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INFLUENCE OF CRITICAL PARAMETERS ON AN UNSTEADY STATE MHD FLOW IN A POROUS CHANNEL WITH EXPONENTIALLY DECREASING SUCTION

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Abstract: The behavior of the velocity for a fixed Reynolds number and different values of angle of inclination has been presented in these graphs. In each of these situations, it is noticed that as the magnetic intensity increases, the velocity decreases. Further, as the angle of inclination increases, the velocity decreases. In all these figures, it is observed that there is a wide spread dispersion of velocity profiles which are very closed to boundary plate initially. Further, it is observed that for constant values of magnetic intensity and Reynolds number. As the angle of inclination increases, the velocity decreases. Further, in all above situations the velocity profiles are perfectly parabolic and the no slip conditions are satisfied at both the boundaries. it is seen that as the 'Re' increases, the skin friction in the boundary layer region increases. Further, as the angle of inclination of the boundary surface increases, the skin friction decreases and finally converges to zero. For a slight change in the applied magnetic intensity, a marginal change in the skin friction is noticed in the boundary layer region. Also, it is observed that as the angle of inclination increases,

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the skin friction is observed to be increasing. The nature of profiles in both the cases do not differ qualitatively. For a small change in the applied magnetic intensity, the skin friction decreases. it is seen that as the magnetic field increases, the skin friction varies linearly. As the 'Re' increases, the skin friction on the boundary increases. Not much of change in the profiles of the skin friction is noticed.

Keywords: boundary layer; exponentially decreasing function; Reynolds number; skin friction; flow rate.

2010 AMS Subject Classification: 76A02, 35Q35.

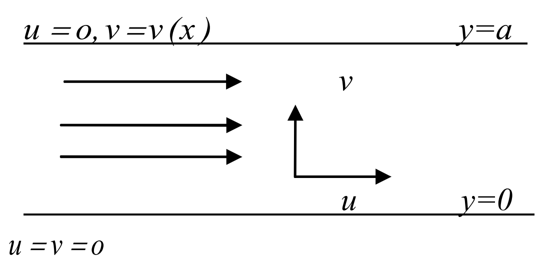
1. INTRODUCTION

Flow through porous media can be considered as an ordered flow in a disordered geometry. Fluid through porous medium involves the porous matrix and the fluid under consideration. In several problems related to geophysical, petroleum, chemical engineering problems and biomedical situations the problem assumes greater significance. In all such situations, one has to consider the final total gross effect of the phenomenon that is being represented by the nature of the flow which is largely compared to the characteristic features of the flow phenomenon. The driving force that is necessary to move the specific volume of the fluid with certain velocity must be in equilibrium with resistive force that is generated due to the internal friction between the fluid and the bounding surface.

The flow of incompressible viscous fluid with parallel rigid boundaries given by the uniform suction/injection at the walls was first discussed by Berman [1]. There after the work of Berman was further analyzed by Sellars [2] for high suction Reynolds number. Thereafter, the flow in a circular pipe for small suction/injection was analyzed by Yuan and Finklestein [3]. The analyzed solution was obtained as an asymptotic expression subsequently, the flow in the Renal tubes through a circular tube with uniform cross section and permeable boundary with radial velocity at the walls with exponentially decreasing function of actual distance was discussed by Masse [4]. In the absence of any heat source, the MHD flow in a vertical parallel channel was discussed by Yu [5]. Thereafter, the theory by considering the laminar flow in a porous pipe with uniform suction and injection was studied by Terril and Thomas [6]. Thereafter, the problem of fully developed flow between two vertical plates was examined by Gupta [7]. In a series of two papers, various aspects of blood flow in a pulmonary channel with a view of understanding the flow and the corresponding dispersion of fluids flowing in the lungs was discussed by Fung [8] and Tang [9].The flow pattern using physical conditions rather than Beavers and Joseph slip conditions were considered in their investigations are found to be independent of stability analysis. Ramana Murthy and Chandrasekhar [10] analyzed the characteristic behavior of boundary layer phenomenon in MHD flow with exponentially decreasing suction. Recently Raman Murthy *et a*l [11] presented a detailed analysis of viscoelastic fluid of second order type between two parallel plates with the lower plate having natural permeability. It is observed that the skin friction on the upper plate is almost linear. However, the situation on the lower plate is not same as above. The case of linear analysis by considering viscoelastic property of the fluid over an inclined porous plate was investigated by Ramana Murthy and Kavitha [12].

In all such analysis, it is observed that as the viscoelasticity increases, there is a decreasing trend in the velocity profiles while the velocity profiles are significantly distributed and are found to be parabolic with the inclusion of viscoelastic term in the governing equations of the motion. Some examples in living organisms are related to fluid transport mechanisms out of which the blood flow in a circulatory system.

2. MATHEMATICAL FORMULATION



Schematic diagram of the problem.

A steady flow of viscous electrically conducting fluid through a long channel under the influence of externally applied homogeneous magnetic field applied is considered. It is assumed that the fluid is incompressible with negligible electrical conductivity and the electromagnetic force is very minute. A rectangular cartesian coordinate system is considered. Y-axis is considered distance measured in the normal section of geometry. Further, u and v are the velocity components considered in the x and y directions while P is the pressure gradient.

The governing equations of continuity and momentum are given as follows. The section of geometry is considered to be varying from horizontal to the vertical mode.

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*}+\frac{\partial^2 u^*}{\partial y^2}-\frac{\sigma_e B_0^2}{\rho}u^* \tag{2}$$

Where $B=(0,B_0)$ is the magnetic field vector. The appropriate boundary conditions are

$$u^*=0 \quad \text{on} \quad y=0 \tag{3}$$

$$u^*=0 \quad \text{on} \quad y=a \tag{4}$$

Where a is the width of the channel, σ_e is the electrical conductivity, ρ the fluid density, ν is the kinematic viscosity. Introducing the following non dimensional quantities:

$$u=\frac{u^*}{U}, v=\frac{v^*}{U}, x=\frac{x^*}{a}, y=\frac{y*}{a}, p=\frac{p^*}{\rho U^2}, t=\frac{t^*}{(a^2/v)}, Re=\frac{Ua}{v} \tag{5}$$

and substituting into equation (1) and equation (2), we obtain

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{6}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{Re}\left\{\frac{\partial^2 u}{\partial y^2}\right\}+Mu+Gcos\alpha \tag{7}$$

The boundary conditions reduce to

$$u=0 \; on \; y=0 \tag{8}$$

$$u=0 \; on \; y=1 \tag{9}$$

Under the assumptions that the flow is laminar and has the velocity components are $[u(y,t),0,0]$ and the plate is large enough always along x-axis the equation of motion is given by:

$$\frac{\partial u}{\partial x}=0 \tag{10}$$

$$\frac{\partial u}{\partial t} = \frac{1}{Re}\left(\frac{\partial^2 u}{\partial y^2}\right) + Mu + Gcos\alpha \tag{11}$$

Where $\alpha =$ Angle of inclination

And the boundary conditions reduce to

$$u = 0 \ on \ y = 0 \tag{12}$$

$$u = 0 \ on \ y = 1 \tag{13}$$

3. SOLUTION OF THE PROBLEM

Let the solution of the equation (10) and equation (11) be of the form

$$u(y,t) = u_0(y)e^{-i\omega t} \tag{14}$$

Substituting equation (14) and equation (15) in equation (11) we obtain:

$$\frac{d^2 u_0}{dy^2} + p^2 u_0 = -q^2 \tag{15}$$

The boundary conditions transformed to:

$$u_0 = 0 \quad \text{on} \quad y = 0 \tag{16}$$

$$u_0 = 0 \quad \text{on} \quad y = 1 \tag{17}$$

The solution of equation (15) satisfying boundary conditions (16) and (17) is

$$u(y,t) = \frac{\operatorname{Re} G\cos\alpha}{p^2}\left[\frac{\sin p(1-y) + \sin py}{\sin p} - 1\right] \tag{18}$$

Where $p = \sqrt{\operatorname{Re}(i\omega + M)}$

$q = \sqrt{\operatorname{Re} G\cos\alpha e^{i\omega t}}$

And skin friction $\left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{\operatorname{Re} G\cos\alpha}{p}\left(\frac{1-\cos p}{\sin p}\right)$ (19)

Flow rate $\int_0^1 u dy = 2\frac{\operatorname{Re} G\cos\alpha}{p}\left(\frac{1-\cos p}{\sin p}\right) - \frac{\operatorname{Re} G\cos\alpha}{p^2}$ (20)

4. RESULTS AND CONCLUSIONS

1. The effect of magnetic intensity on the velocity profiles has been illustrated in figure1, figure2, figure 3 and figure4.

