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EDGE DOMINATION IN FUZZY GRAPHS USING STRONG ARCS

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Abstract: This paper introduces the concept of strong edge dominance in fuzzy graphs. We determine the strong edge domination number $\gamma'(G)$ of some types of fuzzy graphs. We present some common bounds related to the strong edge domination number. Besides, this new domination is discussed in the join of fuzzy graphs.

Keywords: fuzzy graph; strong arc; edge domination; edge domination number; join of fuzzy graphs; complete fuzzy graph.

2010 AMS Subject Classification: 05C72.

1. INTRODUCTION

The study of the dominating set in the graph was started by Berge [1] and Ore [6]. Rosenfeld introduced the notion of fuzzy graphs and several fuzzy analogs of graph theoretic concepts such as path, cycle, and connectedness [7]. A. Somasundaram and S. Somasundaram present the concept of domination in fuzzy graphs [5]. K.R. Bhutani and A. Rosenfeld introduced the concept of strong arcs in fuzzy graphs [10]. O.T. Manjusha and Sunitha discussed some concepts in domination and total domination in fuzzy graphs using strong arcs [4]. In this paper, we present the concept of edge domination in fuzzy graphs using strong arcs**.** For graph-theoretic terminology, we refer to Harary,1969 [8].

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2. PRELIMINARIES

Fuzzy graph G(σ,μ) is a pair of function $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that $\mu(u,v) \le \sigma(u) \wedge \sigma(v)$ for all u, v in V. The fuzzy graph $H(\tau,\rho)$ is called a fuzzy subgraph of G(σ,μ) if $\tau(u) \le \sigma(u)$ for all u in V and $\rho(u,v) \le \mu(u,v)$ for all $u, v\ in\ V$.. The underlying crisp graph of a fuzzy graph G(σ,μ) is denoted by $G^* = (\sigma^*, \mu^*)$, where $\sigma^* = \{u \in V \mid \sigma(u) > 0\}$ and $\mu^* = \{(u,v) \in V \times V \mid \mu(u,v) > 0\}$. A fuzzy graph G(σ, μ) is said to be bipartite if the node set V can be partitioned into two non-empty sets V_1 and V_2 such that $\mu(v_1, v_2) = 0$ if $v_1, v_2 \in V_1$ or $v_1, v_2 \in V_2$. Further if $\mu(v_1, v_2) > 0$ for all $v_1 \in V_1$ and $v_2 \in V_2$ then G is called complete bipartite graph. A complete bipartite graph on 'n' vertices is denoted by F(K_n). An arc (u, v) of the fuzzy graph G(σ,μ) is called an effective edge if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ and effective edge neighborhood of u $\in V$ is $N_e(u) = \{v \in V: edge(u,v)\ is\ effective\}$. A path ρ in a fuzzy graph G(σ,μ) is a sequence of distinct nodes v_0, v_1, v_2,…v_n such that $\mu(v_{i-1}, v_i) > 0$ where $1 \le i \le n$ and n is called the length of ρ. The strength of the path ρ is defined to be $\wedge_{i=i}^{n} \mu(v_{i-1}, v_i)$. A single vertex considers as a path of length zero. The strongest path joining any two nodes u, v is a path corresponding to maximum strength between u and v. The strength of the strongest path is denoted by $\mu^\infty(u,v)$ and it is defined by $\mu^\infty(u,v) = \sup\{\mu^k(u,v) \mid$ k = 1, 2, 3…∞}. An arc (u, v) of a fuzzy graph G(σ,μ) is called a strong arc if $\mu^\infty(u,v) = \mu(u,v)$, otherwise it is called a non-strong arc. Strong neighborhood of u $\in V$ is $N_s(u) = \{v \in V: arc(u,v)\ is\ strong\}$. $N_s[u] = N(u) \cup \{u\}$ is the closed neighborhood of u. Let G(σ,μ) be a fuzzy graph and u, v be two nodes of G(σ,μ). We say that u dominates v if the arc (u, v) is strong. A subset D of V is called a strong vertex dominating set of G (σ,μ) if for every $v \in V - D$, there exists $u \in D$ such that u dominates v. A strong vertex dominating set D is called a minimal strong vertex dominating set if no proper subset of D is a strong vertex dominating set. The minimum cardinality taken over all minimal strong vertex dominating set is called strong domination number, denoted by $\gamma(G)$ and the corresponding dominating set is called minimum strong vertex dominating set.

