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A δ- SHOCK MAINTENANCE MODEL FOR A DETERIORATING SYSTEM WITH IMPERFECT DELAYED REPAIR UNDER PARTIAL PRODUCT PROCESS

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Abstract. There are some systems in real life that cannot be repaired immediately, if they fail, but repairs are not always delayed. In different words, once the system fails, the repair is sometimes delayed and the repair is sometimes immediate. This kind of repair is referred to as the imperfect delayed repair(IDR). In a δ-shock model, a shock is a deadly shock, if the time interval between two successive shocks is smaller than the specified threshold δ. In this paper, a δ-shock maintenance model for a deteriorating system with imperfect delayed repair under partial process is studied. A replacement policy N is adopted by which the system is replaced by an identical new one at the time following the N-th failure. The long-run average cost per unit time is then evaluated and the corresponding optimal policy is determined analytically. A numerical example is given.

Keywords: geometric process; partial product process; replacement policy; δ- shock.

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1. INTRODUCTION

The study of a maintenance model for a simple repairable system is a fundamental and important problem in reliability. In the earliest study, the common assumption is that the system is

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as good as new after repair. This is termed as perfect repair. This assumption, however, is not always true. In reality, most repairable systems are deteriorating due to the aging and cumulative wear impacts. Barlow and Hunter (1960) first introduced a minimal repair model that does not change the age of the system. Brown and Proschan (1983) investigated an imperfect repair model in which a repair is perfect with probability p and a minimum repair with probability $1-p$. Besides, it is more reasonable for these deteriorating repairable systems to assume that the system's successive operating times after repair are shorter and shorter, while the system's consecutive repair times after failure are longer and longer. The monotone process model would therefore be the most suitable model for a deteriorating system. Lam (1988) introduced a *geometric process* for these phenomena and Babu, Govindaraju and Rizwan (2018) introduced a *partial product process*.

Lam and Zhang (2004) studied the δ-shock model for a repairable and deteriorating system under geometric process. In this model, the system breaks down due to two successive shocks that are too close to one another.

Zhang and Wang (2017) studied the imperfect delayed repair in which the repair is delayed with probability θ and is undelayed with probability $1-\theta$ for a repairable and deteriorating system under extended geometric process.

In this paper, a δ-shock maintenance model for a deteriorating system with imperfect delayed repair under partial process is studied.

The preliminary definitions and results about partial product process are given below.

Definition 1.1. Let $\{X_n,\ n=1,2,3,...\}$ be a sequence of non-negative independent random variables and let $F(x)$ be the distribution function of X_1. Then $\{X_n,\ n=1,2,3,...\}$ is called a partial product process, if the distribution function of X_{i+1} is $F(\beta_i x)$ $(i=1,2,3,...)$ where $\beta_i>0$ are constants and $\beta_i=\beta_0\beta_1\beta_2...\beta_{i-1}$.

Lemma 1.1. For real β_i $(i=1,2,3,...)$, $\beta_i=\beta_0^{2^{i-1}}$.

Then the distribution function of X_{i+1} is $F\left(\beta_0^{2^{i-1}}x\right)$, for $i=1,2,3,...$

Lemma 1.2. Given a partial product process $\{X_n,\ n=1,2,3,...\}$,

(i) if $\beta_0>1$, then $\{X_n,\ n=1,2,3,...\}$ is stochastically decreasing.

(ii) if $0<\beta_0<1$, then $\{X_n,\ n=1,2,3,...\}$ is stochastically increasing.

(iii) if $\beta_0 = 1$, then $\{X_n,\ n = 1,2,3,...\}$is a renewal process.

Lemma 1.3. Let $E(X_1) = \lambda,\ Var(X_1) = \sigma^2$. Then for $i = 1,2,3,...$

$E(X_{i+1}) = \frac{\lambda}{\beta_0^{2^{i-1}}}$ and $Var(X_{i+1}) = \frac{\sigma^2}{\beta_0^{2^i}}$.

Definition 1.2. An integer valued random variable N is said to be *stopping time* for the sequence of independent random variables $X_1, X_2, \ldots$,if the event $\{N = n\}$is independent of $X_{n+1}, X_{n+2}, \ldots$, for all $n = 1,2,....$

Theorem 1.1 Wald's equation. If $X_1, X_2, \ldots$ are independent and identically distributed random variables having finite expectations and if N is the stopping time for $X_1, X_2, \ldots$ such that $E[N] < \infty$, then $E\left[\sum_{n=1}^{N} X_n\right] = E[N]E[X_1]$.

