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A STUDY ON FRACTIONAL DIFFERINTEGRATIONS IN ASSOCIATION WITH I - FUNCTION

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Abstract: In the present paper, the author has proved three theorems on the fractional differintegrations of I - Function in association with different functions of one variable. Corollary and some examples are also given.

Key words: Goursat's theorem; I -function; analytic function; fractional derivative.

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1. INTRODUCTION

Definitions of the fractional derivatives and integral of the function of single variable:

(i) Goursat's theorem (Cauchy's theorem) for the function of single variable is:

$$f^{(n)}(z)=\frac{n!}{2\pi i}\int_{\gamma}\frac{f(\zeta)}{(\zeta-z)^{n+1}}d\zeta \qquad (n\in N\cup\{0\}, z\in D) \tag{1.1}$$

where $f(z)$ is analytic in a domain D, which is surrounded with a piecewise smooth closed Jorden curve γ, in the ζ -plane.

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(ii) (Derivative). If $f(z)$ is an analytic (regular) function and it has no branch point inside $C(=\{\underset{-}{C},\underset{+}{C}\})$ and on C, and

$$_{\underset{-}{C}}f_\nu = {}_{\underset{-}{C}}f_\nu(z) = \frac{\Gamma(\nu+1)}{2\pi i}\int_{\underset{-}{C}} \frac{f(\zeta)}{(\zeta-z)^{\nu+1}}d\zeta \tag{1.2}$$

$$= \frac{\Gamma(\nu+1)}{2\pi i}\int_{-\infty}^{(0+)} \eta^{-(\nu+1)} f(z+\eta)d\eta, \quad (\zeta - z = \eta) \tag{1.3}$$

$$(\zeta \neq z, -\pi \leq \arg(\zeta - z) \leq \pi, \nu \notin Z^-)$$

$$_{\underset{+}{C}}f_\nu = {}_{\underset{+}{C}}f_\nu(z) = \frac{\Gamma(\nu+1)}{2\pi i}\int_{\underset{+}{C}} \frac{f(\zeta)}{(\zeta-z)^{\nu+1}}d\zeta \tag{1.4}$$

$$= \frac{\Gamma(\nu+1)}{2\pi i}\int_{\infty}^{(0+)} \eta^{-(\nu+1)} f(z+\eta)d\eta, \quad (\zeta - z = \eta) \tag{1.5}$$

$$(\zeta \neq z, -\pi \leq \arg(\zeta - z) \leq \pi, \nu \notin Z^-)$$

$$f_{-n} = {}_C f_{-n} = \lim_{\nu \to -n} {}_C f_\nu \quad (n \in Z^+, C = \{\underset{-}{C},\underset{+}{C}\}), \tag{1.6}$$

Where $\underset{-}{C}$ and $\underset{+}{C}$ are integral curves as shown in Fig. 1 and Fig. 2 (that is $\underset{-}{C}$ is a curve along the cut joining two points z and $-\infty + i\lim(z)$, and $\underset{+}{C}$ is a curve along the cut joining two points z and $\infty + i\lim(z)$, then $f_\nu = {}_C f_\nu(z) = \{{}_{\underset{-}{C}} f_\nu(z), {}_{\underset{+}{C}} f_\nu(z)\}(\nu > 0)$ is the fractional derivative of order ν of the function $f(z)$, if f_ν exists.