3. EDGE DOMINATION USING STRONG ARC

Definition 3.1. Let v be a vertex of a fuzzy graph G(σ,μ).We define $\sigma^s(v) = \max\{\mu(v,x)/ x \epsilon V\}$ that is $\sigma^s(v)$ is the maximum of the weights of the edges incident at v.

Definition 3.2. Let $G = (\sigma,\mu)$ be a fuzzy graph. Let e_i and e_j be two edges of G. We say that

e_i dominates e_j if e_i is a strong arc and e_i is adjacent to e_j. A subset $D'$of E(G) is called strong edge dominating set if for every $e_j \in E(G) - D'$ there exist $e_i \in D'$ such that e_i dominates e_j.

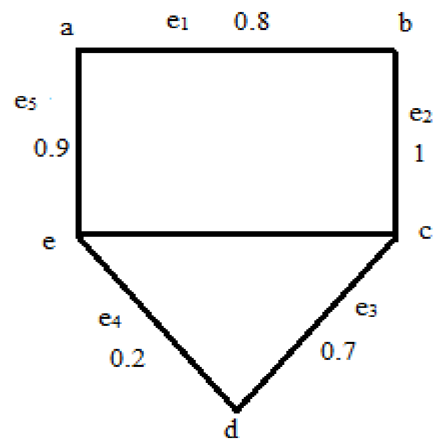


figure (i)

Example 3.3. In figure (i), the edges (a, b), (b, c), (c, d) & (a, e) are strong arcs and the edge (d, e) is a non strong arc. Here $D^{'} = \{(a, e), (b, c)\}$ is an edge dominating set.

Definition 3.4. An edge dominating set $D^{'}$ of G is called minimal edge dominating set if no proper subset of $D^{'}$ is an edge dominating set.

Example. In figure (i), $\{e_2, e_5\}, \{e_1, e_3\}$ and $\{e_3, e_5\}$ are minimal edge dominating sets.

Definition 3.5. The smallest number of edges in any edge dominating set of G is called its edge domination number and its denoted by $\gamma^{'}(G)$. An edge dominating set $D^{'}$ of G such that $|D^{'}| = \gamma^{'}(G)$ is called minimum edge dominating set.

Remark 3.6.

1. The set of all strong arcs of G is an edge dominating set of G.
2. The set E(G) need not be an edge dominating set because E(G) may contain non-strong arcs.

Definition 3.7. A fuzzy subgraph $H = (\tau, \rho)$ is said to be a spanning subgraph of G = (σ, μ) if $\tau(u) = \sigma(u)$ for all u in V. In this case, the two graphs have the same fuzzy node set, but they differ only in the arc weights.

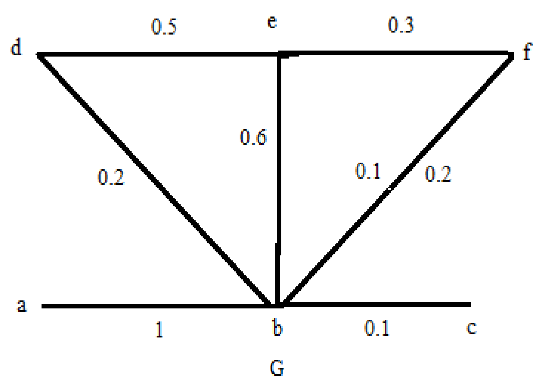
Theorem 3.8. Let $G = (\sigma, \mu)$ be a fuzzy graph and $H = (\sigma^{'}, \mu^{'})$ be a spanning subgraph of G. Let (x, y) be an arc in H such that $\mu^{'}(x, y) = \mu(x, y)$. If (x, y) is a strong arc in G then it must be a strong arc in H.