Now we make the following assumptions about the maintenance model for a deteriorating system with imperfect delayed repair.

2. Model Description

We consider the maintenance model for a deteriorating system and make the following assumptions.

A1. At the beginning, a new simple repairable system with IDR is installed. Whenever the system fails, it may be repaired or replaced by a new and identical one.

A2. The system is subject to a sequence of shocks. The shocks will arrive according to a renewal process with rate ζ. If the system is repaired for n times $(n = 0,1,2,...)$, the threshold of deadly shock will be $\alpha^n\delta$ where $\alpha \geq 1$ is the rate and δ is the threshold of deadly shock for a new system. This means that when the time to the first shock is less than δ or the interval time between two successive shocks after the n-th repair is less than $\alpha^n\delta$, the system will fail. The system is closed during the repair so that any shock that occurs when the system is under repair is ineffective.

A3. Let X_n be the operating time of the system following the $(n-1)$-st repair, let F_n be the distribution function of X_n and let $E(X_n) = \lambda_n$.

A4. Let Y_1 be the repair time after the first failure and let $G(y)$ be the distribution function of Y_1. For $i = 1,2,3,...$, let Y_{i+1}be the repair time after the $(i+1)$-st failure. Following Babu etal (2018) the distribution function of Y_{i+1} is $G\left(\gamma_0{}^{2^{i-1}}x\right)$, where $0 < \gamma_0 \leq 1$ is a constant. That is, the consecutive repair times $\{Y_n,\ n = 1,2,3,...\}$ form an increasing partial product process. If

$\gamma_0 = 1$, then it is a renewal process. Moreover, assume that $E(Y_1) = \mu \geq 0$. Here $\mu = 0$ means that the expected repair time is negligible.

A5. When the system fails, the repair is delayed with probability θ and is undelayed with probability $1-\theta$.

A6. Let D_n be the delayed repair (DR) time after the n-th failure, and assume that $\{D_n,\ n = 1,2,3,...\}$ is a sequence of independent random variables with identical distribution function. Also, let $E(D_n) = \nu \geq 0,\ n = 1,2,3,...$, while $\nu = 0$ means that the DR time is negligible.

A7. X_n, Y_n and $D_n,\ n = 1,2,3,...$ are independent of each other.

A8. Let Z be the replacement time with $E(Z) = \tau$.

A9. The repair cost rate is c, the reward rate is r and the replacement cost of the system comprises two parts: one part is the basic replacement cost R, the other part is the cost proportional to the length of replacement time Z at rate c_P.

A10. The renewal process, the partial product process and replacement time Z are independent.

A11. The system can neither incur cost nor bring income during the waiting for the repair.

A12. The n-th cycle of the system $(n = 1,2,3,...)$ is the time interval between the completion of the $(n-1)$-st repair and the completion of the n-th repair. Let A_n denote the event that the repair is delayed in n-th cycle of the system and let $\overline{A_n}$ denote the event that the repair is undelayed in n-th cycle of the system.

First, we need to evaluate the values of λ_n. For this, let W_{n1} be the arrival time of the first shock after the $(n-1)$-st repair. In general, let W_{nk} be the interarrival time between the $(k-1)$-st and k-th shocks after the $(n-1)$-st repair. Let $E(W_{11}) = \lambda$. Assume that $\{W_{ni},\ i = 1,2,...\}$ are independent and identically distributed (i.i.d.) sequences for all n.

Let $M_n,\ n = 1,2,3,...$, be the number of shocks following the $(n-1)$-st repair until the first deadly shock occurred. Then,

$$M_n = \min\left\{m \mid W_{n1} \geq \alpha^{n-1}\delta, ..., W_{n\ m-1} \geq \alpha^{n-1}\delta,\ W_{nm} < \alpha^{n-1}\delta\right\} \tag{1}$$

and

$$X_n = \sum_{i=1}^{M_n} W_{ni} \tag{2}$$

Obviously, M_n follows a geometric distribution with parameter

$$p_n = P\left(W_{nm} < \alpha^{n-1}\delta\right) \tag{3}$$

with $q_n = 1 - p_n$ and

$$E\left(M_n\right) = \frac{1}{p_n} \tag{4}$$

As M_n is a stopping time for $\{W_{ni},\ i = 1, 2, ...\}$, from equations (2) and (4) and by Wald's equation, we have

$$\lambda_n = E\left(X_n\right) = E\left(M_n\right) E\left(W_{n1}\right) = \frac{\lambda}{p_n} \tag{5}$$

Since $\alpha \geq 1$, from equations (3) and (5), we have the following Lemma.