The behavior of the velocity for a fixed Reynolds number and different values of angle of inclination has been presented in these graphs. In each of these situations, it is noticed that as the magnetic intensity increases, the velocity decreases. Further, as the angle of inclination increases, the velocity decreases. In all these figures, it is observed that there is a wide spread dispersion of velocity profiles which are very closed to boundary plate initially.

As we move away far from the plate, the velocity profiles converge. It is seen that the velocity profiles are parabolic in nature in all above situations.

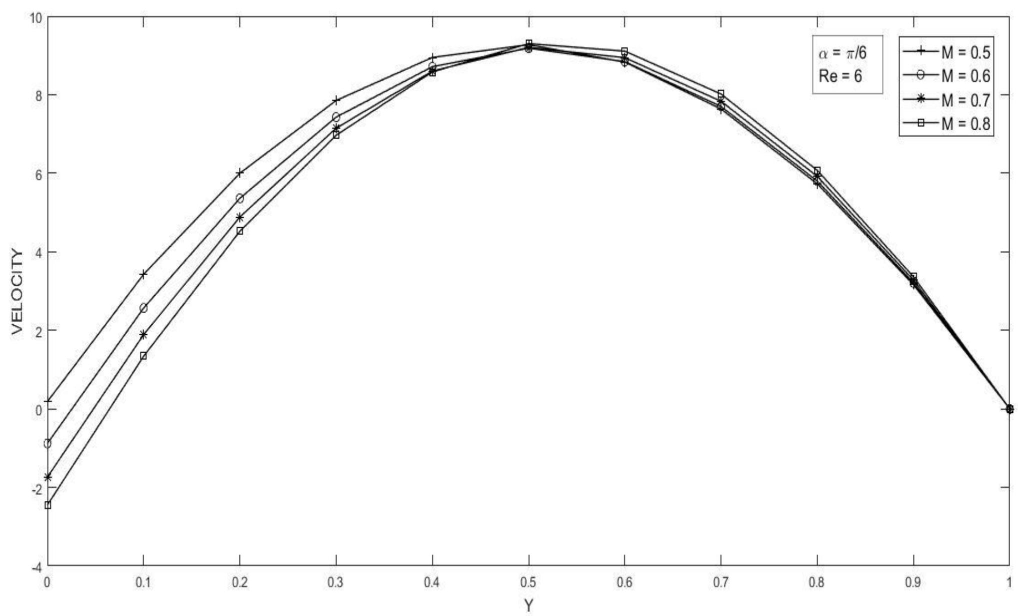


Fig -1: Influence of magnetic field on velocity

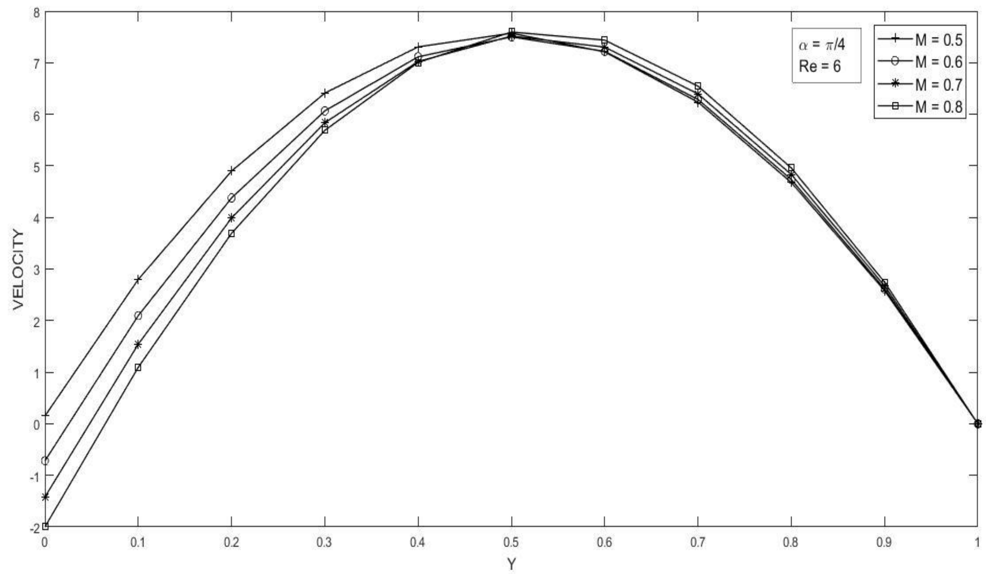


Fig-2: Effect of magnetic field on velocity

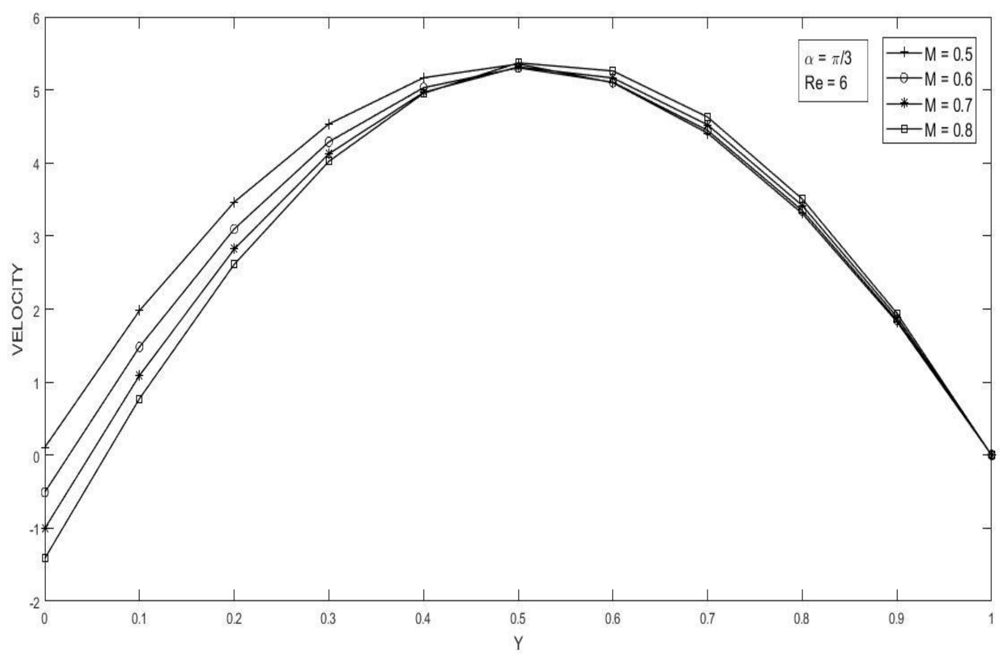


Fig-3: Influence of magnetic field on velocity

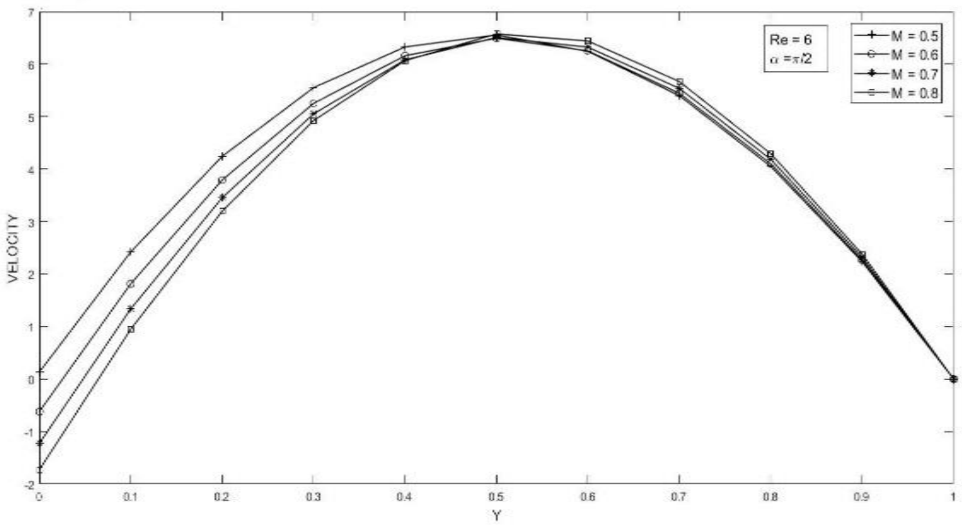


Fig-4: Effect of magnetic field on velocity

2. The effect of the angle of inclination for a fixed Reynolds number and for different values of magnetic intensity, the velocity profiles are plotted as figure 4, figure 5, figure 6and figure 7.

In each of these illustrations, it is observed that for constant values of magnetic intensity and Reynolds number. As the angle of inclination increases, the velocity decreases. Further, in all above situations the velocity profiles are perfectly parabolic and the no slip conditions are satisfied at both the boundaries.