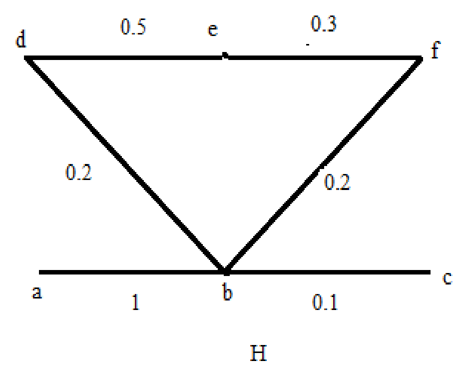
Proof. Let (x, y) be an arc in H. Suppose it is not a strong arc in H then there exist a path in H say $\rho' : x = x_0, x_1, \ldots\ldots x_n = y$ such that $s'_\rho > \mu(x,y)$. This path ρ' must also be in G. Since $\mu'(e) \leq \mu(e)$, for any edge e in H, we have s'_ρ in G $> \mu'(x,y) = \mu(x,y)$. This is a contradiction.

Remark 3.9.

1. The converse of the theorem 3.8 need not be true
2. A non-strong arc in $G = (\sigma, \mu)$ need not be a non-strong arc in $H = (\sigma', \mu')$
3. Let G be a fuzzy graph having an edge dominating set D'. If a spanning subgraph H of G contains all the elements of D' then $\gamma'(H) \leq \gamma'(G)$.
4. In general, we cannot compare $\gamma'(G)$ with $\gamma'(H)$.

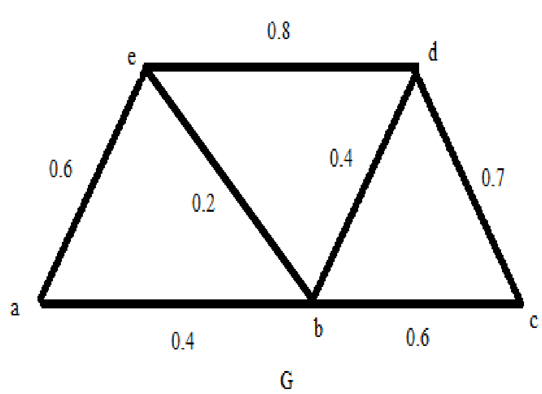
Example 3.10.

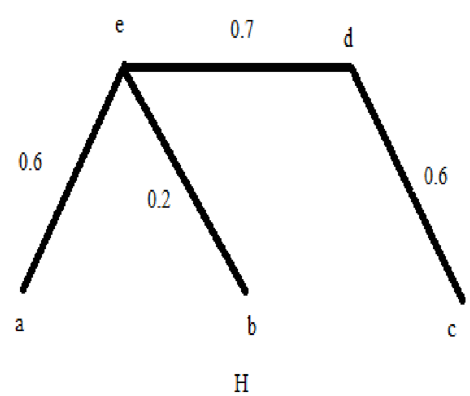




Here $\gamma'(G)$ = 1 and $\gamma'(H)$ = 2. Therefore $\gamma'(G) < \gamma'(H)$

Example 3.11.





Here $\gamma'(G)$ = 2 and $\gamma'(H)$ = 1. Therefore $\gamma'(G) > \gamma'(H)$

Definition 3.12. Let $G = (\sigma, \mu)$ be a fuzzy graph having a strong vertex dominating set D. Let $u \in D$. We define the set $d_s(u) = \{v \in V - D | (u, v) is\ a\ strong\ arc\}$

Theorem 3.13. Let $G = (\sigma, \mu)$ be a fuzzy graph having no isolated vertices then $\gamma(G) \leq 2\gamma'(G)$.

