymptotically stable, and there exists a positive unstable equilibrium; when $0 < \theta \leq 1, 0$ is unstable and there exists a negative and locally asymptotically stable equilibrium;
- (2) $a < 0, b < 0$. When $\theta < 0$ with $|\theta| \leq 1, 0$ unstable; when $0 < \theta \leq 1, 0$ is locally asymptotically stable, and there exists a positive unstable equilibrium;

(3) $a > 0, b < 0$. When $\theta < 0$ with $|\theta| \leq 1, 0$ is unstable and there exists a locally asymptotically stable negative equilibrium; when $0 < \theta \leq 1, 0$ is stable, and a positive unstable equilibrium appears;

(4) $a < 0, b > 0$. When θ changes from negative to positive, 0 changes its stability from stable to unstable. Correspondingly a negative unstable equilibrium becomes positive and locally asymptotically stable.

In order to apply the Center Manifold Theory (CMT), it is necessary to make the following changes to the state variables, we let $S = x_1, I = x_2, C = x_3, R = x_4, E = x_5, L = x_6, F = x_7$. The system (1) can now be written in the form $\frac{df}{dx} = f(x)$, where $x = (x_1, x_2, x_3, x_4, x_5, x_6, x_7)$. The system (1) therefore becomes

$$(8) \quad \left. \begin{aligned} \dot{x}_1 &= \Pi + \omega x_4 - \beta x_7 x_1 - \mu x_1, \\ \dot{x}_2 &= \beta x_7 x_1 - (\mu + \gamma_1 + \alpha) x_2, \\ \dot{x}_3 &= \alpha x_2 - (\mu + \gamma_2) x_3, \\ \dot{x}_4 &= \gamma_1 x_2 + \gamma_2 x_3 - (\mu + \omega) x_4, \\ \dot{x}_5 &= N_i \tau_1 x_2 + N_c \tau_2 x_3 - (v_e + \rho) x_5, \\ \dot{x}_6 &= \rho x_5 - \left(v_l + \frac{\varepsilon \delta}{1 + x_6} \right) x_6, \\ \dot{x}_7 &= \frac{\varepsilon \delta x_6}{1 + x_6} - v_f x_7. \end{aligned} \right\}$$

The basic reproduction of the system (1) is established in a compact form to be

$$(9) \quad R_0 = \frac{\beta \Pi (\alpha \delta \rho \tau_2 \varepsilon N_c + \gamma_2 \delta N_1 \rho \tau_1 \varepsilon + \delta \mu N_1 \rho \tau_1 \varepsilon)}{\mu (\gamma_1 + \mu) (\gamma_2 + \mu) v_f (v_e + \rho) (\delta \varepsilon + v_l)}.$$

Suppose, we choose $\theta = \beta$ as the bifurcation parameter so that when $\mathcal{R}_0 = 1$, we have

$$(10) \quad \theta = \frac{\mu (\gamma_1 + \mu) (\gamma_2 + \mu) v_f (v_e + \rho) (\delta \varepsilon + v_l)}{\Pi (\alpha \delta \rho \tau_2 \varepsilon N_c + \gamma_2 \delta N_1 \rho \tau_1 \varepsilon + \delta \mu N_1 \rho \tau_1 \varepsilon)}$$

The system (8) with the bifurcation point θ , has a simple zero eigenvalue. Thus, it enables us to use the Center Manifold Theory to analyse the stability of the system (8) near $\beta = \theta$. Therefore

a right eigenvector w associated with zero eigenvalue has components

$$\begin{aligned}
 (11) \quad w_1 &= -\left(\frac{(\alpha + \mu + \omega + \gamma_1)}{\mu + \omega} + \frac{\alpha \omega}{(\mu + \omega)(\mu + \gamma_2)}\right), \quad w_2 = 1, \quad w_3 = \frac{\alpha}{\mu + \gamma_2}, \\
 w_4 &= \frac{(\alpha \gamma_2 + \gamma_1(\mu + \gamma_2))}{(\mu + \omega)(\mu + \gamma_2)}, \quad w_5 = \frac{(N_1(\mu + \gamma_2)\tau_1 + \alpha N_c \tau_2)}{(\mu + \gamma_2)(\rho + v_e)}, \\
 w_6 &= \frac{\rho(N_1(\mu + \gamma_2)\tau_1 + \alpha N_c \tau_2)}{(\mu + \gamma_2)(\rho + v_e)(\delta \varepsilon + v_l)}, \quad w_7 = \frac{\delta \varepsilon \rho(N_1(\mu + \gamma_2)\tau_1 + \alpha N_c \tau_2)}{(\mu + \gamma_2)(\rho + v_e)v_f(\delta \varepsilon + v_l)}.
 \end{aligned}$$