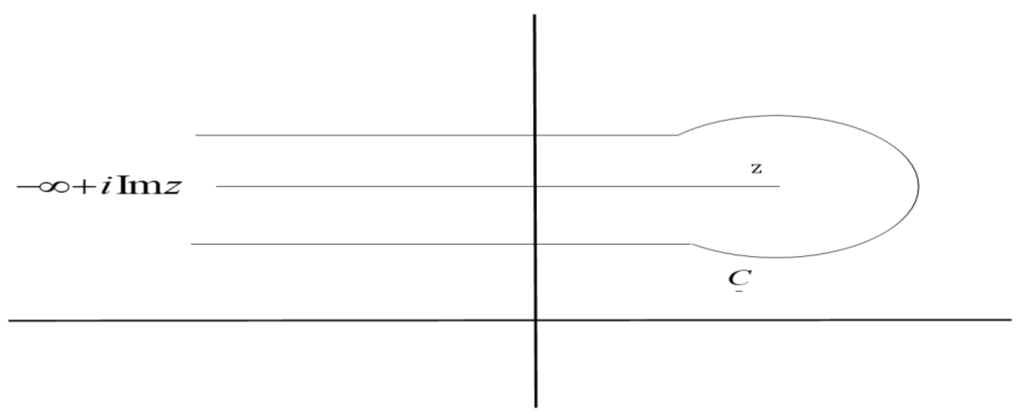


Fig. 1

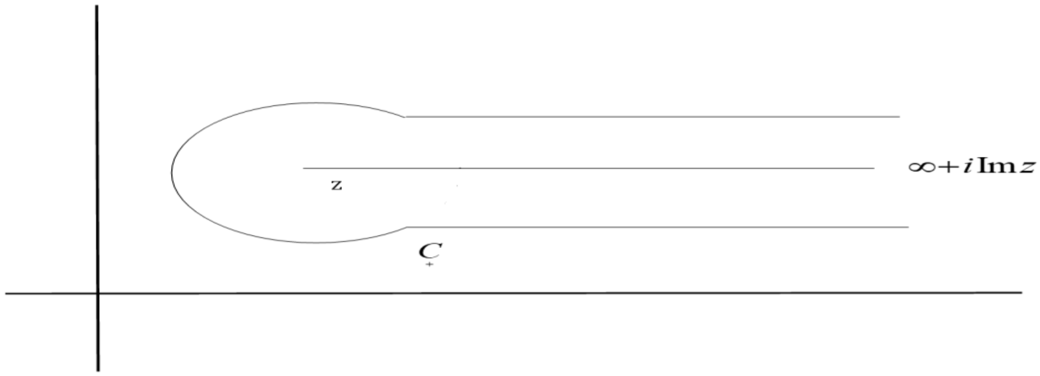


Fig. 2

Definition 2 (Integral). $f_\nu(\nu<0)$ is the fractional integral of order $|\nu|$. That is, the derivative of fractional order $-\nu(\nu>0)$ is the fractional integral of order $\nu(\nu\in R)$, if f_ν exists.

Formal unification of derivative and integral of the function of single variable:

If $f(z)$ is the analytic function and it has no branch point inside C and on $C(C=\{C_{-},C_{+}\})$, and

$$f_\nu = {}_C f_\nu(z) = \{{}_{C_-} f_\nu(z), {}_{C_+} f_\nu(z)\} \tag{1.7}$$

Then

$$f_\nu \text{ is } \begin{cases} \textit{derivative} & \textit{for } \nu>0 \\ \textit{original} & \textit{for } \nu=0 \\ \textit{integral} & \textit{for } \nu<0 \end{cases} \tag{1.8}$$

for $\nu\in R$, and

$$f_\nu \text{ is } \begin{cases} \textit{derivative} & \textit{for } \mathrm{Re}(\nu)>0 \\ \textit{original} & \textit{for } \nu=0 \\ \textit{integral} & \textit{for } \mathrm{Re}(\nu)<0 \end{cases} \tag{1.9}$$

for $\nu\in C$, if f_ν exists.

And in case of $\mathrm{Re}(\nu)=0$, f_ν is only formal differintegration regardless of $\mathrm{Im}(\nu)\ge 0$ or $\mathrm{Im}(\nu)\le 0$.

That is , we have no derivative and integral for $\nu=$pure imaginary.

