Available online at http://scik.org
J. Math. Comput. Sci. 10 (2020), No. 1, 150-156
https://doi.org/10.28919/jmcs/4301
ISSN: 1927-5307

SOME FAMILIES OF 4-TOTAL DIFFERENCE CORDIAL GRAPHS

R. PONRAJ[1,*], S. YESU DOSS PHILIP[2], R. KALA[2]

[1]Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627412, Tamilnadu, India

[2]Department of Mathematics, Manonmaniam Sundarnar University, Abishekapatti, Tirunelveli-627012, Tamilnadu, India

Abstract. Let G be a graph. Let $f : V(G) \to \{0,1,2,\ldots,k-1\}$ be a map where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called k-total difference cordial labeling of G if $\left|t_{df}(i) - t_{df}(j)\right| \leq 1$, $i, j \in \{1,2,\ldots,k\}$ where $t_{df}(x)$ denotes the total number of vertices and the edges labeled with x. A graph with admits a k-total difference cordial labeling is called k-total difference cordial graphs. In this paper we investigate the 4-total difference cordial labeling behaviour of some graphs like $J_{n,n} \cup K_{1,n}$,$J_{n,n} \cup B_{n,n}$,$J_{n,n} \cup P_n$ etc.

Keywords: path; bistar; jelly fish; union of graphs; corona of graphs.

2010 AMS Subject Classification: 05C78.

1. Introduction

In this paper we consider here finite, simple and undirected graphs only. The notion of k-total difference cordial graph was introduced in [4]. In [4, 5], 3-total difference cordial labeling behaviour of path, complete graph, comb, armed crown, crown, wheel, star etc have been investigated and also in [6], 4-total difference cordial labeling of path, star, bistar, comb, crown etc., have been invetigated. In [7], 4-total difference cordial labeling of $P_n \cup K_{1,n}$, $S(P_n \cup K_{1,n})$,

*Corresponding author
E-mail address: ponrajmaths@gmail.com
Received September 14, 2019

$P_n \cup B_{n,n}$ etc., have been invetigated. In this paper we investigate 4-total difference of cordial labeling of union some graphs like $J_{n,n} \cup K_{1,n}$,$J_{n,n} \cup B_{n,n}$,$J_{n,n} \cup P_n$.

2. PRELIMINARIES

Definition 2.1. Let G be a graph. Let $f : V(G) \to \{0,1,2,\ldots,k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called k-total difference cordial labeling of G if $\left|t_{df}(i) - t_{df}(j)\right| \leq 1$, $i, j \in \{1,2,\ldots,k\}$ where $t_{df}(x)$ denotes the total number of vertices and the edges labelled with x. A graph with a k-total difference cordial labeling is called k-total difference cordial graph.

Definition 2.2. The *corona* of G_1 with G_2,$G_1 \odot G_2$ is the graph obtained by taking one copy of G_2 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2.

Definition 2.3. The *Bistar* $B_{m,n}$ is the graph obtained by joining the two central vertices of $K_{1,m}$ and $K_{1,n}$.

Definition 2.4. The *union* of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.

Definition 2.5. The jelly fish is the graph $J_{n,n}$ with vertex set t $V(J_{n,n})$ = $\{u_i, v_i, u, v, x, y\}$ $1 \leq i \leq n$ and edge set $E(J_{n,n})$ = $\{uu_i, vv_i, ux, uy, vx, vy : 1 \leq i \leq n\}$.

3. MAIN RESULTS

Theorem 3.1. $J_{n,n} \cup K_{1,n}$ is 4-total difference cordial for all n.

Proof. Take the vertex set and edge set of $J_{n,n}$ as in definition 3.4. Let $K_{1,n}$ be the star. Let w be the central vertex of $K_{1,n}$ and $w_1, w_2, \ldots, w_n$ be the pendent vertices adjacent to w. Clearly,$\left|V(J_{n,n} \cup K_{1,n})\right| + \left|E(J_{n,n} \cup K_{1,n})\right| = 6n + 10$.

Case 1. $n \geq 4$.