Similarly, the corresponding left eigenvector v associated with zero eigenvalue has components

$$\begin{aligned}
 (12) \quad v_1 = v_4 &= 0, \quad v_2 = 1, \quad v_3 = \frac{N_c(\alpha + \mu + \gamma_1)\tau_2}{\mu N_1 \tau_1 + N_1 \gamma_2 \tau_1 + \alpha N_c \tau_2}, \\
 v_5 &= \frac{(\alpha + \mu + \gamma_1)(\mu + \gamma_2)}{\mu N_1 \tau_1 + N_1 \gamma_2 \tau_1 + \alpha N_c \tau_2}, \quad v_6 = \frac{(\alpha + \mu + \gamma_1)(\mu + \gamma_2)(\rho + v_e)}{\rho(\mu N_1 \tau_1 + N_1 \gamma_2 \tau_1 + \alpha N_c \tau_2)}, \\
 v_7 &= \frac{(\alpha + \mu + \gamma_1)(\mu + \gamma_2)(\rho + v_e)(\delta \varepsilon + v_l)}{\delta \varepsilon \rho(\mu N_1 \tau_1 + N_1 \gamma_2 \tau_1 + \alpha N_c \tau_2)}.
 \end{aligned}$$

We now compute \mathbf{a} and \mathbf{b} as outlined in [5]. From the system (8), the non-zero partial derivatives of $f(x)$ associated with \mathbf{a} are given by

$$(13) \quad \frac{\partial f_2}{\partial x_1 \partial x_7} = \theta.$$

Thus, the expression for \mathbf{a} is given by

$$\begin{aligned}
 (14) \quad \mathbf{a} &= v_2 w_1 w_7 \frac{\partial f_2}{\partial x_1 \partial x_7}, \\
 &= -\left(\frac{(\mu + \omega)(\mu + \gamma_2) + \gamma_1(\mu + \gamma_2) + \alpha \omega \delta \varepsilon}{(\mu + \gamma_2)(\mu + \omega)} \frac{1}{v_f}\right) w_6 < 0.
 \end{aligned}$$

We finally compute the value of \mathbf{b} . The non-zero partial derivatives of $f(x)$ associated with \mathbf{b} is given by

$$(15) \quad \frac{\partial f_2}{\partial x_7 \partial \theta} = \frac{\Pi}{\mu}.$$

Therefore the expression for \mathbf{b} is given by

$$(16) \quad \mathbf{b} = v_2 w_7 \frac{\partial f_2}{\partial x_7 \partial \theta} = \frac{\Pi \delta \varepsilon}{\mu v_f} w_6 > 0.$$

Since, $\mathbf{a} < 0$ and $\mathbf{b} > 0$, from item 4 in [5] we conclude that E_1 is locally asymptotically stable for $R_0 > 1$ close to $R_0 = 1$.

The bifurcation diagrams using a selected set of parameters are given in Figure 2 and Figure

3. We note that from the bifurcation diagrams that certain parameters have an influence on the

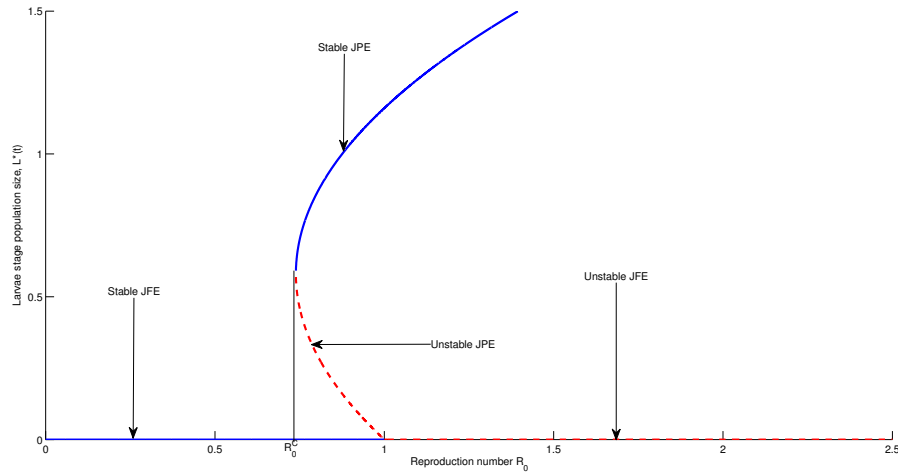


FIGURE 2. Variation of the equilibrium level of L showing the backward bifurcation of the system (1). The following parameters were used $\beta = 0.656, \gamma_1 = 0.4146, \gamma_2 = 0.0956, \tau_1 = 0.021, \tau_2 = 0.087, N_c = 0.087, N_i = 0.000001, \delta = 0.025, \varepsilon = 0.025, \Pi = 0.0365, v_f = 0.0015, v_l = 0.0016, v_e = 0.043, \omega = 0.15, \mu = 0.000012, \rho = 0.0055, \alpha = 0.3408$

bifurcation and they bring about a forward bifurcation when their values are increased. These include v_f , in which when its value is increased from 0.0015 to 0.002107 and v_e in which when its value is increased from 0.043 to 0.093 they bring about a forward bifurcation as shown in the Figure 3. A similar effect is obtained when the mortality of the larvae is increased. This means that jiggers controls can be enhanced by increasing the removal of the eggs, larvae and fleas from the environment. While flea removal is not economically viable, the control of larvae through larvacides and improvement of hygiene has proved to be effective in the control of jiggers.