Proof. Let D' be the minimum edge dominating set of G. Let
D $= \{u_i \backslash u_i\ is\ an\ end\ vertex\ of\ any\ edge\ in\ D'\ \}$. Let $y \in V - D$. Since G has no isolated vertices we have $N_s[y] \neq \emptyset$. Let v $\in N_s[y]$. Suppose $N_s[y] \cap D = \emptyset$, then $v \notin D$. Also $y \notin D$. That is the vertices v and y are not the end vertices of any edge in D'. This implies no edge in D' dominates the edge (v, y). This is a contradiction. Hence $N_s[y] \cap D \neq \emptyset$. Consequently, D is a strong vertex dominating set. Also $|D| \leq 2|D'|$. Therefore $\gamma(G) \leq |D| \leq 2|D'| \leq 2\gamma'(G)$.

Theorem 3.14. Let $G = (\sigma, \mu)$ be a fuzzy graph without isolated vertices. If $\Delta(G) \leq 2$ then $\gamma'(G) \leq \gamma(G)$.

Proof. Let $D = \{u_1, u_2, \ldots\ldots u_n\}$ be a minimum strong vertex dominating set of G. Arrange the elements of D such that $d_s(u_i) \leq d_s(u_{i+1})$. Since D is minimal, $d_s(u_i) \neq 0$ for $1 \leq i \leq n$. Also, since $\Delta(G) \leq 2$, we have either $d_s(u_i) = 1$ or $d_s(u_i) = 2$. If $d_s(u_i) = 1$ select the vertex $v_i \in V - D$ such that (u_i, v_i) is a strong arc. Let u_k be the first vertex such that $d_s(u_k) = 2$. Therefore each $u_i \in D$, has two strong neighbors in V-D, where $k \leq i \leq n$. Let $d_s(u_i) = \{v_i', v_i''\}$. Define $u_i' = \begin{cases} v_i' & if\ d(v_i') \geq d(v_i'') \\ v_i'' & Otherwise \end{cases}$. For each vertex u_i in D where $k \leq i \leq n$, select the vertex v_i in V-D as

$$v_i = \begin{cases} u_i' & if\ u_i'\ is\ not\ adjacent\ to\ u_{i-n}', 1 \leq n \leq i - k \\ d_s(u_i) - \{u_i'\} & Otherwise \end{cases}.$$

Consider the set $D' = \{(u_i, v_i)\ 1 \leq i \leq n\}$. Let (x, y) be any edge in $(G) - D'$.

Case (i) Let $x \in D$ or $y \in D$. Then $x = u_j$ or $y = u_j$ for some j, $1 \leq j \leq n$. Therefore (x, y) is adjacent to the edge (u_j, v_j) in D'.Hence D' dominate the edge (x, y).

Case (ii) Let $x \in V - D$ and $y \in V - D$. Since D is minimum strong vertex dominating set, there exist vertices u_m and u_n such that (u_m, x) and (u_n, y) are strong arcs. Suppose m = n, then either $x = v_m$ or $y = v_m$. Hence (x, y) is adjacent to the edge (u_m, v_m) in D'. Therefore let $m \neq n$. If we assume $y \neq v_n$ then $d_s(u_n) = 2$ and so $n \geq k$. Since $y \neq v_n$ by the choice of v_n, y is adjacent to some vertex v_{n-i} for some i, $1 \leq i \leq n - k$. Since $\Delta(G) \leq 2$

and y is adjacent to the vertex x in V-D, we have $x = v_{n-i}$. Therefore (x, y) is adjacent to the edge (u_{n-i}, v_{n-i}) in D'. Hence D' is an edge dominating set of G. Since $|D'| = n$, we have $\gamma'(G) \leq n = \gamma(G)$.

Remark 3.15. Let $G = (\sigma, \mu)$ be a fuzzy graph having no isolated vertices. If $\Delta(G) \leq 2$ then $\gamma'(G) \leq \gamma(G) \leq 2\gamma'(G)$.