Following results will be used:

(i) ([3];p.16, eq.(1))

$$\left(e^{-az}\right)_\nu = e^{-i\pi\nu}a^\nu e^{-az} \quad \text{for } a\neq 0(z,\nu\in C) \tag{1.10}$$

(ii) ([3];p.18, eq.(6))

$$\left(e^{az}\right)_{\nu} = a^{\nu} e^{-az} \quad \text{for } a \neq 0 (z, \nu \in C) \tag{1.11}$$

(iii) ([3];p.19, eq.(11))

$$\left(a^{z}\right)_{\nu} = (\log a)^{\nu} a^{z} \quad \text{for } a \neq 0 (z, \nu \in C) \tag{1.12}$$

(iv) ([3];p.20, eq.(1))

$$\left(\cosh az\right)_{\nu} = (-ia)^{\nu} \cosh(az + i\frac{\pi}{2}\nu) \quad \text{for } a \neq 0 (z, \nu \in C) \tag{1.13}$$

(v) ([3];p.20, eq.(2))

$$\left(\sinh az\right)_{\nu} = (-ia)^{\nu} \sinh(az + i\frac{\pi}{2}\nu) \quad \text{for } a \neq 0 (z, \nu \in C) \tag{1.14}$$

(vi) ([3];p.21, eq.(1))

$$\left(\cos az\right)_{\nu} = (a)^{\nu} \cos(az + \frac{\pi}{2}\nu) \text{ for } a \neq 0 (z, \nu \in C) \tag{1.15}$$

(vii) ([3];p.22, eq.(2))

$$\left(\sin az\right)_{\nu} = (a)^{\nu} \sin(az + \frac{\pi}{2}\nu) \text{ for } a \neq 0 (z, \nu \in C) \tag{1.16}$$

(viii) ([3];p.32, eq.(1))

$$(\log az)_{\nu} = -e^{-i\pi\nu} \Gamma(\nu)\, z^{-\nu} \text{ for } a \neq 0 (z, \nu \in C) \tag{1.17}$$

The I-function given by Saxena [4] will be represented and defined in the following manner:

$$I[Z] = I^{m,n}_{p_i, q_i : r}[Z] = I^{m,n}_{p_i, q_i : r}\left[z \left| \begin{matrix} (a_j, \alpha_j)_{1,n}, (a_{ji}, \alpha_{ji})_{n+1, p_i} \\ (b_j, \beta_j)_{1,m}, (b_{ji}, \beta_{ji})_{m+1, q_i} \end{matrix} \right. \right] = \frac{1}{2\pi\omega} \int_L \chi(\xi) d\xi \tag{1.18}$$

where $\omega = \sqrt{-1}$

$$\theta(\xi) = \frac{\prod_{j=1}^{m} \Gamma(b_j - \beta_j \xi) \prod_{j=1}^{n} \Gamma(1 - a_j + \alpha_j \xi)}{\sum_{i=1}^{r} \left\{ \prod_{j=m+1}^{q_i} \Gamma(1 - b_{ji} - \beta_{ji}\xi) \prod_{j=n+1}^{p_i} \Gamma(a_{ji}, \alpha_{ji}\xi) \right\}} \tag{1.19}$$

$p_{i,} q_i (i = 1, \ldots, r), m, n$ are integers satisfying $0 \leq n \leq p_i$, $0 \leq m \leq q_i, (i = 1, \ldots, r), r$ is finite $\alpha_j, \beta_j, \alpha_{ij}, \beta_{ji}$ are real and $a_{j,} b_j, a_{ji}, b_{ji}$ are complex numbers such that $\alpha_j (b_h + \nu) \neq \beta_h (a_j - \nu - k)$ for $\nu, k = 01, 2, \ldots$

2. MAIN RESULTS

Theorem 1.

$\left(I(e^{-kz})\right)_\nu = e^{-i\pi\nu}(kz^\nu)^{-1} I(e^{-kz})$ for $k \neq 0 (z, \nu \in C)$

Proof: In case of $|\arg k| < \frac{\pi}{2}$

$$\left(I(e^{-kz})\right)_\nu = \underset{+}{C}\left(I(e^{-kz})\right)_\nu = \frac{\Gamma(\nu+1)}{2\pi i}\int_{C_+} \frac{I(e^{-k\zeta})}{(\zeta - z)^{\nu+1}} d\zeta$$

$$= \frac{\Gamma(\nu+1)}{2\pi i}\int_{C_+} \frac{\left\{\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j - \beta_j s)\prod_{j=1}^{n}\Gamma(1 - a_j + \alpha_j s)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1 - b_{ji} - \beta_{ji} s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}, \alpha_{ji} s)\right\}}(e^{-k\zeta})^s ds\right\}}{(\zeta - z)^{\nu+1}} d\zeta$$