Lemma 2.1. λ_n is non-increasing in n.

3. THE REPLACEMENT POLICY N

Definition 3.1. A replacement policy N is a policy in which the system will be replaced at the N-th failure of the system.

The main aim is to find an optimal replacement N^* such that the long-run average cost per unit time is minimized.

Let T_1 be the first replacement time, in general, for $n \geq 2$, let T_n be the time between $(n-1)$-st and n-th replacement. Then, clearly $\{T_n,\ n = 1, 2, ...\}$ forms a renewal process. By the renewal reward theorem, Ross(1983), the long-run average cost per unit time under the replacement policy N is given by

$$\begin{aligned} C\left(N\right) &= \frac{\text{the expected cost incurred in a cycle}}{\text{the expected length of a cycle}} \\ &= \frac{cE\left(\sum\limits_{i=1}^{N-1} Y_i\right) + R + c_p E\left(Z\right) - rE\left(\sum\limits_{i=1}^{N} X_i\right)}{E\left(\sum\limits_{i=1}^{N} X_i\right) + E\left[\sum\limits_{i=1}^{N-1} \left(D_i + Y_i\right)\chi_{A_i}\right] + E\left(\sum\limits_{i=1}^{N-1} Y_i \chi_{\overline{A_i}}\right) + E\left(Z\right)} \end{aligned}$$

where $\chi_A\left(.\right)$ denotes the indicator function. Then,

$$
\begin{aligned}
C(N) &= \frac{cE\left(\sum_{i=1}^{N-1} Y_i\right) + R + c_p E(Z) - rE\left(\sum_{i=1}^{N} X_i\right)}{E\left(\sum_{i=1}^{N} X_i\right) + E\left[\sum_{i=1}^{N-1} (D_i + Y_i)\right]\theta + E\left(\sum_{i=1}^{N-1} Y_i\right)(1-\theta) + E(Z)} \\
&= \frac{c\left(\sum_{i=1}^{N-1} E(Y_i)\right) + R + c_p E(Z) - r\left(\sum_{i=1}^{N} E(X_i)\right)}{\sum_{i=1}^{N} E(X_i) + \theta\left(\sum_{i=1}^{N-1} E(D_i)\right) + \sum_{i=1}^{N-1} E(Y_i) + E(Z)} \\
&= \frac{c\left(\sum_{i=1}^{N-1} E(Y_i)\right) + R + c_p \tau - r\left(\sum_{i=1}^{N} E(X_i)\right)}{\sum_{i=1}^{N} E(X_i) + \theta(N-1)\nu + \sum_{i=1}^{N-1} E(Y_i) + \tau} \\
&= \frac{(c+r)\sum_{i=1}^{N-1} E(Y_i) + R + (c_p + r)\tau + r\theta(N-1)\nu}{\sum_{i=1}^{N} E(X_i) + \theta(N-1)\nu + \sum_{i=1}^{N-1} \mu_i + \tau} - r \qquad (6) \\
&= \frac{(c+r)\left(E(Y_1) + \sum_{i=2}^{N-1} \mu_i\right) + R + (c_p + r)\tau + r\theta(N-1)\nu}{\sum_{i=1}^{N} E(X_i) + \theta(N-1)\nu + \left(E(Y_1) + \sum_{i=2}^{N-1} \mu_i\right) + \tau} - r \\
&= \frac{(c+r)\mu\left(1 + \sum_{i=2}^{N-1} \frac{1}{\gamma_0^{2^{i-2}}}\right) + R + (c_p + r)\tau + r\theta(N-1)\nu}{\sum_{i=1}^{N} \lambda_i + \theta(N-1)\nu + \mu\left(1 + \sum_{i=2}^{N-1} \frac{1}{\gamma_0^{2^{i-2}}}\right) + \tau} - r \qquad (7)
\end{aligned}
$$