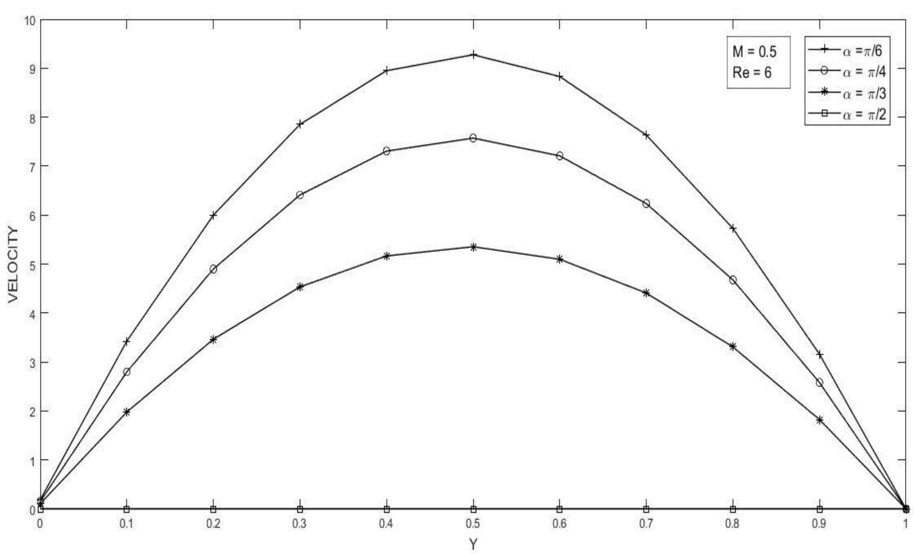


Fig-5: Effect of angle of inclination on velocity

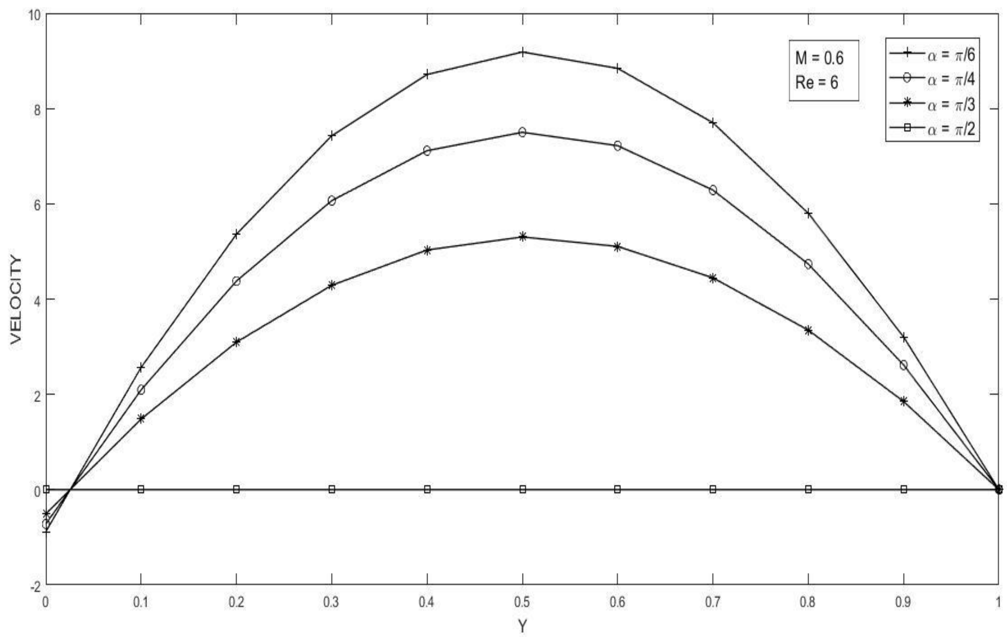


Fig-6: Influence of angle of inclination on velocity

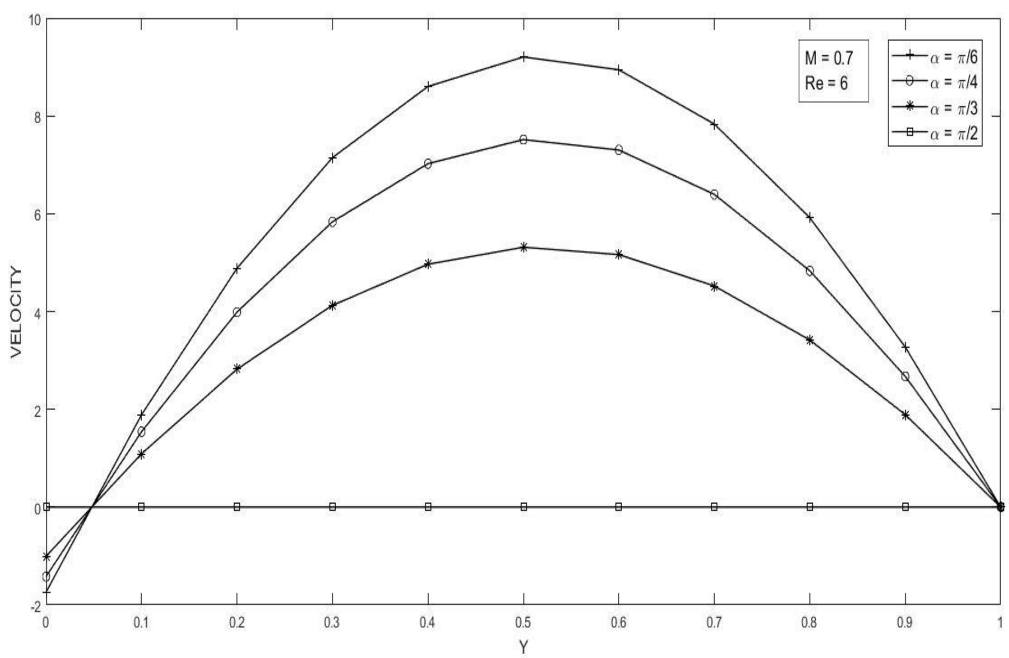


Fig-7: Influence of angle of inclination on velocity profiles

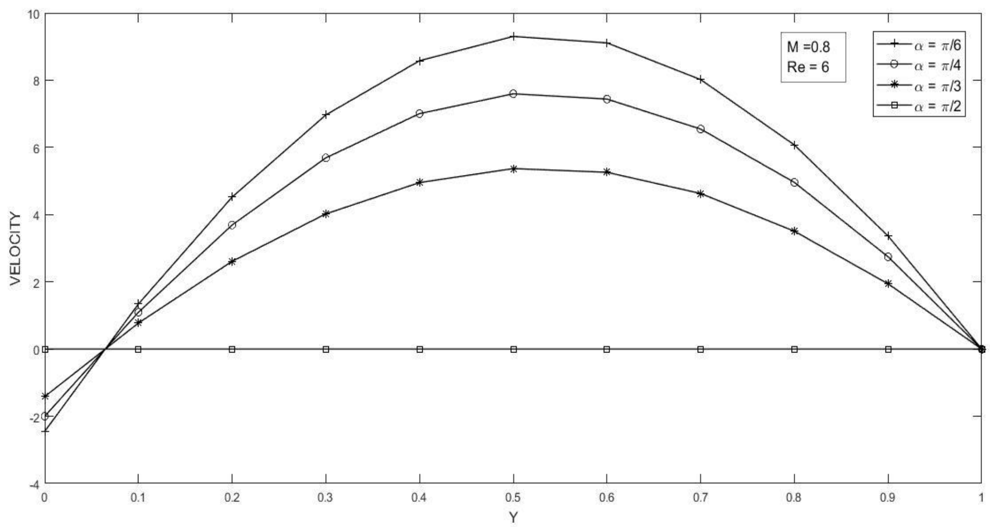


Fig-8: Effect of angle of inclination on velocity profiles

3. The contribution of Reynolds number (Re) on the skin friction is shown in figure 9 and figure 10 for M = 0.5 and M = 0.6 respectively. In both these figures it is seen that as the Re increases, the skin friction in the boundary layer region increases. Further, as the angle of inclination of the boundary surface increases, the skin friction decreases and finally converges