Fix the labels $1,1$ and 1 to the vertices u_1, u_2 and u_3 and also fix the labels $1,1$ and 1 to the vertices v_1, v_2 and v_3. Assign the labels $3,3,3$ and 3 to the vertices u, x, y and v. Next assign the labels 3 and 1 to the vertices u_4 and u_5. Next consider the two vertices u_6 and u_7. Assign the label 3 and 1 respectively to the vertices u_6 and u_7. Continue in this pattern until we reach the vertex u_n. Note that the vertex u_n receive the label 3 or 1 if n is even or odd. Assign the label 3 and 1 to the vertices v_4 and v_5. Next consider the two vertices v_6 and v_7. Assign the label 3 and 1 respectively to the vertices v_6 and v_7. Proceeding in this pattern until we reach the vertex v_n. Clearly the vertex v_n receive the label 3 or 1 if n is even or odd. We now consider the vertices of the star $K_{1,n}$. Fix the label 3 to the central vertex w. Next assign the labels 3 and 1 to the vertices w_1 and w_2. Next consider the two vertices w_3 and w_4. Assign the label 3 and 1 respectively to the vertices w_5 and w_6. Proceeding like this untill we reach the vertex w_n. Note that the vertex w_n receive the label 3 or 1 if n is even or odd.

Case 2. $n = 2$ and 3.

Table 1 gives a 4-total difference cordial labeling for this case.

n	u_1	u_2	u_3	u	v	x	y	v_1	v_2	v_3	w	w_1	w_2	w_3
2	1	1		3	3	3	3	1	1		3	3	1	
3	1	1	1	3	3	3	3	1	1	1	3	3	1	3

TABLE 1

The table 2 shows that this vertex labeling is a 4-total difference codial labeling.

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
n is odd	$\frac{3n+5}{2}$	$\frac{3n+5}{2}$	$\frac{3n+5}{2}$	$\frac{3n+5}{2}$
n is even	$\frac{3n+6}{2}$	$\frac{3n+4}{2}$	$\frac{3n+4}{2}$	$\frac{3n+6}{2}$

TABLE 2

□

Theorem 3.2. $J_{n,n} \cup B_{n,n}$ is 4-total difference cordial for all n.

Proof. Take the vertex set and edge set of $J_{n,n}$ as in definition 3.4. Let $B_{n,n}$ be the bistar. Let w, z $(1 \le i \le n)$ be the central vertices of $B_{n,n}$ and w_i, z_i $(1 \le i \le n)$ be the pendent vertices adjacent to w and z respectively. Clearly, $|V(J_{n,n} \cup B_{n,n})| + |E(J_{n,n} \cup B_{n,n})| = 8n + 12$. Fix the labels $1, 1, 1$ and 1 to the vertices u_1, u_2, v_1 and v_2. Next assign the label 3 to vertices $u_3, u_4, \ldots, u_n$

and $v_3, v_4, \ldots, v_n$. We now consider the vertices of the bistar $B_{n,n}$. Fix the labels $3, 3$ and 3 to the vertices w, z and w_1. Next assign the label 1 to the vertices $w_2, w_3, \ldots, w_n$ and $z_1, z_2, \ldots, z_n$. Clearly $t_{df}(0) = t_{df}(1) = 3n = t_{df}(2) = t_{df}(3) = 2n+3$. □

Theorem 3.3. $C_n \cup K_{1,n}$ is 4-total difference cordial for all n.

Proof. Let C_n be the cycle $u_1u_2\ldots u_nu_1$. Let $K_{1,n}$ be the star. Let v be the central vertex of $K_{1,n}$ and $v_1, v_2, \ldots, v_n$ be the pendent vertices adjacent to v. First we consider the cycle C_n. Assign the label 3 to the all cycle vertices $u_1, u_2, \ldots, u_n$. We now consider the star $K_{1,n}$. Assign the label 3 to the central vertex v. Next assign the label 1 to all the pendent vertices $v_1, v_2, \ldots, v_n$. Clearly $t_{df}(0) = t_{df}(1) = 3n = t_{df}(2) = t_{df}(3) = n$.

□

Theorem 3.4. $J_{n,n} \cup (P_n \odot K_1)$ is 4-total difference cordial for all values of n.

