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ON THE CONSTRUCTION OF FUZZY INTRINSIC EDGE MAGIC GRAPHS

M. KALIRAJA[1,*] AND M. SASIKALA[2]

[1]Department of Mathematics, H.H. The Rajah's college (Autonomous), Pudukkottai - 622 001, Tamil Nadu, India

[2]Department of Mathematics, Arul Anandar College (Autonomous), Madurai - 625 514, Tamil Nadu, India.

Abstract: In this paper, we have discussed the idea of fuzzy intrinsic edge-magic graphs with edge-magic persistent. Further, the fuzzy intrinsic edge magic labelling graphs like fuzzy paths and fuzzy cycles are also discussed. We have focused some theorems on fuzzy intrinsic edge-magic graphs. It ought to be note that the necessary and sufficient conditions have been given for all the above mentioned graphs. In addition, we have introduced the pseudo edge-magic graphs.

Keywords: fuzzy intrinsic edge-magic labeling; fuzzy intrinsic edge-magic graphs; mock constant; pseudo edge-magic graphs.

2010 AMS Subject Classification: 05C72, 05C78.

1. INTRODUCTION

Fuzzy set was firstly introduced by [1]. Then various researches added productive concepts to develop fuzzy sets theory like [3] and [8]. In 1987 Bhattacharya has succeeded to develop the connectivity notions between fuzzy bridge and fuzzy cut nodes [5]. A fuzzy graph contains

*Corresponding author
E-mail address: mk.maths009@gmail.com
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many properties similar to crisp graph due to generalization of crisp graphs but it diverge at many places.

A crisp graph G is an order pair of vertex-set V and edge set E such that $E \subseteq V \times V$. In addition $v = |V|$ is said to order and $e = |E|$ is called size of the graph G respectively. In a crisp graph, a bijective function $\rho : V \cup E \to N$ that produced a unique positive integer (To each vertex and/or edge) is called a labelling [4]. Introduced the notion of magic graph that the labels vertices and edges are natural numbers from 1 to $|V| + |E|$ such that sum of the labels of vertices and the edge between them must be constant in entire graph [6]. Extended the concept of magic graph with added a property that vertices always get smaller labels than edges and named it super edge magic labelling. Numerous other authors have explored diverse types of different magic graphs [7], [11] & [12]. The subject of edge-magic labelling of graphs had its origin in the work of Kotzig and Rosa on what they called magic valuations of graphs [2]. These labelling are currently referred to as either edge-magic labelling or edge-magic total labelling.

Fuzzy graphs are generalization of graphs. In graphs two vertices are either related or not related to each other. Mathematically, the degree of relationship is either 0 or 1. In fuzzy graphs, the degree of relationship takes values from [0, 1]. A fuzzy graph has ability to solve uncertain problems in a wide range of fields. The first definition of a fuzzy graph was introduced by Kaufmann in 1973. Azriel Rosenfield in 1975 [3] developed the structure of fuzzy graphs and obtained analogs of several graph theoretical concepts. In [10], NagoorGani et. al. introduced the concepts of fuzzy labelling graphs, fuzzy magic graphs. A fuzzy graph contains many properties similar to crisp graph due to generalization of crisp graphs but it diverge at many places. In this paper we have developed the concept of fuzzy intrinsic edge magic graphs and also we introduced some general form of edge-magic persistent of above graphs. Throughout this paper we only focused on undirected fuzzy graphs.

2. PRELIMINARIES

Definition 2.1:

A fuzzy graph $G=(\sigma,\mu)$ is a pair of functions $\sigma: V \to [0,1]$ and $\mu: V\times V \to [0,1]$ where for all $u,v \in V$, we have $\mu(u,v) \le \sigma(u)\Lambda\sigma(v)$.

Definition 2.2:

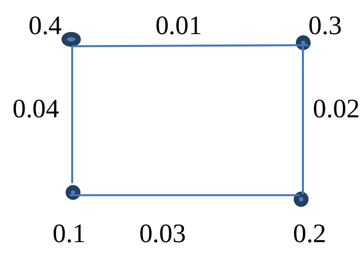
A path P in a fuzzy graph is a sequence of distinct nodes $v_1, v_2, v_3.....v_n$ such that $\mu(v_i, v_{i+1}) > 0; 1 \le i \le n;$ here $n \ge 1$ is called the length of the path P. The consecutive pairs (v_i, v_{i+1}) are called the edge of the path.