3. MAIN RESULTS

NUMERICAL SIMULATIONS

3.1. Parameter estimation. Since not so much has been done mathematically to model jiggers infestation, parameters that relate to transmission and progression rates are difficult to find.

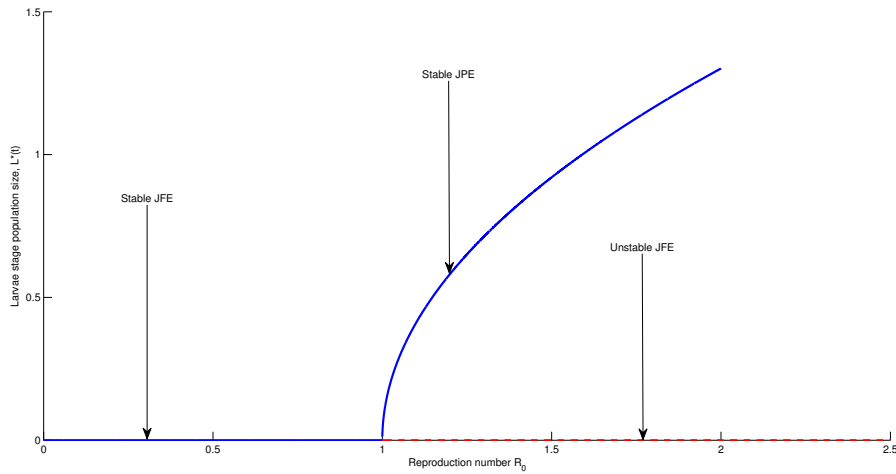


FIGURE 3. Variation of the equilibrium level of L showing the forward bifurcation of the system (1). The following parameters considered were same as the ones used in Figure 2 except for $v_f = 0.002107$ and $v_e = 0.093$

In this section, we assumed majority of the parameters using information collected on the populations in Kenya. Some have been acquired from the literature mostly from [1] and also with reference to the population infested with jiggers in Murang’a County in Kenya. Some of the demographic parameters used in our model simulation are described as follows:

- The demographic data released by the Central Intelligence Agency [31], estimated life expectancy at birth to be 63.52 years in 2014 and 63.4 years in 2015. This can then be estimated from 50 to 70 years. Thus the natural death rate of humans is estimated as $0.0000291 \leq \mu \leq 0.0000548$ per day.
- According to the World Fact Book by Central Intelligence Agency [31], the average birth rate in Kenya was estimated to be 28.27 births per 1000 population in the year 2014 and 26.4 per 1000 population in 2015. Therefore the birthrate is estimated to be $2.85 \leq \Pi \leq 7.95$ per day.
- From Ahadi Kenya Trust [1], it takes 3 – 4 days for the eggs to hatch on the ground. Therefore we approximate the rate at which the eggs develop into larvae to be, $0.001 \leq \rho \leq 0.03$ per day. After the eggs have hatched onto the ground, it takes 3 – 4 weeks for them to go through the larvae and pupa stage to become adults. Thus the rate at which the larvae develops into a pupa can also be estimated as $0.03333 \leq \delta \leq 0.5$ per day.

- A proportion p of the infested persons become chronically infested. We thus consider $0 \leq p \leq 1$.
- The natural death rates of the vector, that is, v_e, v_l and v_f can be approximated from the life cycle of the flea given by Ahadi Kenya Trust [1] as follows; $0.003 \leq v_e \leq 0.02, 0.038 \leq v_l \leq 0.081$ and $0.000006 \leq v_f \leq 0.1$ per day.

The parameter values used in our simulation are given in the Table 3 below for illustrative purposes.

TABLE 3. Parameter values estimates used in the model for jigger infestation and their sources.

Parameter	Range	Point value	Source
Π	2.85 – 7.95	5.36	[31]
μ	$2.91e^{-5} - 5.48e^{-5}$	$5.4e^{-5}$	[31]
γ_1	$4.85e^{-2} - 6.37e^{-2}$	$5.03e^{-2}$	Estimated
γ_2	0.548 – 0.913	0.731	Estimated
β	0.0-0.0001	$8.5e^{-6}$	Estimated
τ_1	0.0001-0.002	0.001	Estimated
τ_2	$1.0e^{-3} - 3.0e^{-3}$	$1.8e^{-3}$	Estimated
N_c	50–150	100	Estimated
N_i	60–120	90	Estimated
δ	0.001-0.03	0.016665	[1]
ε	0.01-0.4	0.2	[1]
v_f	0.000006-0.1	0.09	[1]
v_l	0.038-0.081	0.049525	[1]
v_e	0.003-0.02	0.005	[1]
ω	0.11-0.89	0.555	Estimated
ρ	0.001-0.03	0.0126665	[1]
α	0-1	0.2	Estimated