$$= \frac{1}{2\pi i}\int_L \theta(s) ds \left\{\frac{\Gamma(\nu+1)}{2\pi i}\int_{C_+}\frac{e^{-ks\zeta}}{(\zeta - z)^{\nu+1}} d\zeta\right\}$$

$$= \frac{1}{2\pi i}\int_L \theta(s) e^{-i\pi\nu}(ks)^\nu e^{-ksz} ds = e^{-i\pi\nu}(kz^\nu)^{-1} I(e^{-kz})$$

Case II. $\frac{\pi}{2} < |\arg z| < \pi$, we have

$$\left(I(e^{-kz})\right)_\nu = \underset{-}{C}\left(I(e^{-kz})\right)_\nu = \frac{\Gamma(\nu+1)}{2\pi i}\int_{C_-} \frac{I(e^{-k\zeta})}{(\zeta - z)^{\nu+1}} d\zeta$$

$$= \frac{\Gamma(\nu+1)}{2\pi i}\int_{C_-} \frac{\left\{\frac{1}{2\pi i}\frac{\prod_{j=1}^{m}\Gamma(b_j - \beta_j s)\prod_{j=1}^{n}\Gamma(1 - a_j + \alpha_j s)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1 - b_{ji} - \beta_{ji} s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}, \alpha_{ji} s)\right\}}\int_L (e^{-k\zeta})^s ds\right\}}{(\zeta - z)^{\nu+1}} d\zeta$$

$$= \frac{1}{2\pi i}\int_L \theta(s) ds \left\{\frac{\Gamma(\nu+1)}{2\pi i}\int_{C_-}\frac{e^{-ks\zeta}}{(\zeta - z)^{\nu+1}} d\zeta\right\} = e^{-i\pi\nu}(kz^\nu)^{-1} I(e^{-kz})$$

Case III. $|\arg z| = \frac{\pi}{2}$

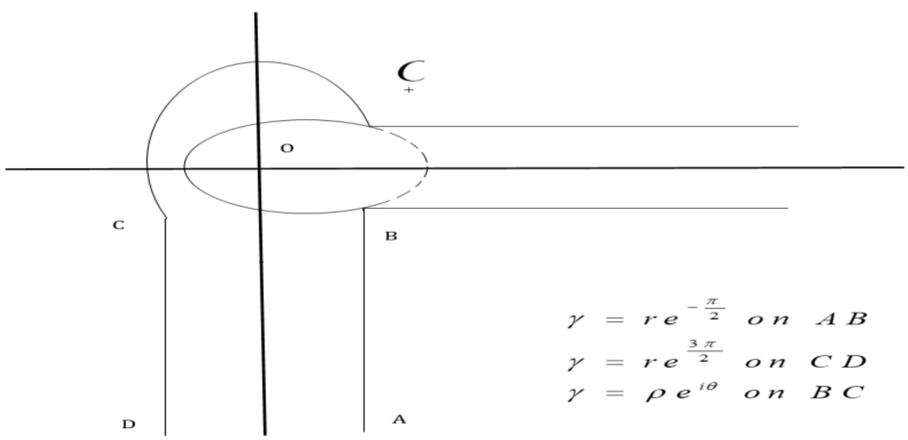
$$\left(I(e^{-kz})\right)_\nu = \underset{+}{C}\left(I(e^{-kz})\right)_\nu = \frac{\Gamma(\nu+1)}{2\pi i}\int_{\underset{+}{C}} \frac{I(e^{-k\zeta})}{(\zeta - z)^{\nu+1}}\, d\zeta$$

$$= \frac{\Gamma(\nu+1)}{2\pi i}\int_{\underset{+}{C}} \frac{\left\{ \frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j - \beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}-\beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji},\alpha_{ji}s)\right\}}(e^{-k\zeta})^s ds \right\}}{(\zeta - z)^{\nu+1}} d\zeta$$

$$= \frac{1}{2\pi i}\int_L \theta(s)ds\left\{\frac{\Gamma(\nu+1)}{2\pi i}\int_{\underset{+}{C}} \frac{e^{-ks\zeta}}{(\zeta - z)^{\nu+1}}\, d\zeta\right\}$$

