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ALGORITHM TO COMPUTE TRANSITIVE CLOSURE OF FUZZY SOFT RELATION

ANJU S. MATTAM*, SASI GOPALAN

Department of Mathematics, Cochin University of Science and Technology,Kerala, India

Abstract. Fuzzy soft set is a mapping from a parameter set to the collection of fuzzy subset of universal set. In this paper fuzzy soft relation is presented based on the cartesian product of fuzzy soft sets and the notion of fuzzy soft equivalence relation is introduced. The basic properties of these relations are studied to explain the concept of transitive closure of a fuzzy soft relation. Existence and uniqueness of transitive closure of a fuzzy soft relation is established and an algorithm to compute the transitive closure of a fuzzy soft relation is also given.

Keywords: fuzzy soft set; fuzzy soft relation; composition of Fuzzy soft relation; fuzzy soft transitve relation.

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1. Introduction

Theory of fuzzy sets and fuzzy relation first developed by Zadeh [31] has been applied to many branches of mathematics . Fuzzy equivalence relation introduced by Zadeh as a generalization of the concept of an equivalence relation has been widely studied in [9],[3],[13],[22],[21], [8] as way to measure the degree of distinguishability or similarity between the objects of a given universe of discourse. And it have been shown to be useful in different context such as

*Corresponding author
E-mail address: anju.rajath@gmail.com
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fuzzy control [27], appropriate reasoning [32], fuzzy cluster analysis[6]. Depending on the authors and the context in which they appeared, it have received other names such as similarity relations, indistinguishability operators [5], many valued equivalence relations[18], etc. Later V. Murali[20]studied the cuts of fuzzy equivalence relation and lattice theoretic properties of fuzzy equivalence relation. In 1999 Molodtsov[19]proposed the novel concept of soft theory which provides a completely new approach for modelling vagueness and uncertainity.Theory of soft set has gained popularity among the researchers working in diverse areas. It is getting richer with new developments. Application of soft set theory can be seen in [24], [33],[26],[29],[10], [12],[15],[17]. Relations in soft set have been studied in[30],[19]. Structures of soft set have been studied by many authors[24],[25],[7].

Recently Ali et al.[2] have shown that a collection of soft set with reference to so called new operations give rise to many algebraic structures and form certain complete modular lattice structures. The theory of fuzzy soft set[14], fuzzification of the notion of soft set has the ability of hybridization. In this reagard fuzzy soft set and their applications has been investigated by many authors[16],[23],[4],[1],[28],[11]. Fuzzy soft set is the parametrized collection of fuzzy sets. Collection of fuzzy soft set form a complete modular lattice structures with respect to certain binary operations defined on them [34].

Based on these concept fuzzy soft relation is introduced and its theoretical properties are studied. Composition of Fuzzy soft relations provides both a general and flexible method for the designing of fuzzy logic controller and more generally for the modelling of any decision making process.

This paper is organized in the following manner. In section II some basic definitions related to fuzzy soft sets are given. These basic concepts are required in later section. In section III fuzzy soft relation and fuzzy soft equivalence relation is defined and different operations like union, intersection, composition that can be defined on the collection of fuzzy soft relation is introduced and its properties are studied. Fuzzy soft relation stores data in terms of relation between parameters which we define by membership function. Section IV is devoted to the study of transitive closure of fuzzy soft relation.The existence of the transitive closure is shown to be guaranteed for any fuzzy soft relation. In section V an algorithm to compute the transitive closure of a fuzzy soft relation is given and it is implemented usig an example.

2. Preliminaries

Through out this paper X refers to an initial universe, $\wp$ is a set of parameters in relation to objects in X. Parameters are often attributes, characteristics or properties of objects. I^X denote the set of all fuzzy subsets of X and P,Q $\subset \wp$.

2.1. Definition [14]. The pair (f,P) is called a fuzzy soft set over X if f is a mapping given by $f:P \rightarrow I^X$ i.e. f_p: X $\rightarrow$ [0,1]. Let (f,P) and (g,Q) be two fuzzy soft set over X. Then (f,P) is called fuzzy soft subset of (g,Q) denoted by (f,P) $\subseteq$(g,Q) if P $\subseteq$ Q and $f_p(x) \leq g_p(x)$, $\forall$ p $\in$ P.