to zero. For a slight change in the applied magnetic intensity, a marginal change in the skin friction is noticed in the boundary layer region.

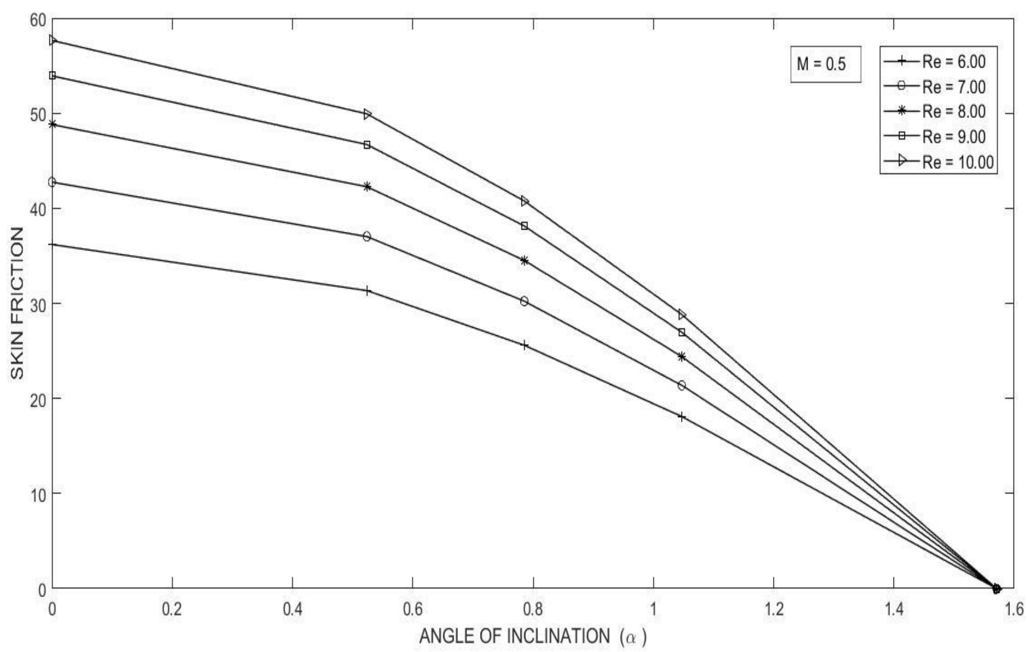


Fig-9: Effect of 'Re' on skin friction

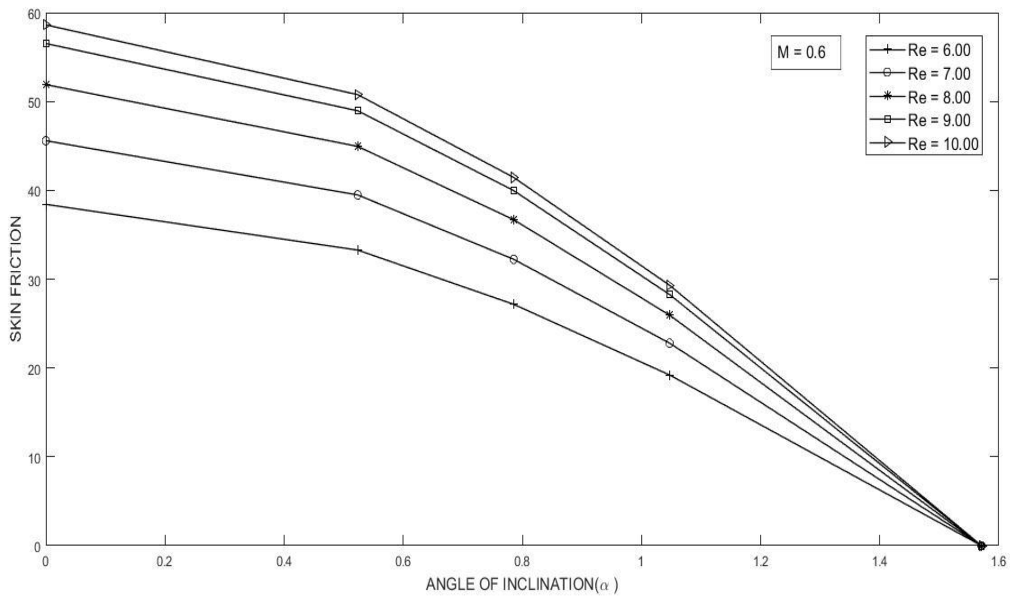


Fig-10: Influence of 'Re' on skin friction

4. The influence of the angle of inclination with respect to the 'Re' for the applied magnetic intensity M = 0.5 and M = 0.6 is illustrated in figures 11 and 12 respectively. In both these figures, it is observed that as the angle of inclination increases, the skin friction is observed to be increasing. The nature of profiles in both the cases do not differ qualitatively. For a small change in the applied magnetic intensity, the skin friction decreases.