(put $\zeta - z = \eta, ks\eta = \xi, 0 \le |\arg\eta| \le 2\pi$)

$$= \frac{1}{2\pi i}\int_L \theta(s)ds.(ks)^\nu e^{-ksz}\,\frac{\Gamma(\nu+1)}{2\pi i}\int_{\infty e^{-i\frac{\pi}{2}}}^{(0+)} \xi^{-(\nu+1)}e^{-\xi}d\xi,\ (\phi = \arg k = -\frac{\pi}{2}) \quad (2.1)$$



and

$$\int_{\infty e^{-i\frac{\pi}{2}}}^{(0+)} \xi^{-(\nu+1)}e^{-\xi}d\xi = \left(\int_{AB} + \int_{CD} + \int_{BC}\right)\xi^{-(\nu+1)}e^{-\xi}d\xi$$

$$= \int_\infty^0 \left(re^{-i\frac{\pi}{2}}\right)^{-(\nu+1)} e^{-re^{-i\frac{\pi}{2}}} e^{-i\frac{\pi}{2}}dr + \int_0^\infty \left(re^{-i\frac{3\pi}{2}}\right)^{-(\nu+1)} e^{-re^{-i\frac{3\pi}{2}}} e^{-i\frac{3\pi}{2}}dr$$

$$+\lim_{\rho\to 0}\int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} \left(\rho e^{i\theta}\right)^{-(\nu+1)} e^{-(\rho e^{i\theta})}\rho i e^{i\theta}d\theta$$

$$=-2ie^{-i\frac{\pi}{2}\nu}\sin\pi\nu\Gamma(-\nu)e^{-i\frac{\pi}{2}\nu}=-2\pi ie^{-i\pi\nu}\frac{\sin\pi\nu}{\pi}\Gamma(-\nu)=\frac{2\pi ie^{-i\pi\nu}}{\Gamma(\nu+1)}$$

From (2.1), we get

$$=e^{-i\pi\nu}(kz^{\nu})^{-1}I(e^{-kz}).$$

Theorem 2.

$$\left(I(e^{kz})\right)_{\nu}=\Gamma(\nu+1)(kz^{\nu})^{-1}I(e^{kz})\text{ for }k\neq 0(z,\nu\in C)$$

Proof: In case of $|\arg k|<\frac{\pi}{2}$

$$\left(I(e^{kz})\right)_{\nu}=\underset{-}{C}\left(I(e^{kz})\right)_{\nu}=\frac{\Gamma(\nu+1)}{2\pi i}\int_{\underset{-}{C}}\frac{I(e^{k\zeta})}{(\zeta-z)^{\nu+1}}d\zeta$$

$$=\frac{1}{2\pi i}\int_{L}\theta(s)ds\left\{\frac{\Gamma(\nu+1)}{2\pi i}\int_{\underset{+}{C}}\frac{e^{ks\zeta}}{(\zeta-z)^{\nu+1}}d\zeta\right\}$$

(put $\zeta-z=\eta, ks\eta=\xi, 0\leq|\arg\eta|\leq 2\pi$)

$$=\frac{1}{2\pi i}\int_{L}\theta(s)ds.(ks)^{\nu}e^{ksz}\frac{\Gamma(\nu+1)}{2\pi i}\int_{-\infty e^{-i\frac{\pi}{2}}}^{(0+)}\xi^{-(\nu+1)}e^{-\xi}d\xi,\ (\phi=\arg k)$$

$$=\frac{1}{2\pi i}\int_{L}\theta(s)ds.(ks)^{\nu}e^{ksz}\frac{\Gamma(\nu+1)}{2\pi i}\int_{-\infty}^{(0+)}\xi^{-(\nu+1)}e^{-\xi}d\xi,\ (|\phi|<\frac{\pi}{2})$$

For $|\arg k|<\frac{\pi}{2}$, $\int_{-\infty}^{(0+)}\xi^{-(\nu+1)}e^{-\xi}d\xi=\frac{2\pi i}{\Gamma(\nu+1)}$

We arrive at the required result.