2.2. Definition [14]. Union of two fuzzy soft sets (f,P) and (g,Q) over X is defined as the fuzzy soft set (h,C)=(f,P) $\cup$ (g,Q) where C=P$\cup$Q and for all c$\in$C

$$h_c(x) = \begin{cases} f_c(x) & \text{if } c \in P - Q \\ g_c(x) & \text{if } c \in Q - P \\ f_c(x) \vee g_c(x) & \text{if } c \in P \cap Q \end{cases}$$

2.3. Definition [14]. Intersection of two fuzzy soft sets (f,P) and (g,Q) over X is defined as the fuzzy soft set (h,C)=(f,P) $\cap$ (g,Q) where C=P$\cap$Q and for all c$\in$ C, $h_c(x)=f_c(x) \wedge g_c(x)$

2.4. Definition [14]. Let (f,P) and (g,Q) be two fuzzy soft sets over a universe X. Then cartesian product of (f,P) and (g,Q) is defined as (f,P) x (g,Q) = (h, P x Q) where h: P x Q$\rightarrow I^X$ and $h_{(p,q)}(x) = \min(f_p(x),g_q(x))$, $\forall$ (p,q) $\in$ P x Q .

2.5. Example. Consider the various investment avenues as x_1 - bank deposit, x_2 - Insurance, x_3 - postal savings, x_4 - shares and stocks, x_5 - mutual funds, x_6 - gold, x_7 - real estate as the universal state X, and factors influencing investment decision such as e_1 - safety of funds, e_2 - liquidity of funds, e_3 - high returns, e_4 - maximum profit in minimum time period, e_5 - stable returns, e_6 - easy accessiblity, e_7 - tax concession, e_8 - minimum risk of parameters.

Decision maker P is good at the parmeters e_1 and e_5. Decision maker Q is good at the parameters e_3 and e_4. This information can be expressed by two fuzzy soft sets (f,P) and (g,Q) respectively.

$$(f,P) = \left\{ \begin{array}{l} e_1 = \{\frac{x_1}{1}, \frac{x_2}{0.9}, \frac{x_3}{1}, \frac{x_4}{0.2}, \frac{x_5}{0.3}, \frac{x_6}{0.8}, \frac{x_7}{0.4}\} \\ \\ e_5 = \{\frac{x_1}{1}, \frac{x_2}{1}, \frac{x_3}{1}, \frac{x_4}{0.1}, \frac{x_5}{0.1}, \frac{x_6}{0.3}, \frac{x_7}{0.7}\} \end{array} \right\}$$

and

$$(g,Q) = \left\{ \begin{array}{l} e_3 = \{\frac{x_1}{0.5}, \frac{x_2}{0.5}, \frac{x_3}{0.5}, \frac{x_4}{0.7}, \frac{x_5}{0.6}, \frac{x_6}{0.8}, \frac{x_7}{0.9}\} \\ \\ e_4 = \{\frac{x_1}{0.4}, \frac{x_2}{0.2}, \frac{x_3}{0.4}, \frac{x_4}{0.8}, \frac{x_5}{0.6}, \frac{x_6}{0.8}, \frac{x_7}{0.9}\} \end{array} \right\}$$

A typical element of (h,P x Q) will look like

$$h(e_1,e_3) = \left\{ \frac{x_1}{0.5}, \frac{x_2}{0.5}, \frac{x_3}{0.5}, \frac{x_4}{0.2}, \frac{x_5}{0.3}, \frac{x_6}{0.8}, \frac{x_7}{0.4} \right\}.$$

3. Fuzzy Soft Relation

Fuzzy soft Relation is a suitable tool for describing correspondance between the parameters in a fuzzy soft set, which makes the theory of fuzzy soft set a hot subject for research.It plays an important role in modeling and decision making of systems.

3.1. Definition. Fuzzy Soft Relation R from (f,P) to (g,Q) is a fuzzy soft subset of (f,P) x (g,Q). If R is a fuzzy soft subset of (f,P) x (f,P) then it is called a Fuzzy Soft Relation on (f,P). If R is a Fuzzy Soft Relation on (f,P) then $R^{-1}_{pq} = R_{qp}$, $\forall$ (p,q) $\in$ PxP.

3.2. Definition. Let R_1 and R_2 be two Fuzzy Soft Relations from (f,P) to (g,Q) and (g,Q) to (h,S) respectively. Composition of R_1 and R_2 denoted by $R_1 \circ R_2$ is a Fuzzy Soft Relation from (f,P) to (h,S) defined as

$(R_1 \circ R_2)_{ps} = \bigvee_{q \in Q}((R_1)_{pq} \wedge (R_2)_{qs})$ where (p,q) $\in$ PxQ and (q,s) $\in$ QxS.