3.2. Sensitivity analysis. In this section we carry out sensitivity analysis of the parameters to the model output in order to determine the important parameters that can be targeted so as to control the jigger infestation. Sensitivity analysis is defined as the study of how uncertainty in the output of a system can be a portioned to different sources of uncertainty in the model parameters [2]. It is a technique for systematically varying model inputs and determining their effect on the model output. We perform the sensitivity analysis by computing the Partial Rank Correlation Coefficients (PRCC) with 1000 simulations per run for each of the parameter values sampled by the Latin Hypercube Sampling (LHS) scheme which belongs to the Monte Carlo class of sampling methods [28]. The LHS is defined as a statistical method for generating a sample of possible collections of parameter values from a multidimensional distribution [2]. Parameters with positive PRCCs will increase the model output variable when they are increased thus the number of the new infestations may increase if the model output variable is that of infested humans. The parameters with negative PRCCs values decrease the model output variable when they are increased.

Applying the approach in [15], the PRCCs between the reproduction number R_0 and each of the parameters in table 2 are derived. Using 1000 simulations per run of the Latin Hypercube Sampling (LHS) scheme [21], the established PRCCs are derived and represented in the Tornado plot, Figure 4.

The model parameter with highest influence on R_0 according to the PRCCs results shown in Figure 4 is β followed with $\delta, N_i, \tau_1, \varepsilon, \Pi, \gamma_2$ and ρ respectively, which are positively correlated. This parameters increase(decrease) R_0 when they are increased(decreased).

The parameters $\mu, v_f,$ and v_e have the highest negative influence on jigger infestation. They are negatively correlated.

Thus based on the result of sensitivity analysis we remark that, the parameters with most influence on R_0 are the rate at which the susceptible become infested (β), natural death rate of the eggs (v_e) and the natural death rate of the flea (v_f). R_0 increases with increase in β whereas it decreases with increase in v_e and v_f . This shows that increasing the death rate of the flea

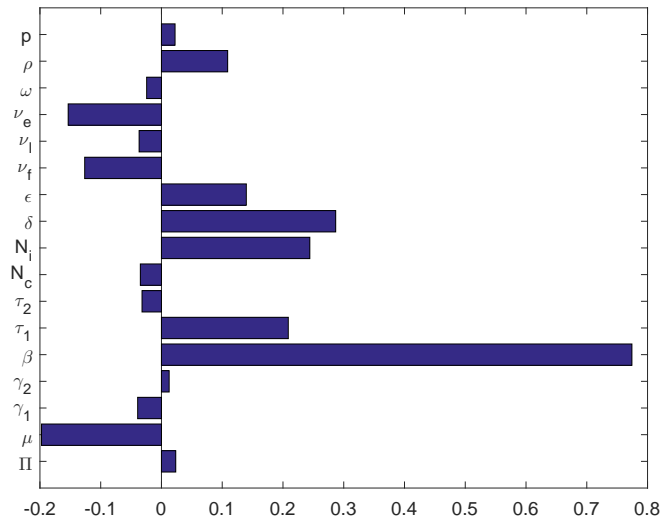


FIGURE 4. Tornado plots of Partial Rank Correlation Coefficients (PRCCs) plot showing the effects of parameters on R using the parameter ranges given in Table 3.

and eggs is likely to eradicate jigger infestation. Also decreasing the rate at which susceptible become infested would eradicate the jigger menace.

3.3. Simulation results.

Parameter variation effects. We now present some simulation results driven by observing the time series plots of chosen state variables for different parameters of interest. We begin by considering the potential impact of increasing the mortality of adult fleas on the number of chronically infested individuals. This is synonymous to considering an intervention aimed at increasing the mortality of adult fleas. The simulation results in Figure 5 show that increasing the death rate of the adult flea, ν_f , results in the reduction of the model reproduction number R_0 , and consequently a reduction in the number of individuals chronically infested..

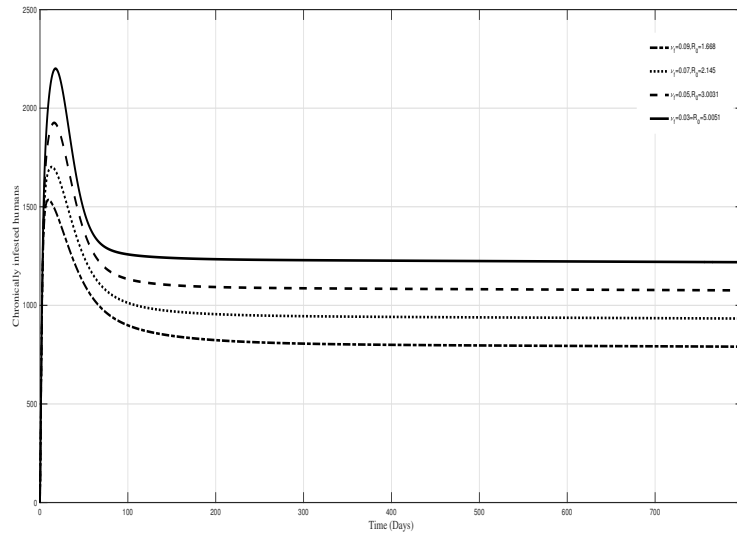


FIGURE 5. Simulation results showing the total number of chronically infested humans with the control associated with the parameter value v_f with the rest of parameters given in Table 3.