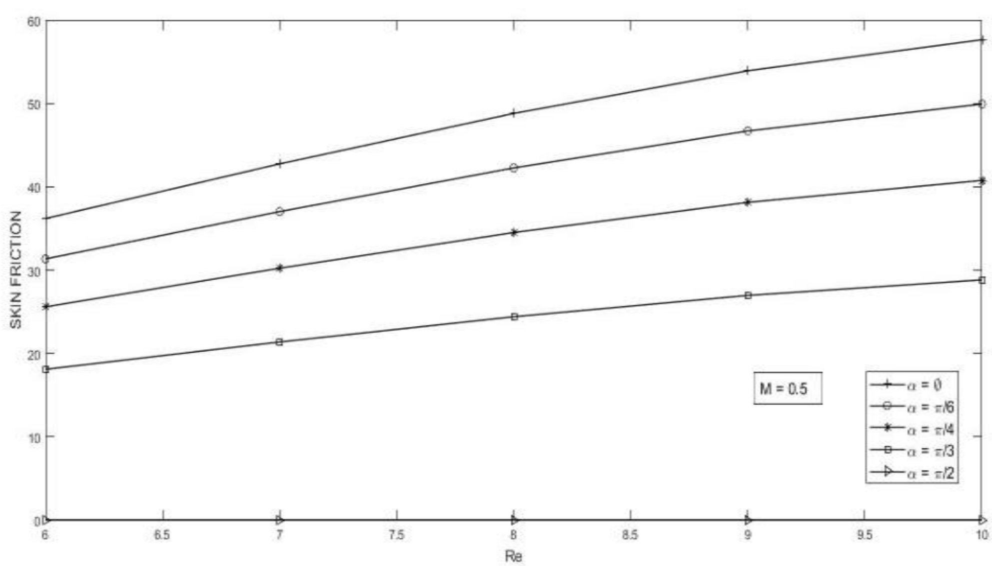


Fig-11: Contribution of the angle of inclination on skin friction

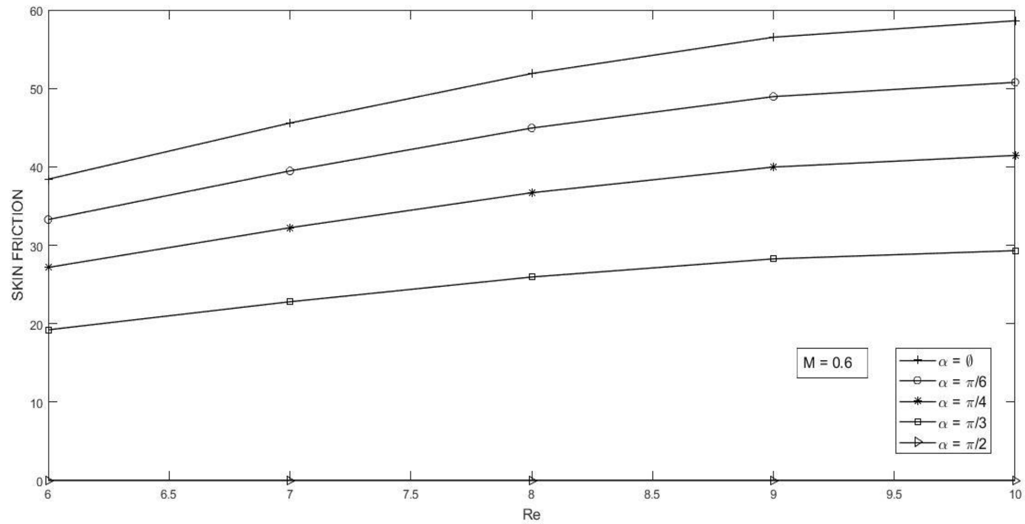


Fig-12: Effect of the angle of inclination on skin friction

5. The contributed effect of magnetic field with respect to the angle of inclination for Re = 6 and Re = 7 is depicted in figure 13 and figure 14 respectively. In both these illustrations, it is observed that as the angle of inclination increases, the skin friction decreases rapidly. This is in confirmation with the real-life experimental results. As the angle of inclination varies from 0 to pi/2, the boundary surface does not experience any friction on the boundary. Further, as the 'Re' increases, a marginal decrease in the skin friction is noticed in the boundary layer region.

Also, the dispersion of the profiles is more significant in the initial stages and such a dispersion narrows down as the angle of inclination increases.

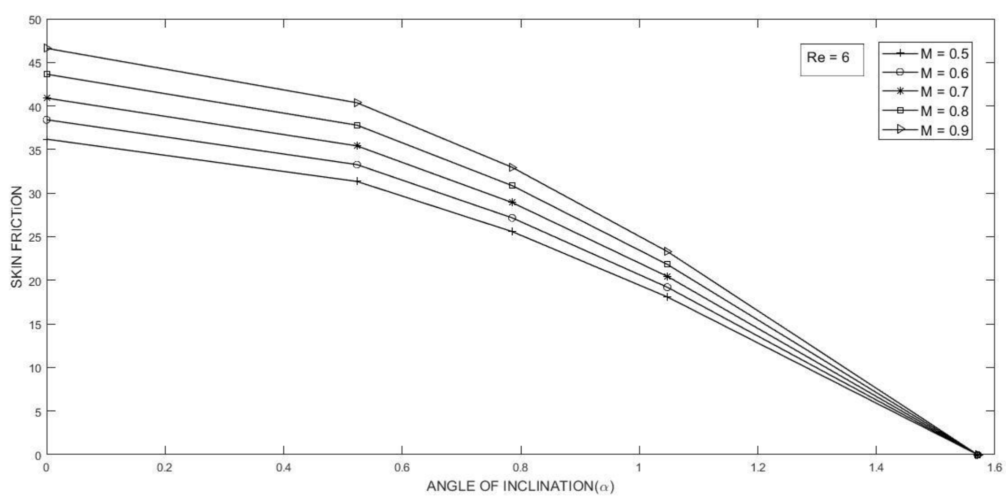


Fig-13: Influence of magnetic field on skin friction

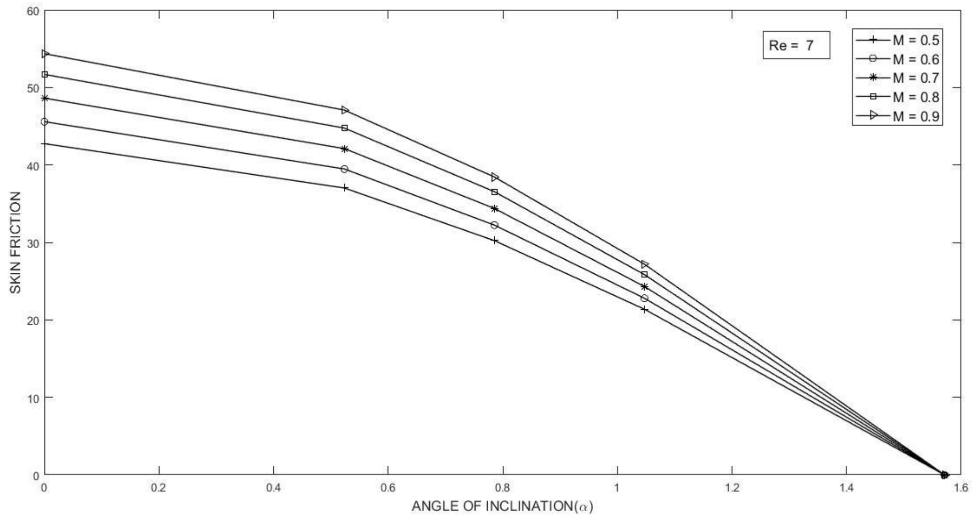


Fig-14: Effect of magnetic field on skin friction

6. The effect of the angle of inclination with respect to the Reynolds number Re = 6 and Re = 7 is shown in figure 15 and figure 16 respectively. For a constant value of Re, as the magnetic intensity increases the skin friction decreases. Further, from the above figures, it is seen that as the magnetic field increases, the skin friction varies linearly. As the 'Re' increases, the skin friction on the boundary increases. Not much of change in the profiles of the skin friction is noticed in both the illustrations.