Theorem 3.16. Let $F(k_n)$ be a complete fuzzy graph on 'n' vertices then $\gamma'[F(k_n)] = 1 + \gamma'[F(K_{n-2})]$, $n \geq 3$

Proof. In $F(k_n)$ all the edges are strong. Let x = (u, v) be any edge in $F(k_n)$. Since $F(k_n)$ is complete there are n-2 edges incident at u and v other than the edge x. Therefore the edge x dominates 2n-4 edges. The number of edges not dominated by x is $nC_2 - (2n-3) = \frac{n^2-5n+6}{2} = (n-2)C_2$. In $F(k_n)$ other than the vertices u and v we have n-2 vertices and $(n-2)C_2$ edges are not dominated by the edge x = (u, v). Hence the set of all vertices $S = V(F(k_n)) - \{u, v\}$ forms $F(K_{n-2})$. Therefore $\gamma'[F(k_n)] = 1 + \gamma'[F(K_{n-2})]$.

Remark 3.17.

In the following table, we present the fuzzy edge domination number of some complete fuzzy graphs

$F(k_n)$	$F(k_3)$	$F(k_4)$	$F(k_5)$	$F(k_6)$	$F(k_7)$	$F(k_8)$	$F(k_9)$	$F(k_{10})$
$\gamma'[F(k_n)]$	1	2	2	3	3	4	4	5

Theorem 3.18. Let $G = (\sigma, \mu)$ be a fuzzy bipartite graph having no isolated vertices. Let V_1 and V_2 be the partition of V(G), then $\gamma'(G) \leq \min\{|V_1|, |V_2|\}$

Proof. Let V_1 and V_2 be the partition of V(G). Let $u_1, u_2, \ldots . u_m \in V_1$ and $v_1, v_2, \ldots . v_n \in V_2$. Without loss of generality assume $m < n$. We denote the edge joining the points u_i and v_j by x_{ij}. For each u_i, $1 \leq i \leq m$, select the edge x_{ik} such that $\mu(x_{ik}) = \sigma^s(u_i)$, $1 \leq k \leq n$. Evidently, x_{ik} is a strong arc. Let $D' = \{x_{ik}, 1 \leq i \leq m\}$. Let x be any edge in $E(G) - D'$. Then $x = x_{ij}$ for some i and j. Therefore x is adjacent to the edge x_{ik} in D' for some i. That is the edge x is dominated by the edge x_{ik}. Hence D' is an edge dominating set of G. Therefore $\gamma'(G) \leq |D'| \leq m$. If n < m then by the similar argument $\gamma'(G) \leq n$. Hence $\gamma'(G) \leq \min\{|V_1|, |V_2|\}$.

Theorem 3.19. Let G be a complete fuzzy bipartite graph without isolated vertices. Let V_1 and V_2 be the partition of V(G), then $\gamma'(G) = \min\{m, n\}$.

Proof. Let V_1 and V_2 be the partition set of G. Let $u_1, u_2, \ldots . u_m \in V_1$ and $v_1, v_2, \ldots . v_n \in V_2$. Without loss of generality assume $m < n$. We denote the edge joining the points u_i and v_j by x_{ij}. Let D be the set of any m - 1 strong edges of G.

Let $S_1 = \{u_i \in V_1 \,/\, u_i \; is\ an\ end\ vertex\ of\ any\ edge\ in\ D\}$

and $S_2 = \{v_k \in V_2 \,/\, v_k \; is\ an\ end\ vertex\ of\ any\ edge\ in\ D\}$. $|S_1| \leq m - 1$ and $|S_2| \leq m - 1 < n$. Since $|V_1| = m$ and $|V_2| = n$, there exist vertices $u_i \in V_1$ and $v_j \in V_2$ such that $u_i \notin S_1$ and $v_j \notin S_2$. Since G is complete the edge x_{ij} connecting the vertices u_i and v_j is in E(G). This implies the edge x_{ij} is not dominated by any edge in D. Therefore, any set of m-1 edges of G cannot be an edge dominating set of G. Hence $\gamma'(G) > m - 1$. That is $\gamma'(G) \geq m$. By Theorem 3.18, $\gamma'(G) \leq m$. Hence $\gamma'(G) = $ m. Therefore $\gamma'(G) = \min\{|V_1|, |\,V_2|\}$.