We also consider how increasing v_f , impacts the infested humans. The results show a similar trend to the ones observed in Fig. 5. Thus the reduction in v_f increases the number of infested humans, see Figure 6. Overall, increasing adult fleas' mortality is crucial for the control of infestation. If time series data on the number of infested cases (including the chronically infested) was available, this investigation would be critical in determining the levels of interventions aimed at reducing the adult fleas. The release of eggs into the environment by the female fleas burrowed in the host skin is critical for disease propagation. Determining the potential impact of increased eggs' release is thus important. From Figure 7, the variation of the parameter $\tau_3 = N_1 \tau_1$, shows that decreasing the value of τ_3 results in a reduction in the value of the reproduction number R_0 , consequently leading to a reduction in the number of infested humans.

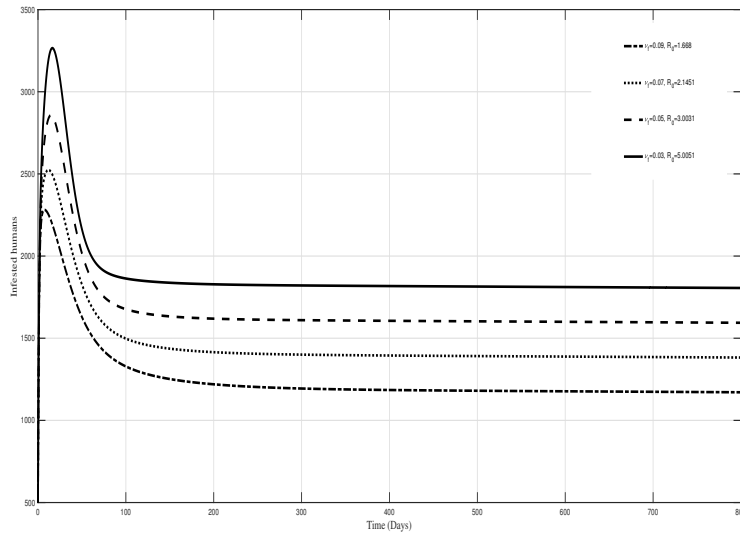


FIGURE 6. Simulation results showing the total number of infected humans with the control associated with the parameter value v_f with the rest of parameters given in Table 3.

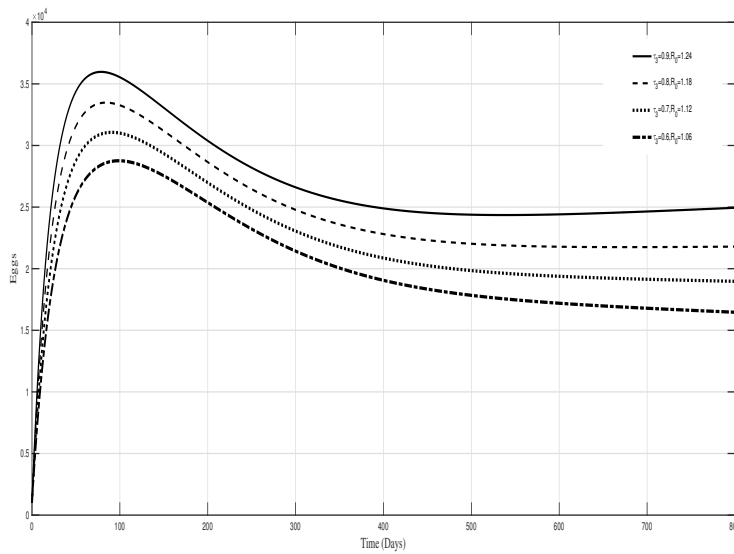


FIGURE 7. Simulation results showing the total number of eggs with the control associated with the parameter value τ_3 with the rest of parameters given in Table 3.

Similarly from Figure 8 the variation of the parameter $\tau_4 = N_2 \tau_2$ leads to the results observed in Figure 7 but for the chronically infested humans. So, the reduction in the number of eggs laid

in the environment is critical for the control of jiggers. The control in the deposition of eggs is synonymous to providing hygiene that ensures, the non-survival of the laid eggs.