3.3. Example. Consider a fuzzy soft set (f,P) over X, Where X $=\{x_1, x_2, x_3, x_4\}$, P $=\{p, q\}$.Following example shows that composition of fuzzy soft relation R and S on (f,P) is not commutative.

TABLE 1. Fuzzy soft Relation R on (f,P).

	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$
$\mathbf{R}_{pp}$	0.76	0.5	0.82	0.64
$\mathbf{R}_{qp}$	0.58	0.075	0.58	0.56
$\mathbf{R}_{pq}$	0.6	0.7	0.65	0.8

TABLE 2. Fuzzy Soft Relation S on (f,P).

	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$
$\mathbf{S}_{pp}$	0.8	0.6	0.87	0.5
$\mathbf{S}_{pq}$	0.5	0.6	0.5	0.6
$\mathbf{S}_{qp}$	0.4	0.6	0.55	0.75

TABLE 3. Fuzzy soft Relation R∘S on (f,P) .

	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$
$\mathbf{R}\circ\mathbf{S}_{pp}$	0.76	0.6	0.82	0.75
$\mathbf{R}\circ\mathbf{S}_{qp}$	0.58	0.075	0.58	0.5
$\mathbf{R}\circ\mathbf{S}_{pq}$	0.55	0.5	0.5	0.6
$\mathbf{R}\circ\mathbf{S}_{qq}$	0.5	0.075	0.5	0.56

TABLE 4. Fuzzy soft Relation S∘R on (f,P).

	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$
$\mathbf{S}\circ\mathbf{R}_{pp}$	0.76	0.5	0.82	0.56
$\mathbf{S}\circ\mathbf{R}_{pq}$	0.6	0.6	0.65	0.5
$\mathbf{S}\circ\mathbf{R}_{qp}$	0.4	0.5	0.55	0.64
$\mathbf{S}\circ\mathbf{R}_{qq}$	0.4	0.6	0.55	0.75

3.4. Definition. If R_1 and R_2 be two fuzzy soft relations defined on the fuzzy soft set (f,P) then their Union and Intersections are defined as follows.

$(R_1 \cup R_2)_{pq}(x) = \max \{(R_1)_{pq}(x), (R_2)_{pq}(x)\}$

$(R_1 \cap R_2)_{pq}(x) = \min \{(R_1)_{pq}(x), (R_2)_{pq}(x)\}$

$\forall$ x $\in$ X and (p,q) $\in$ P x P.

3.5. Theorem. Let Q, R, S be Fuzzy Soft Relation on (f,P) then

1) $(R^{-1})^{-1} = R$

2) $(R^c)^{-1} = (R^{-1})^c$

3) $R \subseteq S \implies R^{-1} \subseteq S^{-1}$

4) $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$

5)$R \subseteq S \implies R \circ Q \subseteq S \circ Q$

6)$(Q \circ R) \circ S = Q \circ (R \circ S)$

7)$(Q \cup R)^{-1} = Q^{-1} \cup R^{-1}$ and $(Q \cap R)^{-1} = Q^{-1} \cap R^{-1}$

Proof

All the above properties obviously holds using definition 3.1,3.2 and 3.5.

3.6. Corollary. If R and Q be two fuzzy soft relations on (f,P) such that $R \subseteq Q$ then $R^n \subseteq Q^n$.

A new definition of fuzzy soft equivalence relation on a fuzzy soft set (f,P)is proposed which will partition the parameter set into equivalence classes

3.7. Definition. Let (f,P) be a fuzzy soft set over the universal set X and R be a fuzzy soft relation on (f,P) then R is said to be

1) Fuzzy soft reflexive if $\forall$ p,q $\in$ P with p $\neq$ q and $\forall$ x $\in$ X

$R_{pq}(x) \leq R_{pp}(x)$ and $R_{qp} \leq R_{pp}(x)$

2) Fuzzy soft symmetric relation if $R = R^{-1}$

3) Fuzzy soft transitive relation if $R \circ R \subseteq R$

4) Fuzzy soft equivalence relation if it is fuzzy soft reflexive, fuzzy soft symmetric and fuzzy soft transitive.