4. EDGE DOMINATION IN JOIN OF FUZZY GRAPHS

Definition 4.1. Let $G_1(\sigma_1, \mu_1)$, and $G_2(\sigma_2, \mu_2)$ be two fuzzy graphs on V_1 and V_2 respectively, with $V_1 \cap V_2 = \emptyset$. The join of $G_1(\sigma_1, \mu_1)$ and $G_2(\sigma_2, \mu_2)$, denoted by $G_1 + G_2$ is the fuzzy graph $G(\sigma_1 + \sigma_2,\ \mu_1 + \mu_2)$ on $V_1 \cup V_2$, where, $(\sigma_1 + \sigma_2)u = \begin{cases} \sigma_1(u) \;\; if\ u \in V_1 \\ \sigma_2(u) if\ u \in V_2 \end{cases}$

and $(\mu_1 + \mu_2)(uv) = \begin{cases} \mu_1(uv) \;\; if\ u, v \in V_1 \\ \mu_2(uv) \;\; if\ u, v \in V_2 \\ \sigma_1(u) \wedge \sigma_2(v)\ if\ u \in V_1 and\ v \in V_2 \end{cases}$

Example 4.2.

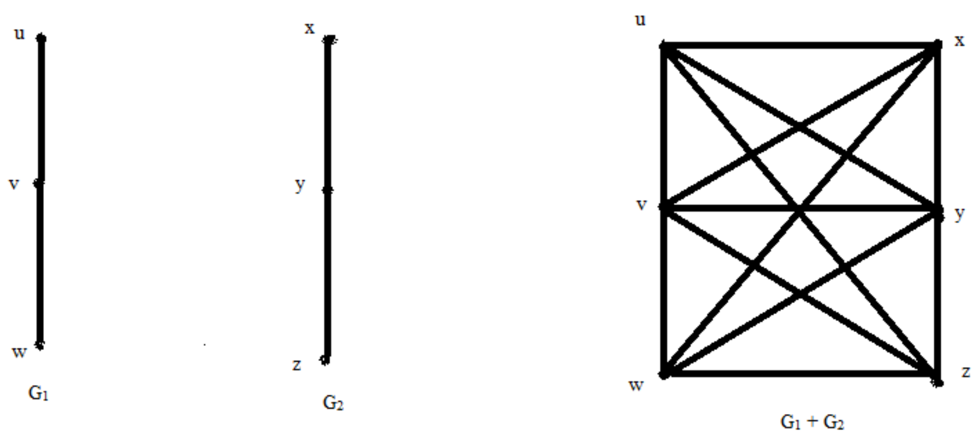


figure 4.1

Let σ_1= (u|0.5,v|0.8,w|0.7) and μ_1= (uv|0.4, vw|0.5). Let σ_2 = (x|0.8,y|0.7,z|0.9) and μ_2= (xy|0.6,yz|0.7)

$\sigma_1 + \sigma_2$ = (u|0.5, v|0.8, w|0.7, x|0.8, y|0.7, z|0.9)

$\mu_1 + \mu_2$ = (uv|0.4, vw|0.5, xy|0.6, yz|0.7, ux|0.5, uy|0.5, uz|0.5, vx|0.8, vy|0.7, vz|0.8, wx|0.7, wy|0.7, wz|0.7)

Theorem 4.3. Let $G_1(\sigma_1, \mu_1)$, and $G_2(\sigma_2, \mu_2)$ be two fuzzy graphs on V_1 and V_2 respectively, with $V_1 \cap V_2 = \emptyset$. Then $\gamma'(G_1 + G_2) \leq \min\{|V(G_1)|, |V(G_2)|\}$.

Proof. Let $G_1(\sigma_1, \mu_1)$ be a fuzzy graph with $|V(G_1)| = m$ and $G_2(\sigma_2, \mu_2)$ be a fuzzy graph with $|V(G_2)| = n$. Let $x_1, x_2, \dots . x_m$ be the vertices of G_1 and $y_1, y_2, \dots . y_n$ be the vertices of G_2. Without loss of generality assume m < n. By the definition of $G_1 + G_2$, any edge of the form (x_i, y_k) is an effective edge and hence it is a strong arc in $G_1 + G_2$. In particular, the edge (x_1, y_1) is a strong arc. This edge (x_1, y_1) dominates the edges of the form (x_1, y_j) and also the edges incident at x_1 and y_1, 1≤ $j \leq n$. Likewise, the edge (x_2, y_2) dominates the edges of the form (x_2, y_j) and also the edges incident at x_2 and y_2, 1≤ $j \leq n$. Continuing these steps all the edges of $G_1 + G_2$ is dominated by the set $D' = \{(x_i, y_i), 1 \leq i \leq m\}$ Therefore $\gamma'(G_1 + G_2) \leq m = \min\{|V(G_1)|, |\, V(G_2)|\}$.