Proof. Take the vertex set and edge set as in definition 3.4. Let P_n be the path $w_1w_2\ldots w_n$ and $z_1, z_2, \ldots, z_n$ be the pendent vertices adjacent to $w_1, w_2, \ldots, w_n$ respectively. First consider the jelly fish $J_{n,n}$. Fix the label 3 to the vertices u, v, x and y. Assign the label 1 to vertices $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$. We now consider the comb $P_n \odot K_1$. Assign the label 3 to all the path vertices $w_1, w_3, \ldots, w_n$.Next we move to pendent vertices $z_1, z_2, \ldots, z_n$. Fix the label 1 to the vertices z_1 and z_2.Next assign the label 3 to the remaining pendent vertices $z_3, z_4, \ldots, z_5$. Clearly $t_{df}(0) = t_{df}(1) = 3n = t_{df}(2) = t_{df}(3) = 2n+2$. □

Example 3.5. A 4-total difference cordial labeling of $J_{6,6} \cup (P_6 \odot K_1)$ is shown in Figure 1

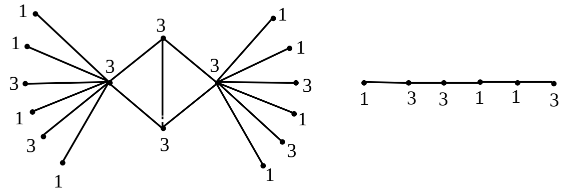

FIGURE 1

Theorem 3.6. $J_{n,n} \cup P_n$ is 4-total difference cordial for all n.

Proof. Take the vertex set and edge set as in definition 3.4. Let P_n be the path $w_1w_2\dots w_n$ be the path vertices. Fix the label 3 to the vertices u, v, x and y and the labels $1, 1$ and 1 to the vertices u_1, u_2 and u_3. Fix the label $1, 1$ and 1 to the vertices v_1, v_2 and v_3. Assign the labels 3 and 1 to the vertices u_4 and u_5. Next consider the two consecutive vertices u_6 and u_7. Assign the labels 3 and 1 to the next two vertices u_6 and u_7. Proceeding like this until we reach the vetex u_n. The vertex u_n receive the label 3 or 1 if n is odd or even. Assign the labels 3 and 1 to the vertices v_4 and v_5. Next consider the two consecutive vertices v_6 and v_7. Assign the labels 3 and 1 to the vertices v_6 and v_7. Proceeding like this untill we reach the vetex v_n. The vertex v_n receive the label 3 or 1 if n is odd or even.We now consider the path P_n.

Case 1. n is even.

Assign the labels $1, 3, 3$ and 1 to the path vertices w_1, w_2, w_3 and w_4. Next consider the four vertices w_5, w_6, w_7 and w_8. Assign the labels $1, 3, 3$ and 1 respectively to the vertices w_5, w_6, w_7 and w_8. We now assign the labels $1, 3, 3$ and 1 to the next four consecutive vertices $w_9, w_{10}, w_{11}, w_{12}$. Next assign the labels $1, 3, 3$ and 1 respectively to the next four consecutive vertices w_{13}, w_{14}, w_{15} and w_{16}. Countinue in this pattern until we reach the vertex u_n. It is easy to verify that the vertex u_n receive the label 1 or 3 according as $n \equiv 0 \pmod 4$ and 3 or $n \equiv 2 \pmod 4$.

Case 2. n is odd.

Assign the labels $1, 3, 3$ to the path vertices w_1, w_2 and w_3. Next consider the four vertices w_4, w_5, w_6 and w_7. Assign the labels $3, 1, 1$ and 3 respectively to the vertices w_4, w_5, w_6 and w_7. We assign the labels $3, 1, 1$ and 3 to the next four consecutive vertices w_8, w_9, w_{10}, w_{11}. Next assign the labels $3, 1, 1$ and 3 respectively to the next four consecutive vertices w_{12}, w_{13}, w_{14} and w_{15}. Countinue in this pattern until reach the vertex u_n. It is easy to check that the vertex u_n receive the label 1 when $n \equiv 1 \pmod 4$ and 3 if $n \equiv 3 \pmod 4$ or $n = 3$. The Table 3 given below shows that this labeling method is a 4-total difference cordial labeling. □

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
n is even	$\frac{3n+4}{2}$	$\frac{3n+4}{2}$	$\frac{3n+4}{2}$	$\frac{3n+4}{2}$
n is odd	$\frac{3n+3}{2}$	$\frac{3n+3}{2}$	$\frac{3n+5}{2}$	$\frac{3n+5}{2}$

TABLE 3

Theorem 3.7. $C_n \cup (P_n \odot K_1)$ is 4-total difference cordial for all values of n.