In case of $\frac{\pi}{2}\leq|\arg k|\leq\pi$, we have

$$\left(I(e^{kz})\right)_{\nu}=\underset{+}{C}\left(I(e^{kz})\right)_{\nu}$$

By using similar lines, we can prove the result easily.

Corollary:

$$\left(I(k^{z})\right)_{\nu}=\Gamma(\nu+1)(\log kz^{\nu})^{-1}I(k^{z})\text{ for }k\neq 0(z,\nu\in C)\,()$$

Proof: We can write as

$$\left(I(k^z)\right)_\nu = \left(I(e^{z\log k})\right)_\nu$$

Some Examples:

(i) $I(e^{-5z})_{\frac{1}{2}} = e^{-i\frac{\pi}{2}}(5z^{\frac{1}{2}})^{-1}I(e^{-5z}) = -\frac{i}{5\sqrt{z}}I(e^{-5z})$

(ii) $I(e^{-5z})_{-\frac{1}{2}} = -\frac{i\sqrt{z}}{5}I(e^{-5z})$

(iii) $I(e^{3z})_{\frac{1}{2}} = \frac{\sqrt{\pi}}{6\sqrt{z}}I(e^{3z})$

(iv) $I(e^{-3z})_{-\frac{1}{2}} = \frac{\sqrt{\pi z}}{3}I(e^{-3z})$

(v) $I(k^z)_{\frac{1}{2}} = \frac{\sqrt{\pi}}{2\log 3\sqrt{z}}I(k^z) = \frac{\sqrt{\pi}}{\log 9z}I(k^z)$

(vi) $I(k^z)_{-\frac{1}{2}} = \frac{\sqrt{\pi}}{2\log(3/\sqrt{z})}I(k^z) = \frac{\sqrt{\pi}}{\log(\frac{9}{z})}I(k^z)$

Theorem 3.

$$I(z^k)_\nu = e^{-i\pi\nu}z^{-\nu}\frac{\Gamma(\nu-ks)}{\Gamma(-ks)}I(z^k)$$

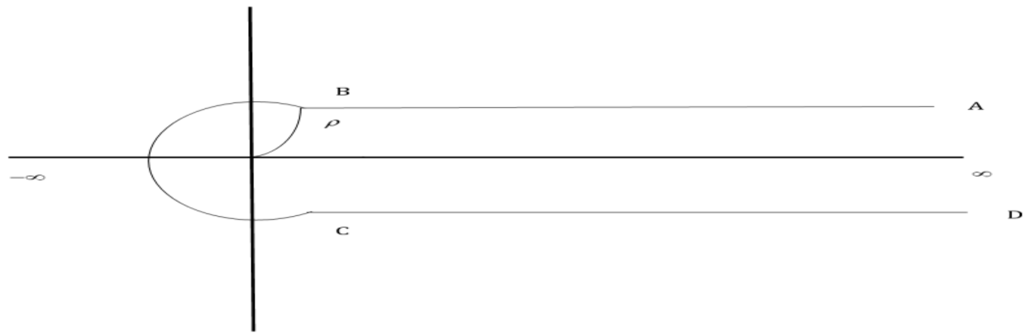
Case I: If $\left|\frac{\Gamma(\nu-ks)}{\Gamma(-ks)}\right| < \infty$, we have then

$$I\left(z^k\right)_\nu = \underset{+}{C}\left(I\left(z^k\right)\right)_\nu = \frac{\Gamma(\nu+1)}{2\pi i}\int_{\underset{+}{C}}\frac{I(\zeta^k)}{(\zeta-z)^{\nu+1}}d\zeta$$

$$= \frac{1}{2\pi i}\int_L \theta(s)ds.\frac{\Gamma(\nu+1)}{2\pi i}\int_{\infty e^{i\phi}}^{(0+)} u^{-(\nu+1)}(1+u)^{ks}z^{ks-\nu}du,\ (\phi = \arg z)$$