4. THE OPTIMAL POLICY N^*

In this section, we shall determine an optimal replacement policy for minimizing $C(N)$. From equation (6),

$C(N+1) - C(N)$

$$
= \frac{(c+r)\sum_{i=1}^{N} \mu_i + R + (c_p + r)\tau + r\theta N\nu}{\sum_{i=1}^{N+1} E(X_i) + \theta N\nu + \sum_{i=1}^{N} \mu_i + \tau} - \frac{(c+r)\sum_{i=1}^{N-1} \mu_i + R + (c_p + r)\tau + r\theta(N-1)\nu}{\sum_{i=1}^{N} E(X_i) + \theta(N-1)\nu + \sum_{i=1}^{N-1} \mu_i + \tau}
$$

$$= \frac{\begin{pmatrix} (c+r)\left[E(Y_N)\left(\sum_{i=1}^{N} E(X_i)+\theta(N-1)\nu+\tau\right)-\sum_{i=1}^{N-1} E(Y_i)\left(E(X_{N+1})+\theta\nu\right)\right] \\ -(R+(c_p+r)\tau)\left[E(X_{N+1})+\theta\nu+E(Y_N)\right] \\ +r\theta\nu\left[\sum_{i=1}^{N+1} E(X_i)+\sum_{i=1}^{N} E(Y_i)+\tau-N\left(E(X_{N+1})+E(Y_N)\right)\right] \end{pmatrix}}{\left(\sum_{i=1}^{N+1} E(X_i)+\theta N\nu+\sum_{i=1}^{N} E(Y_i)+\tau\right)\left(\sum_{i=1}^{N} E(X_i)+\theta(N-1)\nu+\sum_{i=1}^{N-1} E(Y_i)+\tau\right)} \tag{8}$$

Let

$$B(N)=\frac{\begin{pmatrix} (c+r)\begin{pmatrix} E(Y_N)\left[\sum_{i=1}^{N} E(X_i)+\theta(N-1)\nu+\tau\right] \\ -\sum_{i=1}^{N-1} E(Y_i)\left[E(X_{N+1})+\theta\nu\right] \end{pmatrix} \\ +r\theta\nu\left[\sum_{i=1}^{N+1} E(X_i)+\sum_{i=1}^{N} E(Y_i)+\tau-N\left(E(X_{N+1})+E(Y_N)\right)\right] \end{pmatrix}}{(R+(c_p+r)\tau)\left[E(X_{N+1})+\theta\nu+E(Y_N)\right]} \tag{9}$$

That is,

$$B(N)=\frac{1}{(R+(c_p+r)\tau)}\left[(c+r)B_1(N)+r\theta\nu B_2(N)\right] \tag{10}$$

where

$$B_1(N)=\frac{E(Y_N)\left[\sum_{i=1}^{N} E(X_i)+\theta(N-1)\nu+\tau\right]-\sum_{i=1}^{N-1} E(Y_i)\left[E(X_{N+1})+\theta\nu\right]}{E(X_{N+1})+\theta\nu+E(Y_N)} \tag{11}$$

and

$$B_2(N)=\frac{\sum_{i=1}^{N+1} E(X_i)+\sum_{i=1}^{N} E(Y_i)+\tau-N\left(E(X_{N+1})+E(Y_N)\right)}{E(X_{N+1})+\theta\nu+E(Y_N)} \tag{12}$$

Since the denominator of $C(N+1)-C(N)$ is always positive, it is clear that the sign of $C(N+1)-C(N)$ is the same as the sign of its numerator. Therefore, we have the following lemma.

Lemma 4.1.

$$\begin{aligned} C(N+1)>C(N) &\Leftrightarrow B(N)>1 \\ C(N+1)=C(N) &\Leftrightarrow B(N)=1 \\ C(N+1)<C(N) &\Leftrightarrow B(N)<1 \end{aligned} \tag{13}$$