Definition 2.3:

A path P is called a cycle if $v_1 = v_n$ and $n \ge 3$ and a cycle is called a fuzzy cycle if it contains more than one weakest arc.

Definition 2.4:

A bijection ω is a function from the set of all nodes and edges of to [0, 1] which assign each nodes $\sigma^{\omega}(a)$, $\sigma^{\omega}(b)$ and edge $\mu^{\omega}(a,b)$ a membership value such that $\mu^{\omega}(a,b) \le \sigma^{\omega}(a)\ \Lambda\ \sigma^{\omega}(b)$ for all $a,b \in V$ is called fuzzy labeling. A graph is said to be fuzzy labeling graph if it has a fuzzy labelling and it is denoted by G^{ω}.



Fuzzy labelling graph

3. Main results

3. Fuzzy intrinsic edge magic graphs

Definition 3.1:

A fuzzy labelling graph G is said to be fuzzy intrinsic labelling if $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ is bijective such that the membership values of edges and vertices are $\{ z, 2z, 3z, ..., Nz \}$ without any repetition where N is the total number of vertices and edges and let z = 0.1 for N≤6 & z = 0.01 for N>6 .

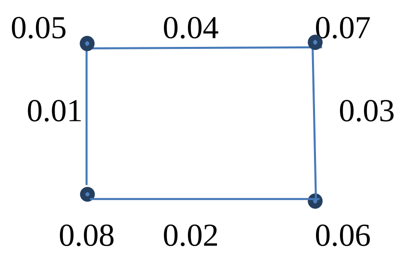


Figure (i)

Definition 3.1(a):

A fuzzy graph is said to be intrinsic graph if it satisfies the intrinsic labelling. For example, the above graph is an intrinsic graph.

Definition 3.2:

A fuzzy intrinsic labelling said to be an edge-magic labelling if it has an intrinsic constant $\lambda_c = \sigma(v_i) + \mu(v_i\, v_j) + \sigma(v_j)$ for all $v_i, v_j \in V$.

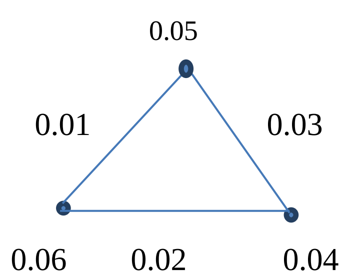


Figure (ii)

Definition 3.3:

A fuzzy graph G is said to be intrinsic edge-magic if it satisfies the intrinsic edge-magic labeling with intrinsic constant $'\lambda_c'$.

Definition 3.4:

An edge-magic constant in a fuzzy intrinsic edge-magic graph is said to be mock constant $'\lambda_m'$ if it is equal to $\sigma(v_i)+\mu(v_i v_j)+\sigma(v_j)$ for some $v_i, v_j \in V$ with $\lambda_c \neq \lambda_m$.

Definition 3.5:

A fuzzy graph is said to be a pseudo-intrinsic edge-magic graph if it contains mock constant $'\lambda_m'$ which is also denoted by $'G_p'$.

Vital condition: For intrinsic edge-magic, the vital condition is that the intrinsic edge-magic graph satisfies only the intrinsic edge magic labelling.

Competent condition: A competent condition for intrinsic edge-magic is that if it has the same intrinsic constant for all edges.

Definition 3.6:

A fuzzy intrinsic edge-magic labeling graph is said to be fuzzy intrinsic edge-magic if it satisfies both vital and competent condition.

Definition 3.7:

A fuzzy graph is said to be a vital graph if it satisfies the vital condition of fuzzy intrinsic graph and also it does not satisfy the competent condition.

Theorem 3.2: A path P_n is fuzzy intrinsic edge-magic if $n \geq 2$ where n is length of P_n.