By putting ($\zeta - z = \eta, \eta = zu$)

$$= \frac{1}{2\pi i}\int_L \theta(s)ds.\frac{\Gamma(\nu+1)}{2\pi i}\int_{\infty}^{(0+)} u^{-(\nu+1)}(1+u)^{ks}z^{ks-\nu}du, (\phi < \frac{\pi}{2}) \qquad (2.2)$$



And

$$\int_{\infty}^{(0+)} u^{-(\nu+1)}(1+u)^{ks}\,du$$

$$=\lim_{\rho\to 0}\left(\int_{AB}+\int_{BC}+\int_{CD}\right)u^{-(\nu+1)}(1+u)^{ks}\,du \tag{2.3}$$

$$(u=re^{i\theta} on\ AB, u=re^{i2\pi} on\, CD, u=\rho e^{i\theta}\ on\, BC)$$

$$=-\int_0^{\infty} r^{-(\nu+1)}(1+r)^{ks}\,dr+e^{-i2\pi\nu}\int_0^{\infty} r^{-(\nu+1)}(1+r)^{ks}\,dr+\lim_{\rho\to 0}\rho^{-\nu}\int_0^{2\pi} e^{-i\theta\nu}\,d\theta$$

$$=(e^{-i2\pi\nu}-1)\int_0^{\infty}\int_0^{\infty} r^{-(\nu+1)}(1+r)^{ks}\,dr,\ \ (\mathrm{Re}(\nu)<0) \tag{2.4}$$

$$e^{-i2\pi\nu}-1=-i2e^{-i\pi\nu}\sin\pi\nu=e^{-i\pi\nu}\frac{2\pi i}{\Gamma(\nu+1)\Gamma(-\nu)} \tag{2.5}$$

And $\int_0^{\infty} r^{-(\nu+1)}(1+r)^{ks}\,dr=\dfrac{\Gamma(-\nu)\Gamma(\nu-ks)}{\Gamma(-ks)}(\mathrm{Re}(ks)<\mathrm{Re}(\nu)<0)$ (2.6)

Applying (2.3), (2.6) into (2.4), we have then

$$\int_{\infty}^{(0+)} u^{-(\nu+1)}(1+u)^{ks}\,du=e^{-i\pi\nu}\frac{\Gamma(\nu-ks)}{\Gamma(-ks)}\frac{2\pi i}{\Gamma(\nu+1)} \tag{2.7}$$

Substituting (2.7) into (2.2), we have then

$$=e^{-i\pi\nu}z^{-\nu}\frac{\Gamma(\nu-ks)}{\Gamma(-ks)}I(z^k)$$

For $\mathrm{Re}(ks)<\mathrm{Re}(\nu)<0, |\arg z|<\dfrac{\pi}{2}, \left|\dfrac{\Gamma(\nu-ks)}{\Gamma(-ks)}\right|<\infty$.

Case II: For $\mathrm{Re}(ks) < \mathrm{Re}(\nu) < 0, \frac{\pi}{2} \leq |\arg z| \leq \pi, \left|\frac{\Gamma(\nu - ks)}{\Gamma(-ks)}\right| < \infty$

In the same way, we have

$$I\left(z^k\right)_\nu = \underset{-}{C}\left(I\left(z^k\right)\right)_\nu$$

$$= \frac{1}{2\pi i}\int_L \theta(s)ds.\frac{\Gamma(\nu+1)}{2\pi i}\int_{-\infty e^{i\phi}}^{(0+)} u^{-(\nu+1)}(1+u)^{ks} z^{ks-\nu} du,\ (\phi = \arg z)$$

$$= \frac{1}{2\pi i}\int_L \theta(s)ds.\frac{\Gamma(\nu+1)}{2\pi i}\int_{\infty}^{(0+)} u^{-(\nu+1)}(1+u)^{ks} z^{ks-\nu} du, (\frac{\pi}{2} < \phi < \pi) = e^{-i\pi\nu} z^{-\nu}\, \frac{\Gamma(\nu-ks)}{\Gamma(-ks)} I(z^k)$$

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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