Then, from equation (11),

$B_1(N+1) - B_1(N)$

$$= \frac{E(Y_{N+1})\left[\sum_{i=1}^{N+1} E(X_i) + \theta N\nu + \tau\right] - \sum_{i=1}^{N} E(Y_i)\left[E(X_{N+2}) + \theta\nu\right]}{E(X_{N+2}) + \theta\nu + E(Y_{N+1})}$$

$$- \frac{E(Y_N)\left[\sum_{i=1}^{N} E(X_i) + \theta(N-1)\nu + \tau\right] - \sum_{i=1}^{N-1} E(Y_i)\left[E(X_{N+1}) + \theta\nu\right]}{E(X_{N+1}) + \theta\nu + E(Y_N)}$$

$$= \frac{\left(\begin{array}{l}\left(\sum_{i=1}^{N+1} E(X_i) + \sum_{i=1}^{N} E(Y_i) + \theta N\nu + \tau\right) \\ \times\left[\left(E(X_{N+1})E(Y_{N+1}) - E(X_{N+2})E(Y_N)\right) + \theta\nu\left(E(Y_{N+1}) - E(Y_N)\right)\right]\end{array}\right)}{\left[E(X_{N+2}) + \theta\nu + E(Y_{N+1})\right]\left[E(X_{N+1}) + \theta\nu + E(Y_N)\right]} \quad (14)$$

Next, from Equation (12),

$B_2(N+1) - B_2(N)$

$$= \frac{\sum_{i=1}^{N+2} E(X_i) + \sum_{i=1}^{N+1} E(Y_i) + \tau - (N+1)\left(E(X_{N+2}) + E(Y_{N+1})\right)}{E(X_{N+2}) + \theta\nu + E(Y_{N+1})}$$

$$- \frac{\sum_{i=1}^{N+1} E(X_i) + \sum_{i=1}^{N} E(Y_i) + \tau - N\left(E(X_{N+1}) + E(Y_N)\right)}{E(X_{N+1}) + \theta\nu + E(Y_N)}$$

$$= \frac{\left(\sum_{i=1}^{N+1} E(X_i) + \sum_{i=1}^{N} E(Y_i) + \tau + N\theta\nu\right) \times \left[E(X_{N+1}) - E(X_{N+2}) + E(Y_N) - E(Y_{N+1})\right]}{\left[E(X_{N+2}) + \theta\nu + E(Y_{N+1})\right]\left[E(X_{N+1}) + \theta\nu + E(Y_N)\right]} \quad (15)$$

Now, from equations (10), (14) and (15), we have

$B(N+1) - B(N)$

$$= \frac{1}{(R + (c_p + r)\tau)}\left[(c+r)\left(B_1(N+1) - B_1(N)\right) + r\theta\nu\left(B_2(N+1) - B_2(N)\right)\right]$$

$$= \frac{\left(\begin{array}{l} (c+r)\left(\sum_{i=1}^{N+1} E(X_i) + \sum_{i=1}^{N} E(Y_i) + \theta N \nu + \tau\right) \left[\begin{array}{l} (E(X_{N+1}) E(Y_{N+1}) - E(X_{N+2}) E(Y_N)) \\ +\theta \nu (E(Y_{N+1}) - E(Y_N)) \end{array} \right] \\ +r\theta \nu \left(\sum_{i=1}^{N+1} E(X_i) + \sum_{i=1}^{N} E(Y_i) + \tau + N\theta \nu\right) [E(X_{N+1}) - E(X_{N+2}) + E(Y_N) - E(Y_{N+1})] \end{array} \right)}{(R + (c_p + r)\tau)[E(X_{N+2}) + \theta \nu + E(Y_{N+1})][E(X_{N+1}) + \theta \nu + E(Y_N)]}$$

$$= \frac{\left(\sum_{i=1}^{N+1} E(X_i) + \sum_{i=1}^{N} E(Y_i) + \theta N \nu + \tau\right) \left[\begin{array}{l} c \left(\begin{array}{l} [E(X_{N+1}) E(Y_{N+1}) - E(X_{N+2}) E(Y_N)] \\ +\theta \nu [E(Y_{N+1}) - E(Y_N)] \end{array} \right) \\ +r([E(X_{N+1}) E(Y_{N+1}) - E(X_{N+2}) E(Y_N)]) \\ +r\theta \nu [E(X_{N+1}) - E(X_{N+2})] \end{array} \right]}{(R + (c_p + r)\tau)[E(X_{N+2}) + \theta \nu + E(Y_{N+1})][E(X_{N+1}) + \theta \nu + E(Y_N)]}$$

This implies that $B(N)$ is non-decreasing in N, because $E(X_n)$is non-increasing in n and $E(Y_n)$is non-decreasing in n.

Using Lemma 4.1, we have the following theorem.