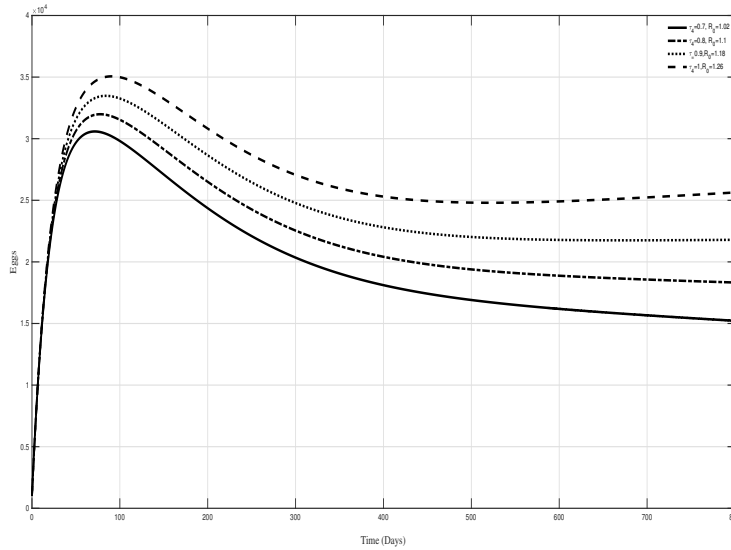


FIGURE 8. Simulation results showing the total number of eggs with the control associated with the parameter value τ_4 with the rest of parameters given in Table 3.

The deposited eggs often develop into adult fleas that eventually infect human, through the cycle of the flea. We also investigated how the potential growth of the adult fleas is related to the deposition of eggs into the environment. We consider eggs deposited by the infested humans and similar trends are observed when eggs deposited by chronically infested humans are considered. From Figure 9, the variation of τ_3 , show the levels of increase of the adult flea as τ_3 is increased.

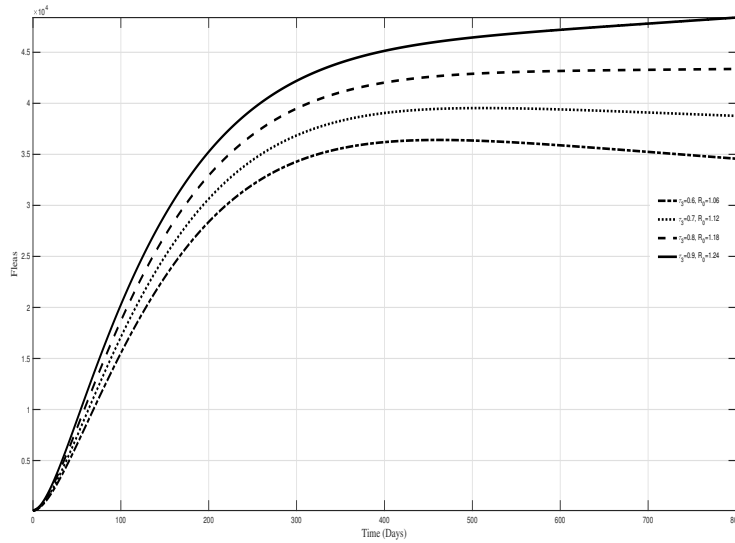


FIGURE 9. Simulation results showing the total number of fleas with the control associated with the parameter value τ_3 with the rest of parameters given in Table 3.

Figure 10 (a) shows the level curves of R_0 with respect to the parameter values β and δ . We find that R_0 increases when both parameters increase, that is, the rate at which larvae develop into adult flea δ and the infestation rate β . Consequently, jiggers control is achieved by the simultaneous control of infestation and killing of larvae. So, any intervention targeting the two parameters, is likely to yield significant results in jiggers control. Figure 10 (b) shows the level of curves of R_0 with respect to the parameter values β and v_f . We find that R_0 is significantly impacted the parameter better when compared to v_f . This important since the killing of the flea is in itself a complicated intervention when compared to controlling the infestation rate. So focus should be on the reduction of the infestation rate in this case.

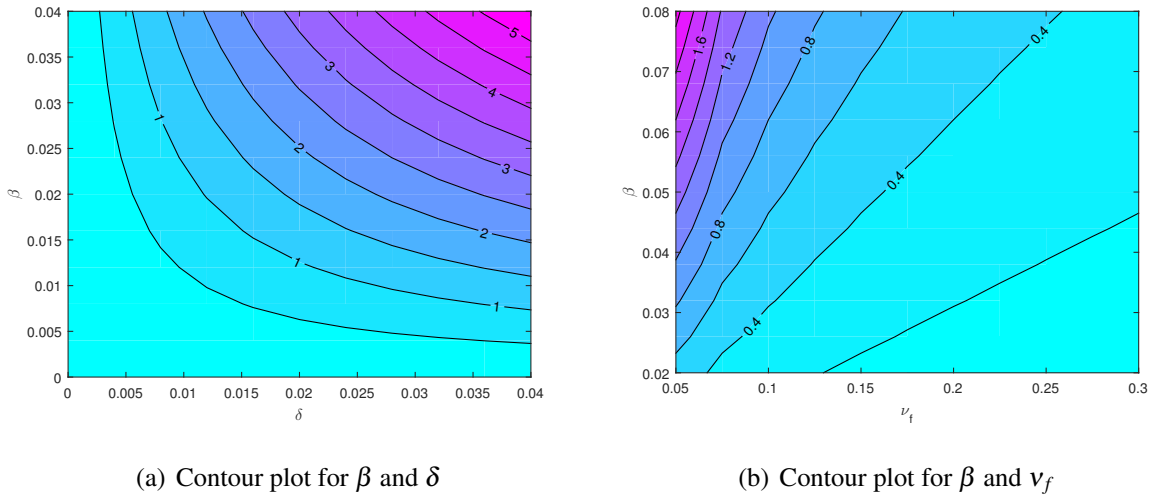


FIGURE 10. shows the contour plots of the reproduction number R_0 with respect to the given parameters.