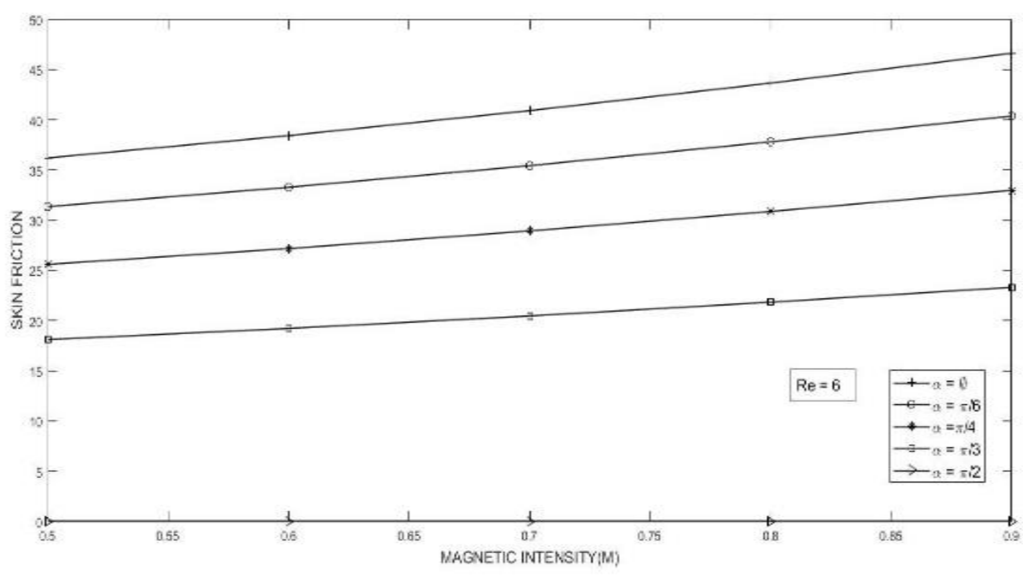


Fig-15: Variation of skin friction with respect to the angle of inclination

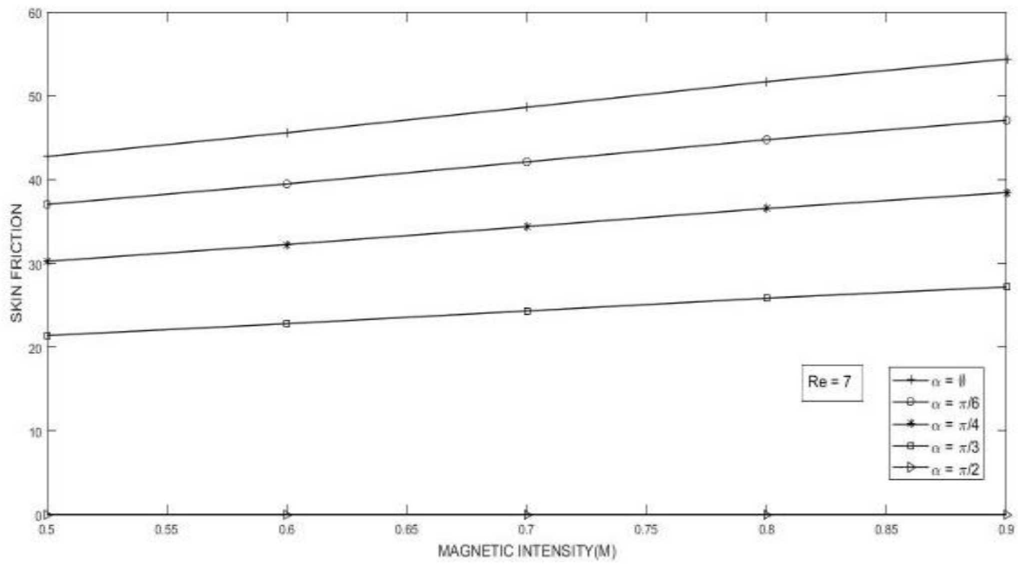


Fig-16: Contribution of the angle of inclination on skin friction

7. Figure 17 and figure 18 shows the influence of the angle of inclination on the flow rate for magnetic field M = 0.5 and M = 0.9 respectively. In both these illustrations it is observed that as the alpha increases, the flow rate increases and then it decreases as shown in figure 18.
However, in both the situations as alpha increases the flow rate decreases. Further, as M increases the profiles of the flow rate seems to be more parabolic.

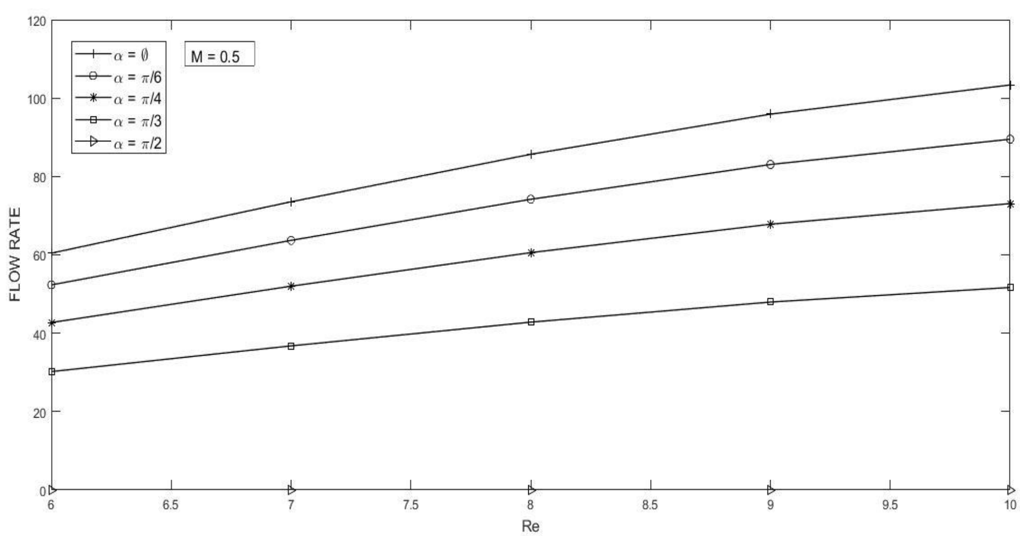


Fig-17: Effect of angle of inclination on flow rate

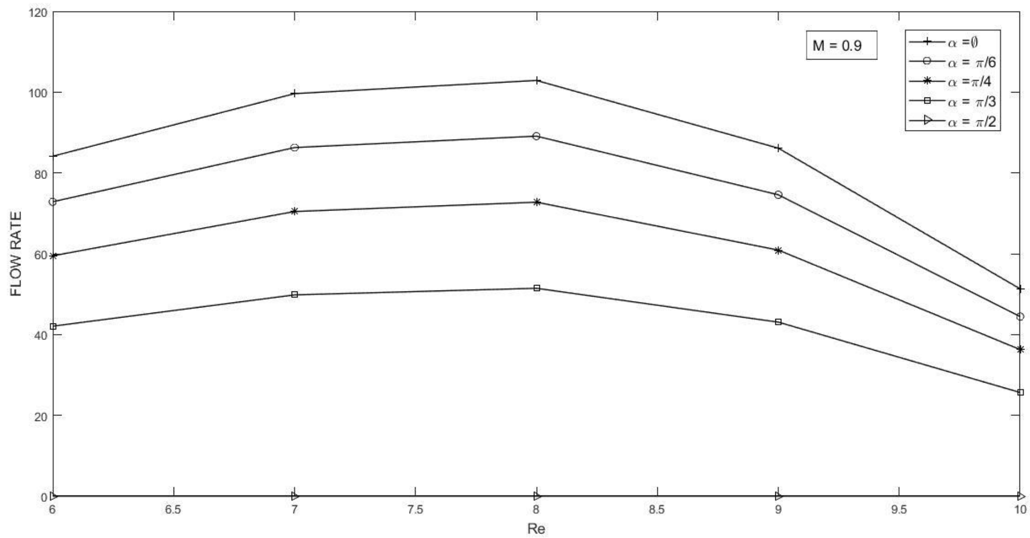


Fig-18: Influence of angle of inclination on flow rate

8. The effect of applied magnetic intensity with respect to the 'Re' for constant values of angle of inclination 0 and pi/3 is shown in figures 19 and 20. In both the cases, as the magnetic intensity increases, the flow rate decreases. The flow rate is found to be linear with respect to Reynolds number for any magnetic field.