Remark 4.4. $\gamma'(G_1) + \gamma'(G_2)$ need not be an edge dominating set of $G_1 + G_2$.

Example 4.5. In figure 4.1, $\gamma'(G_1) = \{(u, v)\}$ and $\gamma'(G_2) = \{(x, y)\}$. But $D' = \{(u, v), (x, y)\}$ is not a strong edge dominating set of $G_1 + G_2$. Because the edge (w, z) is not dominated by any element of D'.

Definition 4.6. Let $G(\sigma, \mu)$ be a fuzzy graph having an edge dominating set D'. We define the solo set of G as S(D') $= \{v \in G / \, v\ is\ not\ an\ end\ vertex\ of\ any\ edge\ in\ D'\}$

Theorem 4.7. Let $G_1(\sigma_1, \mu_1)$, and $G_2(\sigma_2, \mu_2)$ be two fuzzy graphs having minimal edge dominating sets $D_1{}'$ and $D_2{}'$ respectively. Then $D' = \{D_1{}' \cup D_2{}' \cup X\}$ is a strong edge dominating set of $G_1 + G_2$ where $X = \{(x_i, y_i) \in E(G_1 + G_2) / \, x_i \in V(G_1), y_i \in V(G_2) and\ x_i \in S(D_1{}')$

Proof. Let $x_1, x_2, \dots . x_m$ be the vertices of G_1 and $y_1, y_2, \dots . y_n$ be the vertices of G_2. Let (x,y) be any edge in $G_1 + G_2$. Then (x, y) ∈ G_1 or (x, y) ∈ G_2 or (x, y) = (x_i, y_j) for some i and j. If (x, y) ∈ G_1 or (x, y) ∈ G_2 then it is dominated by an element of the set $D_1{}' \cup D_2{}'$. Suppose $(x, y) \notin G_1$ and G_2. Then (x, y) = (x_i, y_j) for some i and j. If $x_i \in S(D_1{}')$ then (x_i, y_j) is dominated by an edge $(x_i, y_i) \in$ X. Suppose $x_i \in V(G_1) - S(D_1{}')$ then the edge (x_i, y_j) is dominated by an element of $D_1{}'$.

Corollary 4.8. Let $G_1(\sigma_1, \mu_1)$, and $G_2(\sigma_2, \mu_2)$ be two fuzzy graphs having minimal edge dominating sets $D_1{}'$ and $D_2{}'$ respectively. Then $D' = \{D_1{}' \cup D_2{}' \cup Y\}$ is a strong edge dominating set of $G_1 + G_2$, where $Y = \{(x_i, y_i) \in E(G_1 + G_2) / x_i \in V(G_1), y_i \in V(G_2) and\ y_i \in S(D_2{}')$

Theorem 4.9. Let $G_1(\sigma_1, \mu_1)$, and $G_2(\sigma_2, \mu_2)$ be two fuzzy graphs. Then $\gamma'(G_1 + G_2) \leq \gamma'(G_1) + \gamma'(G_2) + s$, where $s = \min\{|S(D_1{}')|, |S(D_2{}')|\}$

Proof. It follows by Theorem 4.7 and Corollary 4.8.

5. CONCLUSION

More than thirty domination parameters have been studied by different authors and in this paper, we have introduced the concept of strong edge domination in fuzzy graphs. The edge domination number of complete fuzzy graphs and complete fuzzy bipartite graphs are obtained. We have found some bounds for $\gamma'(G)$. Also, we explored this domination in the join of fuzzy graphs

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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