CONCLUSION

In this paper, we formulated a deterministic model for the dynamics of jigger infestation. We proved that the formulated model is biologically and mathematically well posed in an invariant region Ω . The basic reproduction number was determined using the next generation method. The steady states of the model were determined and the stability analysis carried out.

The analytical results indicated that R_0 is indeed the threshold when the parameter $\nu_f = 0$. However when the natural death rate of the flea is increased, that is $\nu_f > 0$, the dynamics of jigger infestation model exhibits a phenomenon called backward bifurcation where a jigger free equilibrium and two non-trivial equilibria coexists even though the basic reproduction number is below unity. The appearance of backward bifurcation shows that it is not sufficient to decrease the basic reproduction number below unity for eradication of jigger infestation within the community. Thus to effectively control the jigger infestation menace, one has to reduce R_0 below another threshold known as the critical value of the basic reproduction number R_0^c . That is jigger infestation can be eradicated if $R_0 < R_0^c < 1$. In general both analytical and numerical results suggest that the natural death rate of the flea ν_f , is the one responsible for backward bifurcation.

From the numerical results and sensitivity analysis, jigger infestation can be eradicated by reducing the contact rate between humans and the fleas, β and increasing the removal of the fleas, ν_f . Thus, the numerical results show clearly where the strategies can be deployed to reduce disease propagation and at what stage this could be done. From the parameter variation results, clearly if the natural death rate of the adult flea ν_f is increased, the rate of infestations reduces which eventually reduces the jigger menace. Similarly if we reduce the rate at which infested humans produce eggs τ_1 and τ_2 , the number of adult fleas will reduce, thus consequently reducing the rate of infestation.

It is also important to note that the infestations affect children and adults, so the work can be improved by using an age-structured model. Also, in the absence of data, verification of the model and determination of parameter values remains a theoretical consideration. A model that is fitted to data for a particular setting would be ideal, considering the fact that models need to be verified by data and their applications cements their usability. Control strategies, evaluated through mathematical models could also be incorporated in such a model, see also the work in [13]. It is also important to look at the intensity of infections, by looking at the number of fleas per individual as the recovery and treatment levels often depend on such. This is however set in our current research endeavors. Despite these shortcomings, the model presents some interesting insights, that are useful in determining the dynamics of the infestation.

Conflict of Interests

The authors declare that there is no conflict of interests.

REFERENCES

- [1] Ahadi Kenya Trust, Available at <http://www.jigger-ahadi.org>, Accessed August 2016.
- [2] A. Saltelli, M. Ratto, T. Andres, F. Campolongo, J. Cariboni, Gatelli D, Saisana M, Tarantola S, Introduction to Sensitivity Analysis, in *Global Sensitivity Analysis, The Primer*, John Wiley & Sons, Ltd, Chichester, UK, (2007), 1–5.
- [3] B. Kimani, J. Nyagero, L. Ikamari, Knowledge, attitude and practices on jigger infestation among household members aged 18 to 60 years: case study of a rural location in Kenya, *Pan Afr. Med. J.* **13**(S.1) (2012), 7.
- [4] B. Szymanski, T. Caraco, Spatial analysis of vector-borne disease: a four- species model, *Evol. Ecol.* **8**(3) (1994), 299–314.