3.8. Theorem. If R is a fuzzy soft equivalence relation so is R^{-1} and $R \circ R$.

3.9. Theorem. Let (f,P) be a fuzzy soft set over the universal set X then cartesian product R = (f,P)X(f,P) is a fuzzy soft equivalence relation.

Proof

$R_{pq}(x) = \min(f_p(x), f_q(x)) \leq f_p(x)$

$\Longrightarrow R_{pq}(x) \leq R_{pp}(x), \forall$ p,q $\in$ P and $\forall$ x $\in$ X, hence R is a fuzzy soft reflexive relation.

$R_{pq}(x) = \min(f_p(x), f_q(x)) = \min(f_q(x), f_p(x)) = R_{qp}(x) = R^{-1}_{pq}(x) \Longrightarrow$ R is a fuzzy soft symmetric relation.

$(R \circ R)_{pq}(x) = \underset{r \in P}{\forall} \min(R_{pr}(x), R_{rq}(x))$

$= \underset{r \in P}{\forall} \min(f_p(x) \wedge f_r(x), f_r(x) \wedge f_q(x))$

$= \underset{r \in P}{\forall} \min(f_p(x), f_r(x), f_q(x))$

$= \min(f_p(x), \underset{r \in P}{\forall} f_r(x), f_q(x))$

$\leq \min(f_p(x), f_q(x)) = R_{pq}(x)$

$\Longrightarrow$ R is a fuzzy soft transitive relation.

Hence R is a fuzzy soft equivalence relation.

The key point in the work of fuzzy soft equivalence relation is the definition of fuzzy soft transitivity, which is based on composition of fuzzy soft relations. At a glance it is easy to verify whether a given fuzzy soft relation R on (f,P) is fuzzy soft reflexive and fuzzy soft symmetric. Fuzzy soft subset of cartesian product of fuzzy soft sets can be fuzzy soft reflexive and fuzzy soft symmetric but not necessarily a fuzzy soft transitive relation. In the next section we give a method to obtain a fuzzy soft transitive relation close to R , so that we can replace R with the new relation when transitivity is required.To do this an algorithm to compute the transitive closure of a fuzzy soft relation is also given.

4. Transitive closure of a fuzzy soft relation

4.1. Theorem. Fuzzy soft relation R on (f,P) is fuzzy soft transitive if and only if $R^n \subseteq R, \forall$ n $\in$ N where $R^n = R \circ R \circ$.....n times, n$\geq$2

Proof

Suppose that $R^n \subseteq R \ \forall$ n $\in$ N. In particular for n=2, $R^2 \subseteq R \Longrightarrow R \circ R \subseteq R$. Hence R is a fuzzy soft transitive relation.Conversly suppose that R is a fuzzy soft transitive relation.

ie. R $\circ$ R $\subseteq$ R.

$R^n \subseteq$ R can be proved using mathematical induction. By our assumption result is true when n=2. Suppose that $R^n \subseteq$ R, for n $\in$ N. R^{n+1}= $R^n \circ$ R$\subseteq$ R $\circ$ R $\subseteq$ R. Hence the result.

4.2. Definition. Transitive closure of a fuzzy soft relation R on (f,P) denoted by $\hat{R}$ is the smallest fuzzy soft relation containing R.

Next theorem shows that any fuzzy soft relation on a fuzzy soft set(f,P) has a Transitive closure and it is unique.

4.3. Theorem. Consider an arbitrary fuzzy soft set (f,P) then any fuzzy soft relation R on (f,P) has a unique transitive closure.

Proof

Let Ω be the collection of all fuzzy soft transitive relations T on (f,P) such that R$\subseteq$T. By theorem 3.9 the above collection is non empty.

Define $T^*{}_{pq}(x) = \inf_{T\in\Omega} T_{pq}(x)$. Now we have to prove that T^* is a fuzzy soft transitive relation containing R. Since R$\subseteq$T, $\forall$ T$\in \Omega$, T^* is a fuzzy soft relation containing R.

$T^* \subseteq T$, $\forall$ T$\in \Omega$ implies $T^* \circ T^* \subseteq T \circ T$, $\forall$ T$\in \Omega$

$\Longrightarrow T^* \circ T^* \subseteq T$, $\forall$ T$\in \Omega$

$\Longrightarrow (T^* \circ T^*) \subseteq \inf_{T\in\Omega} T)$

$\Longrightarrow T^* \circ T^* \subseteq T^*$

Hence T^* is a fuzzy soft transitive relation. If T_1 and T_2 are two transitive closures of fuzzy soft relation R on(f,P) then according to definition 4.2, $T_1 \subseteq T_2$ and $T_2 \subseteq T_1$, consequently $T_1 = T_2$.