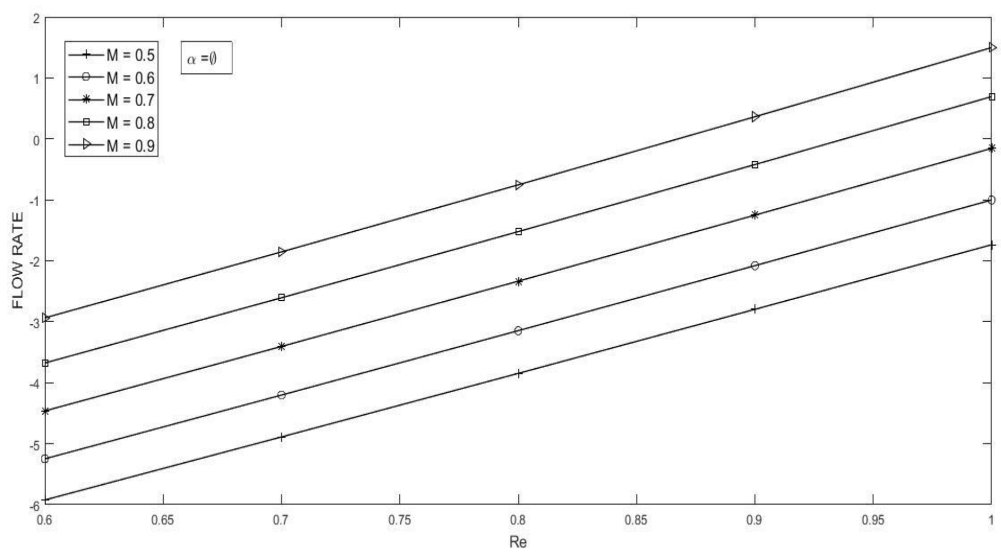


Fig-19: Effect of magnetic intensity on the flow rate

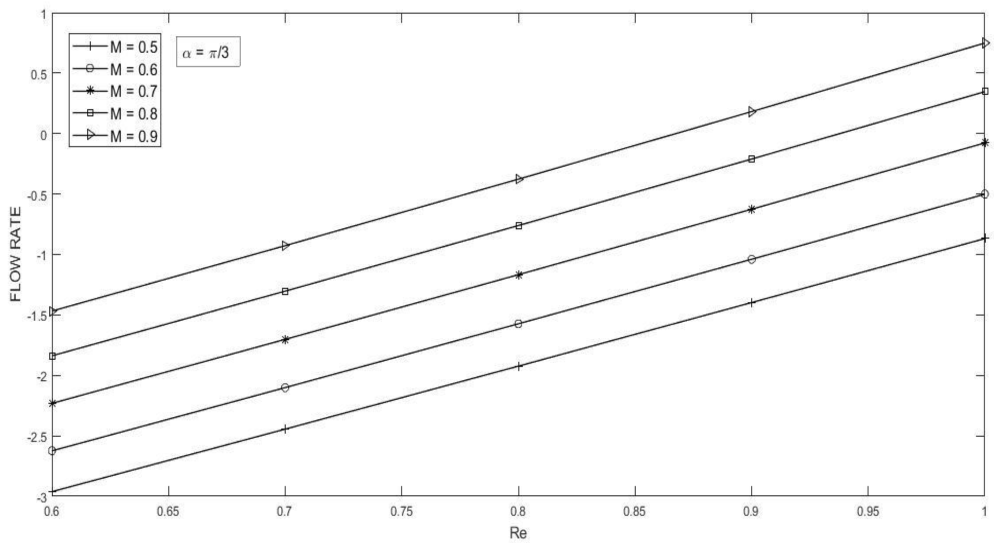


Fig-20: Contribution of magnetic influence on the flow rate

9. The influence of the angle of inclination with respect to applied magnetic intensity is shown in figures 21 and 22. It is seen that as angle of inclination increases, the flow rate decreases.

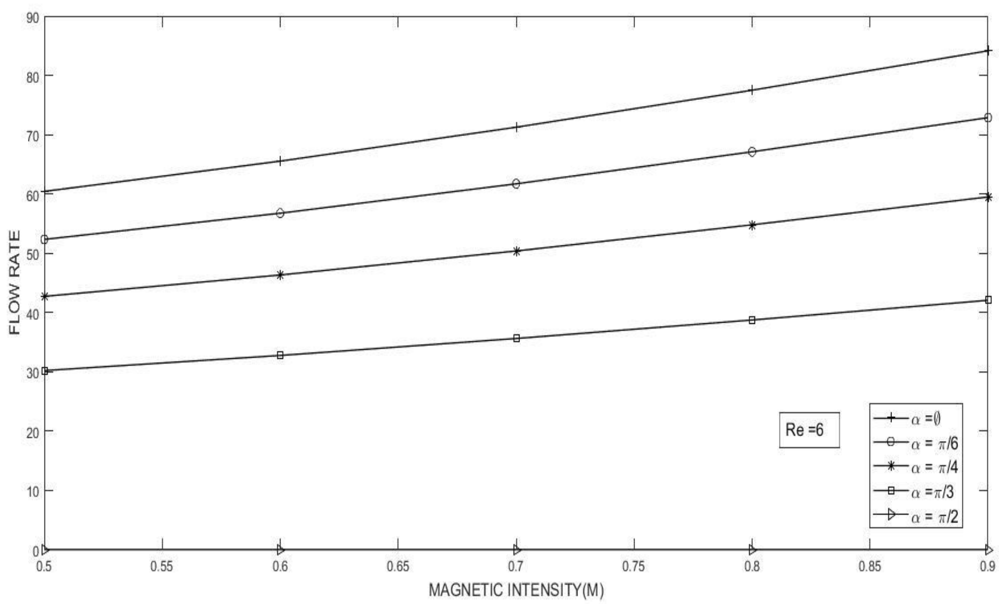


Fig-21: Influence of angle of inclination over flow rate

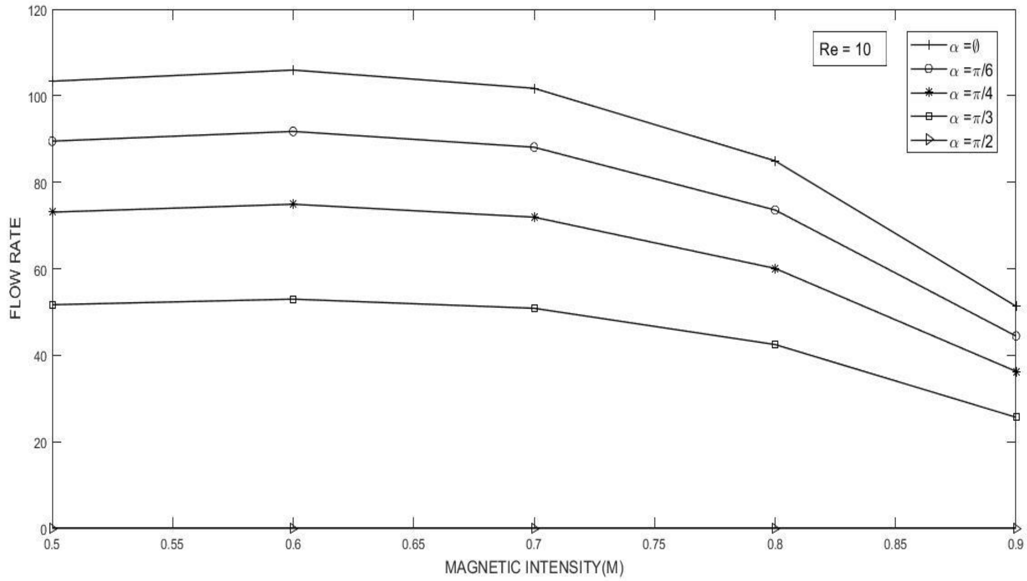


Fig-22: Variation of flow rate with respect to the angle of inclination

10. The effect of the applied magnetic intensity with respect to the angle of inclination is shown in figures 23 and 24. As the angle of inclination increases, the flow rate also increases. Further, the dispersion of the profiles is more significant in the boundary layer region when compared the far-off region.