- [5] C. Castillo-Chavez, B. Song, Dynamical models of tuberculosis and their applications, *Math. Biosci. Eng.* **1**(2) (2004), 361–404.
- [6] D. Pilger, S. Schnalfenberg, Heukelberg et al, Controlling tungiasis in an impoverished community: an intervention study, *PLOS Negl. Trop. Dis.* **2** (10) (2008), e324.
- [7] E. O. Omondi, F. Nyabadza, R. J. Smith, Modeling the impact of mass administration of ivermectin in the treatment of onchocerciasis (river blindness) *Cogent Math. Stat.* **5** (2018), 1429700.
- [8] G. Collins, T. McLeod, N. I. Konfor, C. B. Lamnyam, L. Ngarka, N. L. Ngamnishi, Tungiasis: A neglected health problem in rural Cameroon, *Int. J. Coll. Res. Int. Med. Public Health*, **1**(1)(2009), 2–10.
- [9] H. Feldmeier, E. Sentogo, I. Krantz, Tungiasis (sand flea disease): a parasitic disease with intriguing challenges for a transformed public health, *Eur. J. Clin. Microbiol. Infect.* **32**(2012), 19–26.
- [10] J. Heukelbach, F. A. S. De Oliveira, G. Hesse, H. Feldmeier, Tungiasis: A neglected health problem of poor communities, *Trop. Med. Int. Health*, **6** (2001), 267–272.
- [11] J. Heukelbach, T. Wilcke, G. Hesse, H. Feldmeier, Seasonal variation of tungiasis in an endemic community, *Amer. J. Trop. Med. Hygiene*, **72**(2005), 145–149.
- [12] J. Kahuru, L. Luboobi, N. Yaw, Modeling the dynamics of tungiasis transmission in zoonotic areas, *J. Math. Comput. Sci.*, **2** (2017), 375-399.
- [13] J. Kahuru, L. Luboobi, N. Yaw, Stability analysis of dynamics of tungiasis transmission in endemic areas, *Asian J. Math. App.* **2017**(2017), ama 0385.
- [14] J. N. Mwangi, H. S. Ozwarw, M. M. Gicheru, Epidemiology of Tunga Penetrans infestation in selected areas in Kiharu constituency, Murang'a county, *Tropical Diseases, Travel Med. Vacc.* **1**(1) (2015), 13.
- [15] J. Wu, R. Dihingra, M. Gambhir, J. V. Remais, Sensitivity analysis of infectious disease models: methods, advanced and their application, *J. Royal Soc. Interface*, **10** (2013), no. 86, article 1018.
- [16] K. Jawoko, Jiggers outbreak in Uganda, *Canad. Med. Assoc. J.* **183**(1)(2011), E33–E34.
- [17] K. Joseph, J. Bazile, J. Mutter, S. Shin, A. Ruddle, L. Ivers, E. Lion, P. Fauner, Tungiasis in rural Haiti: a community-based response, *Trans. Royal Soc. Trop. Med. Hygiene*, **100** (10) (2006), 970–974.
- [18] M. Basanez, J. Ricardez-Esquinca, Models for the population biology and control of human onchocerciasis, *Trends Parasitol*, **17**(9) (2001), 430–438.
- [19] M. Martcheva, O. Prosper, Unstable dynamics of vector-borne diseases: Modeling through differential-delay equations, in Rao VSH, Durvasula R (eds.), *Dynamic Models of Infectious Diseases, Volume 1: Vector-Borne Diseases*, Springer, (2012), pp. 43–75.
- [20] M. M. Sache, K. K. Guldbakke, A. Khachemoune, Tunga penetrans: a stowaway from around the world., *J. Eur. Acad. Dermatol. Venereol.* **21**(1)(2007), 11–16.
- [21] M. Stein, Large sample properties of simulations using Latin hypercube sampling, *Technometrics. A J. Stat. Phys. Chem. Eng. Sci.* **29** (1987), no. 2, 143–151.

- [22] M. Thielecke, P. Nordin, N. Ngomi, H. Feldmeier, Treatment of tungiasis with dimeticone: a proof of principle study in rural Kenya, *PLoS Negl. Trop. Dis.* **8** (7)(2014), e3058.
- [23] N. H. Shah, J. Gupta, SEIR Model and simulation for vector-borne diseases, *Appl. Math.* **4** (2013), 13-17.
- [24] O. Diekmann, J. P. A. Heesterbeek, *Mathematical epidemiology of infectious diseases: model building, analysis and interpretation*, John Wiley, (2000).
- [25] O. R. Damazio, M. V. Silvia, Tungiasis in school children in Cuciuma, Santa Caratina State, South Brazil, *Rev. Inst. Med. Trop.* **51**(2) (2009), 103–108.
- [26] R. K. Mucheke, P. M. Maithya, The effects of Tunga Penetrans infestation on academic achievement in public primary schools: Kenyan perspective, *Cradle Knowl. Afr. J. Educ. Soc. Sc. Res.* **2**(1) (2014), 109–120.
- [27] S. K. Ruttoh, D. O. Omondi, Wanyama NI, Tunga penetrans- A silent setback to development in Kenya, *J. Environ. Sci. Eng. B*, **1**(2012), 527–534.
- [28] S. M. Blower, H. Dowlatbadi, Sensitivity and uncertainty analysis of complex models of disease transmission: an HIV model, as an example, *Int. Stat. Rev.* **62**(2)(1994), 229-243.
- [29] S. S. Gibbs, The diagnosis and treatment of tungiasis, *Br. J. Dermatol.* **159**(4) (2008), 981.
- [30] T. E. Cestari, S. Pessato, M. Ramos-e-Silva, Tungiasis and myiasis, *Clin. Dermatol.* **25**(2)(2007), 158–1164.
- [31] The world fact book, Available at <https://www.cia.gov/library/publications/the-world-factbook/fields/2054.html>, Accessed August 2016.
- [32] T. M. Kamau, R. N. Ngechu, Z. T. Haile, J. Mwitari, An exploration of factors associated with jigger infestation (tungiasis) among residents of Murang’a North District, kenya, *Int. J. Health Sci. Res.* **4** (3)(2014), 1–8.
- [33] U. S. Ugbomoiko, I. E. Ofoezie, J. Heukelbach, Tungiasis: High prevalence, parasite load and morbidity in a rural community in Lagos state, Nigeria *Int. Dermatol.* **46**(5)(2007), 475–481.