4.4. Theorem. If R is a fuzzy soft relation on (f,P) then $\hat{R}$ = R^1 U R^2 U....R^n, n$\in \mathbb{N}$ is such that R^n=R^{n+1}

Proof

Since $\hat{R}$ is a fuzzy soft relation containing R it is enough we show that $\hat{R}$ is fuzzy soft transitive.($\hat{R}$ $\circ$ $\hat{R})_{pq}$(x)= $\bigvee_{r\in R}((\hat{R})_{pr}(x) \wedge (\hat{R})_{rq}(x))$ = $\bigvee_{r\in P}((R^i)_{pr}(x) \wedge (R^j)_{rq}(x))$= $(R^i \circ R^j)_{pq}(x)$= $(R^{i+j})_{pq}(x)$

$\leq(\hat{R})_{pq}$(x).Let Q be a fuzzy soft transitive relation containing R then by theorem 4.1 and corollary 3.6, $R^n \subseteq Q, \forall$ n $\in$ N $\implies \hat{R} = R^1 \cup R^2 \cup ....R^n \subseteq$ Q.Hence $\hat{R}$ is the smallest fuzzy soft transitive relation containig R.

4.5. Theorem. Let R and Q be two fuzzy soft relations on(f,P) and

1) if R$\subseteq$ Q then $\hat{R} \subseteq \hat{Q}$

2) if R and Q are fuzzy soft transitive relations such that R $\circ$ Q= Q $\circ$ R

then $(R \hat{\circ} Q)$ = R$\circ$ Q

Proof

Obviously 1 holds, we need only prove 2

Note that R$\circ$ Q $\subseteq (R \hat{\circ} Q)$ (1)

(R$\circ$ Q) $\circ$ (R$\circ$ Q)= R$\circ$(Q $\circ$ R)$\circ$ Q =R$\circ$(R $\circ$ Q)$\circ$ Q

Since R and Q are fuzzy soft transitive relation on (f,P), (R$\circ$ Q) $\circ$ (R$\circ$ Q) $\subseteq$ R$\circ$ Q $\implies$ R$\circ$ Q is a fuzzy soft transitive relation.

By theorem 4.1 $(R\circ Q)^n \subseteq R\circ Q, \forall n \in N \implies (R \hat{\circ} Q) \subseteq (R \circ Q)$ (2)

From (1) and (2) we conclude that $(R \hat{\circ} Q)$ = (R $\circ$ Q)

5. Algorithm to compute the transitive closure of fuzzy soft relation

Let R be a fuzzy soft relation on fuzzy soft set (f,P).We can consider R as a three dimensional matrix of size nxnxm, where n = number of elements of P and m= number of elements of X. The element R_{ijk} denote the fuzzy soft relation between ith and jth parameter of P corresponding to the kth element in X, ie.$0 \leq R_{ijk} \leq 1$

Transitive Algo (a=R,b=R,n,m)

Let $\hat{R}$ be a three dimensional matrix initialize to R

```
for k= 1 to m
  {
  for i=(1 to n)
    {
    for j=(1 to n)
      {
```

```
            c[i][j][k]= max[min(a_iyk,b_yik),∀ y=1 to n]
            }
         }
     }
for i=(1 to n)
{
    for j=(1 to n)
        {
        for k= 1 to m
            c[i][j][k] ≠ a[i][j][k]
            {
                R̂[i][j][k] = max[c[i][j][k],R̂[i][j][k]]
                Transitive Algo (c,b,n,m)
            }
        }
    }
Print R̂[i][j][k]
```

The algorithm to compute the transitive closure of fuzzy soft relation can be explained using the following example.

Let (f,P) be a fuzzy soft set over universal set x=$\{x_1, x_2, x_3, x_4\}$ and parameter set P =$\{p, q, r\}$. Consider fuzzy soft relations R on (f,P). Using the above algoritm we can compute $R^2 = R\circ R$,$R^3 = R^2\circ R$, $R^4 = R^3\circ R$ as follows.

TABLE 5. Fuzzy Soft Relation R On (f,P).