Proof: Let P_n be a fuzzy path graph with 'n' vertices and 'n-1' edges.

Let $z \to (0,1]$ be such that for $z=0.1$ *for* $N \leq 6$ & $z=0.01$ *for* $N > 6$.

The fuzzy intrinsic edge-magic labelling of given path P_n is:

$$\sigma(v_{2i}) = (2n - i)\, z$$
$$\sigma(v_{2i-1}) = (n + 3 - i)\, z$$
$$\mu(v_i\, v_{i+1}) = iz \quad for\, 1 \le i \le N$$

We can consider the above labelling, we gets fuzzy intrinsic constant.

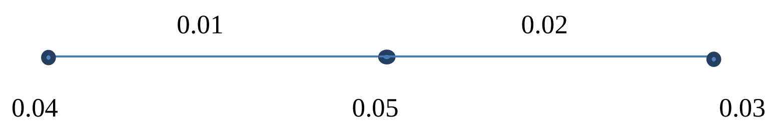
$$\lambda(P_n) = \sigma(v_{2i}) + \mu(v_i\, v_{i+1}) + \sigma(v_{2i-1})$$
$$= (2n - i)z + iz + (n + 3 - i)z$$
$$= (3n + 3 - i)\, z \qquad for\, 1 \le i \le N$$

Here we put particular value of 'i' ,

$$\lambda(P_3) = 0.10 \qquad for\ i = \frac{n+1}{2}$$
$$\lambda(P_4) = 0.13 \qquad for\ i = \frac{n}{2}$$
$$\lambda(P_5) = 0.17 \qquad for\ i = \frac{n+1}{3}$$

Case(i):

If n = 3, then the fuzzy intrinsic constant,

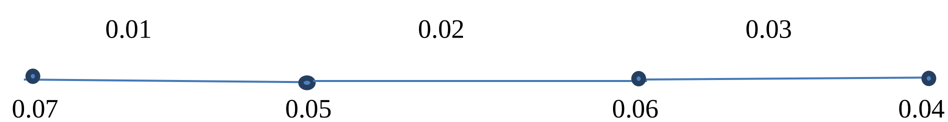


P_3 – *Fuzzy path*

$$\sigma(v_1) + \mu(v_1\, v_2) + \sigma(v_2) = 0.04 + 0.01 + 0.05 = 0.10$$
$$\sigma(v_2) + \mu(v_2\, v_3) + \sigma(v_3) = 0.05 + 0.02 + 0.03 = 0.10$$

Case (ii):

Let n = 4, a path has 4 vertices

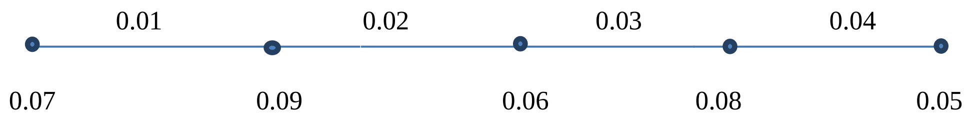


P_4 – *Fuzzy path*

$$\sigma(v_1)+\mu(v_1 v_2)+\sigma(v_2)=0.07+0.01+0.05=0.13$$
$$\sigma(v_2)+\mu(v_2 v_3)+\sigma(v_3)=0.05+0.02+0.06=0.13$$
$$\sigma(v_3)+\mu(v_3 v_4)+\sigma(v_4)=0.06+0.03+0.04=0.13$$

Case (iii) :

If n = 5, then it has fuzzy intrinsic constant



P_5 – *Fuzzy path*

$$\sigma(v_1)+\mu(v_1 v_2)+\sigma(v_2)=0.07+0.01+0.09=0.17$$
$$\sigma(v_2)+\mu(v_2 v_3)+\sigma(v_3)=0.09+0.02+0.06=0.17$$
$$\sigma(v_3)+\mu(v_3 v_4)+\sigma(v_4)=0.06+0.03+0.08=0.17$$
$$\sigma(v_4)+\mu(v_4 v_5)+\sigma(v_5)=0.08+0.04+0.05=0.17$$

For different values of ‘i’ depending upon ‘n’, we get fuzzy intrinsic edge-magic constant. Also a fuzzy path P_n admits a fuzzy intrinsic edge-magic labelling. Hence fuzzy path with two or more vertices are fuzzy intrinsic edge-magic graph with fuzzy intrinsic edge –magic constant:

$$\lambda(P_n)=(3n+3-i)\,z \quad for\, 1\le i\le N \quad \bullet$$

Theorem 3.3:

A graph C_n is fuzzy intrinsic edge-magic if n = 3.