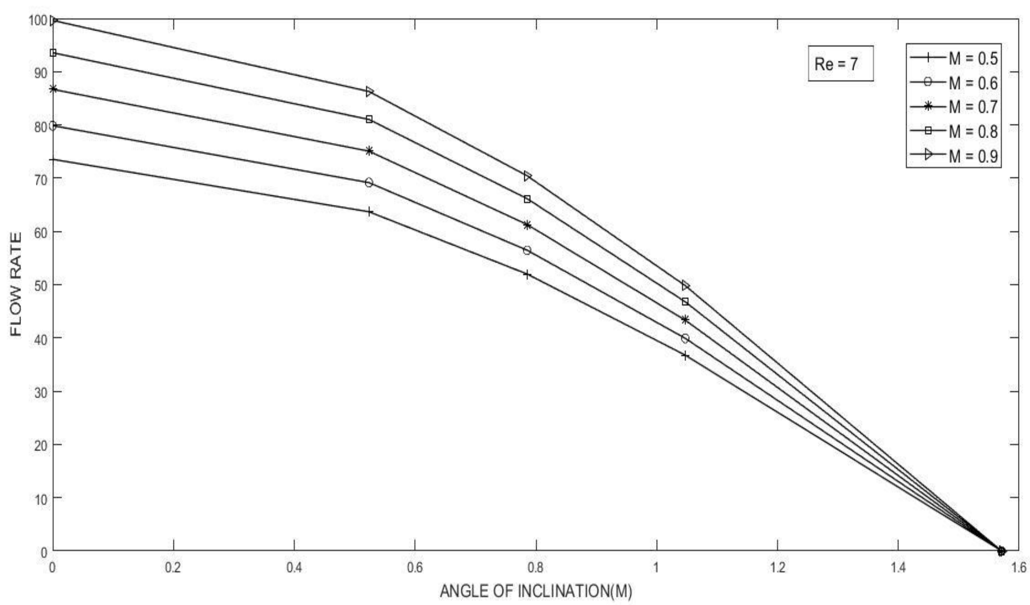


Fig-23: Effect of magnetic field on the flow rate

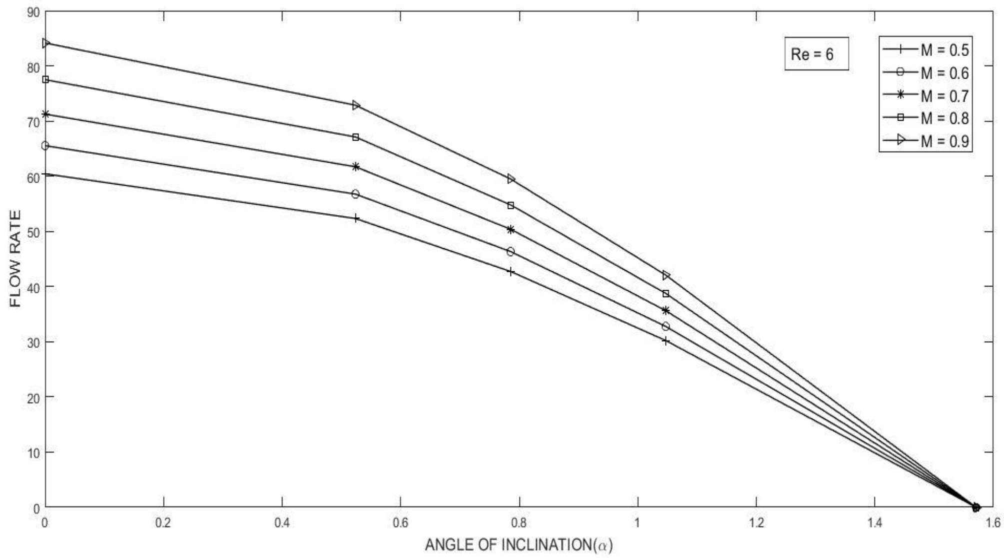


Fig-24: Influence of magnetic intensity on the flow rate

11. Figure 25 and figure 26 depicts the influence of the Reynolds number on the flow rate. As the Reynolds number increases, the flow rate continues to increase. Further the influence of the magnetic field on the flow rate is proved to be perfectly linear. As 'M' increases, the flow rate also increases.

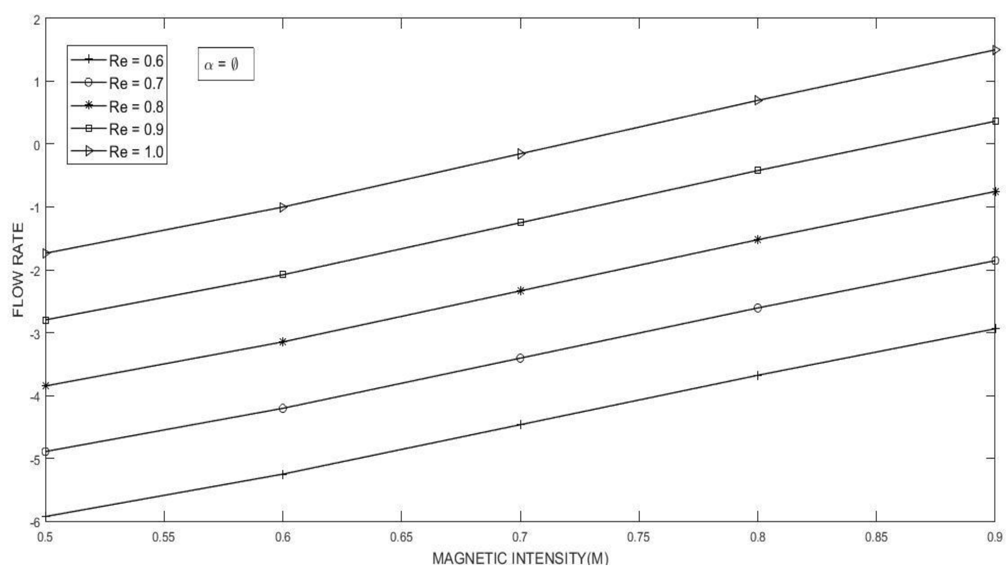


Fig-25: Contribution of Reynolds number on the flow rate

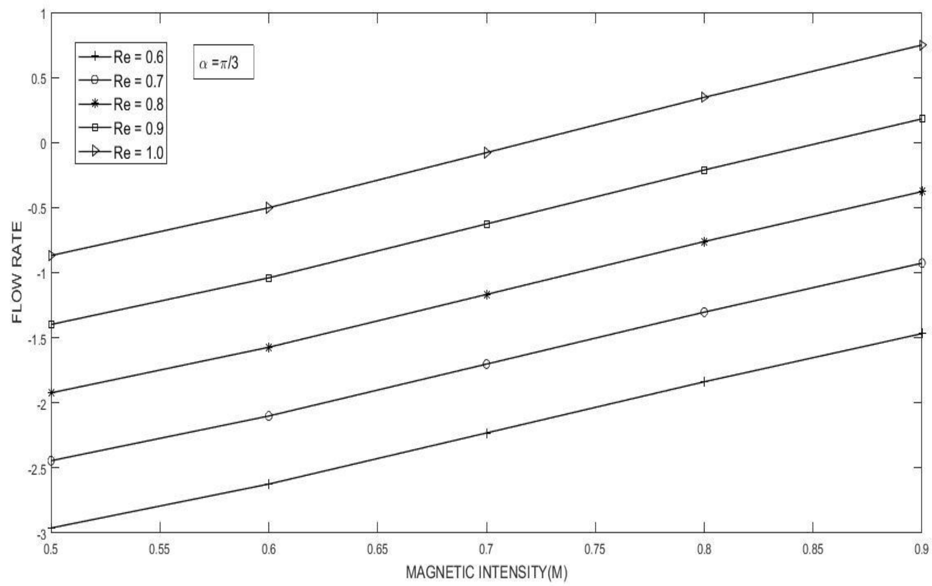


Fig-26: Effect of Reynolds number on the flow rate

Conflict of Interests

The authors declare that there is no conflict of interests.

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