R	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$
$\mathbf{R}_{pp}$	0.76	0.5	0.82	0.64
$\mathbf{R}_{pq}$	0.58	0.075	0.6	0.56
$\mathbf{R}_{rp}$	0.2	0.3	0.67	0.5
$\mathbf{R}_{qr}$	0.6	0.7	0.65	0.8

TABLE 6. Fuzzy Soft Relation $R^2 = R \circ R$ On (f,P).

$\mathbf{R}^2$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$
$\mathbf{R}^2{}_{pp}$	0.76	0.5	0.82	0.64
$\mathbf{R}^2{}_{pq}$	0.58	0.075	0.6	0.56
$\mathbf{R}^2{}_{rp}$	0.2	0.3	0.67	0.5
$\mathbf{R}^2{}_{pr}$	0.58	0.075	0.6	0.56
$\mathbf{R}^2{}_{qp}$	0.2	0.3	0.65	0.5
$\mathbf{R}^2{}_{rq}$	0.2	0.075	0.6	0.5

TABLE 7. Fuzzy Soft Relation $R^3 = R^2 \circ R$ on (f,P).

$\mathbf{R}^3$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$
$\mathbf{R}^3{}_{pp}$	0.76	0.5	0.82	0.64
$\mathbf{R}^3{}_{pq}$	0.58	0.075	0.6	0.56
$\mathbf{R}^3{}_{pr}$	0.58	0.075	0.6	0.56
$\mathbf{R}^3{}_{qp}$	0.2	0.3	0.65	0.5
$\mathbf{R}^3{}_{qq}$	0.2	0.075	0.6	0.5
$\mathbf{R}^3{}_{rp}$	0.2	0.3	0.67	0.5
$\mathbf{R}^3{}_{rq}$	0.2	0.075	0.6	0.5
$\mathbf{R}^3{}_{rr}$	0.2	0.075	0.6	0.5

TABLE 8. Fuzzy Soft Relation $R^4 = R^3 \circ R$ On (f,P).

$\mathbf{R}^4$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$
$\mathbf{R}^4{}_{pp}$	0.76	0.5	0.82	0.64
$\mathbf{R}^4{}_{pq}$	0.58	0.075	0.6	0.56
$\mathbf{R}^4{}_{pr}$	0.58	0.075	0.6	0.56
$\mathbf{R}^4{}_{qp}$	0.2	0.3	0.65	0.5
$\mathbf{R}^4{}_{qq}$	0.2	0.075	0.6	0.5
$\mathbf{R}^4{}_{qr}$	0.2	0.075	0.6	0.5
$\mathbf{R}^4{}_{rp}$	0.2	0.3	0.67	0.5
$\mathbf{R}^4{}_{rq}$	0.2	0.075	0.6	0.5
$\mathbf{R}^4{}_{rr}$	0.2	0.075	0.6	0.5

TABLE 9. Fuzzy Soft Relation $R^5 = R^4 \circ R$ On (f,P).

$\mathbf{R}^5$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$
$\mathbf{R}^5{}_{pp}$	0.76	0.5	0.82	0.64
$\mathbf{R}^5{}_{pq}$	0.58	0.075	0.6	0.56
$\mathbf{R}^5{}_{pr}$	0.58	0.075	0.6	0.56
$\mathbf{R}^5{}_{qp}$	0.2	0.3	0.65	0.5
$\mathbf{R}^5{}_{qq}$	0.2	0.075	0.6	0.5
$\mathbf{R}^5{}_{qr}$	0.2	0.075	0.6	0.5
$\mathbf{R}^5{}_{rp}$	0.2	0.3	0.67	0.5
$\mathbf{R}^5{}_{rq}$	0.2	0.075	0.6	0.5
$\mathbf{R}^5{}_{rr}$	0.2	0.075	0.6	0.5

Since $R^5 = R^4$ algorithm stops computing the composition of fuzzy soft relation R on (f,P) and finally we get the transitive closure of R as $\hat{R} = R \cup R^2 \cup R^3 \cup R^4$

6. CONCLUSION

In this paper basic properties of transitive closure of a fuzzy soft relation is studied. We have established a new minimum durational algorithm for computing the transitive closure of a fuzzy soft relation. In real life situation using this algorithm one can get more affirmative solution in decision making problems. As a future work we will extend the algorithm to compute the minimum fuzzy soft equivalence relation containing the given relation.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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