Theorem 4.1. The optimal replacement policy N^* is determined by

$$N^* = \min \{N |\ B(N) \geq 1\} \tag{16}$$

Moreover, the optimal replacement policy N^* is unique if and only if $B(N^*) > 1$.

5. Numerical Example

Assume that the interarrival times $W_{ni}\,(i \geq 1)$ are independent and identically distributed (i.i.d.) exponential random variables with $E(W_{11}) = \lambda$.

Then from equation (5),

$$\lambda_n = E(X_n) = \frac{\lambda}{1 - e^{-(\alpha^{n-1}\delta)/\lambda}}$$

Let the parameter values be $c = 15,\ r = 45,\ c_p = 10,\ \mu = 10,\ \alpha = 1.05,\ \gamma_0 = 0.9\ , R = 4500,\ \nu = 0.2, \theta = 0.1,\ \delta = 10,\ \lambda = 15$ and $\tau = 10$.

The numerical results of equations (7) and (9) are tabulated in Table 1 and plotted in Figure 1.

TABLE 1. The values of $C(N)$ and $B(N)$ against N

N	$C(N)$	$B(N)$	N	$C(N)$	$B(N)$
1	78.6920	0.1222	**6**	**-7.8984**	**1.4490**
2	25.0724	0.1482	7	-7.0133	2.2525
3	7.3947	0.3569	8	1.4378	2.7329
4	-1.1077	0.5371	9	13.8832	3.0259
5	-5.7360	0.8515	10	14.9984	3.3027

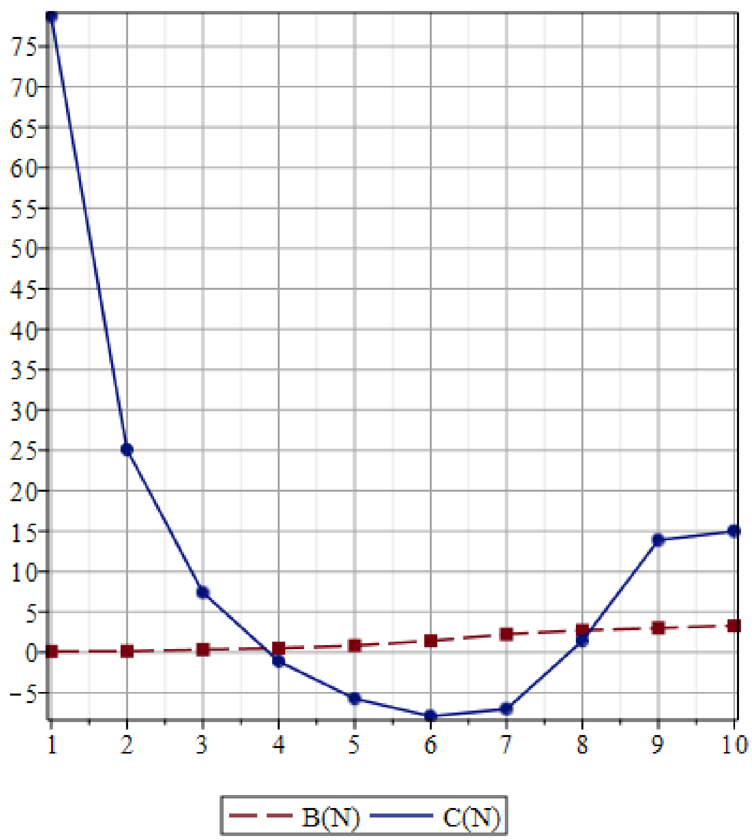

FIGURE 1. The plots of $C(N)$ and $B(N)$ against N

Obviously, $C(6)$= -7.8984 is the minimum of the average cost.

Moreover, $B(6)$=1.4490 > 1, and $6 = \min\{N|B(N) \geq 1\}$.

Therefore, the unique optimal policy is N^*= 6, the system should be replaced at the time of the 6-th failure.

6. Conclusion

By considering a δ-Shock maintenance model for a deteriorating system with imperfect delayed repair under partial product process, an explicit expression for the long-run average cost per unit time under the replacement policy N is determined. An optimal policy N^* for minimizing the long run average cost per unit time is determined analytically. A numerical example is given to explain the methodology used.

Conflict of Interests

The authors declare that there is no conflict of interests.

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