Proof. Let C_n be the cycle $u_1u_2\dots u_nu_1$. Let P_n be the path $v_1v_2\dots v_n$ and $w_1,w_2,\dots,w_n$ be the pendent vertices adjacent to $v_1,v_2,\dots,v_n$ respectively. First we consider the cycle C_n. Assign the label 3 to the all cycle vertices $u_1u_2\dots u_n$. We now consider the comb $P_n \odot K_1$. Fix the label $1,2,2$ to the path vertices v_1,v_2 and v_3. Next we move to pendent vertices w_1,w_2 and w_3. Fix the labels $3,2$ and 1 to the vertices w_1,w_2 and w_3. Next assign the labels 1 and 2 to the vertices v_4 and v_5. Next consider the two vertices v_6 and v_7. Assign the label 1 and 2 respectively to the vertices v_6 and v_7. Continue in this pattern untill we reach the vertex v_n. Note that the vertex u_n receive the label 1 when n is even $n \geq 4$ and 2 if $n \geq 3$. Assign the label 3 and 2 to the vertices w_4 and w_5. Next consider the two vertices w_6 and w_7. Assign the label 3 and 2 respectively to the vertices w_6 and w_7. Continue in this pattern untill we reach the vertex w_n. Note that the vertex w_n receive the label 3 or 2 whwn n is even $n \geq 4$ or odd $n \geq 5$.

The following table 4 establish that this labeling technique is a 4-total difference cordial labeling.

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
n is odd	$\frac{3n+1}{2}$	$\frac{3n-1}{2}$	$\frac{3n-1}{2}$	$\frac{3n-1}{2}$
n is even	$\frac{3n}{2}$	$\frac{3n-2}{2}$	$\frac{3n}{2}$	$\frac{3n}{2}$

TABLE 4

□

Theorem 3.8. $C_n \cup B_{n,n}$ is 4-total difference cordial for all values of n.

Proof. Let C_n be the cycle $u_1u_2\dots u_nu_1$. Let $B_{n,n}$ be the bistar. Let v,w be the central vertex of $B_{n,n}$ and $v_i,w_i (1 \leq i \leq n)$ be the pendent vertices adjacent to v and w. Clearly, $|V(C_n \cup B_{n,n})| + |E(C_n \cup B_{n,n})| = 6n+3$. Assign the label 3 to all the cycle vertices $u_1u_2\dots u_n$. Fix the labels $1,1$ to the vertices v_1,v_2 and w_1,w_2. Next assign the labels 3 and 1 to the vertices v_3 and v_4. Next consider the two vertices v_5 and v_6. Assign the label 3 and 1 respectively to the vertices v_5 and v_6. Continue in this pattern untill we reach the vertex v_n. Note that the vertex v_n receive the label 3 or 1 when n is odd or even.

The following table 5 establish that this labeling pattern is a 4-total difference cordial labeling.

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
n is odd	$\frac{3n+1}{2}$	$\frac{3n+1}{2}$	$\frac{3n+1}{2}$	$\frac{3n+3}{2}$
n is even	$\frac{3n}{2}$	$\frac{3n+2}{2}$	$\frac{3n+2}{2}$	$\frac{3n+2}{2}$

TABLE 5

□

Conflict of Interests

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] I. Cahit, Cordial graphs:A weaker version of graceful and harmonious graphs, Ars Comb., 23(1987), 201-207.

[2] J.A. Gallian, A Dynamic survey of graph labeling, The Electron. J. Comb., 19 (2017), #Ds6.

[3] F.Harary, Graph theory, Addision Wesley, New Delhi (1969).

[4] R. Ponraj, S. Yesu Doss Philip and R.Kala, *k*-total difference cordial graphs, J. Algorithms Comput., 51(2019), 121-128.

[5] R. Ponraj, S. Yesu Doss Philip and R.Kala, 3-total difference cordial graphs, Glob. Eng.Sci. Res., 6(2019), 46-51.

[6] R. Ponraj, S. Yesu Doss Philip and R.Kala, Some results on 4-total difference cordial graphs (communicated).

[7] R. Ponraj, S. Yesu Doss Philip and R.Kala, 4-total difference cordial labeling of union of some graphs (communicated).