Proof:

Let C_n be a cycle graph with ‘n’ vertices and ‘n’ edges.

Let $z\rightarrow(0,1]$ be such that for $z=0.1$ *for* $N\le 6$ & $z=0.01$ *for* $N>6$.

Apply the fuzzy intrinsicedge-magic labelling,

$$\sigma(v_{2i}) = (2n+1-i)\,z \;\; for\, 1 \le i \le n-1$$
$$\sigma(v_{2i-1}) = (n+3-i)\,z \;\; for\, 1 \le i \le n-2$$
$$\mu(v_n\, v_1) = n\,z$$
$$\mu(v_i\, v_{i+1}) = i\,z \qquad for\;\; 1 \le i \le n-1$$

Now, we find the intrinsic constant,

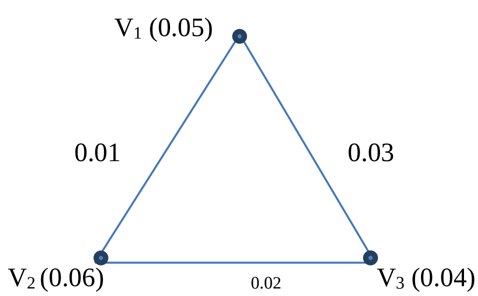
$$\lambda(C_n) = \sigma(v_{2i}) + \mu(v_i\, v_{i+1}) + \sigma(v_{2i-1}) + \mu(v_n v_1)$$
$$= (2n+1-i)z + i\,z + (n+3-i)z + nz$$
$$= (4n+4-i)\,z \qquad for\, 1 \le i \le N$$

$$\lambda(C_n) = (4n+4-i)\,z \qquad for\, 1 \le i \le N$$

Now we assume that n=3, a fuzzy cycle graph with 3 vertices. Let us consider the labeling,

$\sigma(V_1) = 0.05,\ \sigma(V_2) = 0.06,\ \sigma(V_3) = 0.04\,\&$

$\mu(V_1 V_2) = 0.01,\ \mu(V_2 V_3) = 0.02, \mu(V_3 V_1) = 0.03$



$C_3 - Fuzzy\, cycle$

From the above graph, we get

$$\sigma(V_1) + \mu(V_1 V_2) + \sigma(V_2) = 0.12$$
$$\sigma(V_2) + \mu(V_2 V_3) + \sigma(V_3) = 0.12$$
$$\sigma(V_1) + \mu(V_1 V_3) + \sigma(V_3) = 0.12$$

$$\lambda(C_n) = (4n+4-i)\,z \qquad for\, 1 \le i \le N$$

Put n = 3 and particular value if i = n+1 , if n is odd, $\lambda(C_3) = 0.12$ Therefore, fuzzy cycle C_n with 3 vertices is fuzzy intrinsic edge-magic graph.

Theorem 3.4: A cycle graph C_n is pseudo intrinsic edge-magic graph if n>3.

Let C_n be a cycle graph with four vertices.

In the figure (i), three edges have intrinsic constant and only one edge has a mock constant which is not equal to intrinsic constant .In a similar manner, we can easily prove C_n with 4 or greater than 4 vertices which is pseudo intrinsic edge-magic graph.

4. CONCLUSION

In this paper, we discussed the idea of fuzzy intrinsic edge-magic graphs with edge-magic persistent and the fuzzy intrinsic edge magic labelling graphs like fuzzy paths and fuzzy cycles are also discussed. We focused some theorems on fuzzy intrinsic edge-magic graphs. It ought to be note that the necessary and sufficient conditions are given for all the above mentioned graphs.

Conflict of Interests

The authors declare that there is no conflict of interests.

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