Available online at http://scik.org
J. Math. Comput. Sci. 10 (2020), No. 1, 157-170
https://doi.org/10.28919/jmcs/4320
ISSN: 1927-5307

A NOTE ON FUZZY SOFT MATRICES

TRIDIV JYOTI NEOG* AND BIJU KUMAR DUTTA

Department of Mathematics, The Assam Kaziranga University, Jorhat, India

Abstract: The purpose of this paper is to study the notion of fuzzy soft matrices and their operations. A decision making problem using the notion of sum of fuzzy soft matrices has been put forward.

Keywords: fuzzy set; soft set; fuzzy soft set; fuzzy soft matrix.

2010 AMS Subject Classification: 03E72.

1. INTRODUCTION

The theory of fuzzy sets initiated by Zadeh [3] is a very useful tool to deal with uncertainty. In 1999, Molodtsov [2] put forward the concept of soft set as a new mathematical tool to deal with uncertainties. Since its introduction, the notion of soft set has gained respectable attention. In recent times, researchers have been working towards fuzzification of soft set theory. Maji et al. [5] combined fuzzy sets with soft sets and introduced the concept of fuzzy soft sets. These results were later revised and improved by Ahmad and Kharal [1].

Matrices play a vital role within the broad space of science and engineering. Yong Yang and Chenli Ji [8] initiated a matrix illustration of a fuzzy soft set and successfully applied the proposed notion of fuzzy soft matrix in certain deciding issues that contain uncertainty. Neog et al.[7] and Borah et al. [4] extended the notion of fuzzy soft matrices put forward in [8]. In our work, we have put forward the notion of sum, difference and product of fuzzy soft matrices along with some properties and application in decision problem.

*Corresponding author
E-mail address: tridivjyoti@gmail.com
Received September 30, 2019

2. PRELIMINARIES

In this section, we first recall the basic definitions related to fuzzy soft sets which would be used in the sequel.

Definition 2.1 [2]

A pair (*F*, *E*) is called a soft set (over *U*) if and only if *F* is a mapping of *E* into the set of all subsets of the set *U*. In other words, the soft set is a parameterized family of subsets of the set *U*. Every set $F(\varepsilon),\ \varepsilon \in E$, from this family may be considered as the set of ε - elements of the soft set (*F*, *E*), or as the set of ε - approximate elements of the soft set.

Definition 2.2 [5]

A pair (*F*, *A*) is called a fuzzy soft set over *U* where $F : A \to \tilde{P}(U)$ is a mapping from *A* into $\tilde{P}(U)$. Here $\tilde{P}(U)$ represents the fuzzy subsets of *U*.

Definition 2.3 [1]

Let *U* be a universe and *E* a set of attributes. Then the pair (*U*, *E*) denotes the collection of all fuzzy soft sets on *U* with attributes from *E* and is called a fuzzy soft class.

Definition 2.4 [5]

A soft set (*F*, *A*) over *U* is said to be null fuzzy soft set denoted by ϕ if $\forall \varepsilon \in A,\ F(\varepsilon)$ is the null fuzzy set $\bar{0}$ of *U*, where $\bar{0}(x) = 0\ \forall x \in U$.

We would use the notation (ϕ, A) to represent the fuzzy soft null set with respect to the set of parameters *A*.

Definition 2.5 [5]

A soft set (*F*, *A*) over *U* is said to be absolute fuzzy soft set denoted by $\tilde{A}$ if $\forall \varepsilon \in A,\ F(\varepsilon)$ is the absolute fuzzy set $\bar{1}$ of *U* where $\bar{1}(x) = 1\ \forall x \in U$.

We would use the notation (U, A) to represent the fuzzy soft absolute set with respect to the set of parameters *A*.

Definition 2.6 [5]

For two fuzzy soft sets (F, A) and (G, B) in a fuzzy soft class (U, E), we say that (F, A) is a fuzzy soft subset of (G, B), if

(*i*) $A \subseteq B$

(*ii*) For all $\varepsilon \in A$, $F(\varepsilon) \subseteq G(\varepsilon)$ and is written as $(F, A)\ \tilde{\subseteq}\ (G, B)$.

Definition 2.7 [5]

Union of two fuzzy soft sets (F, A) and (G, B) in a soft class (U, E) is a fuzzy soft set (H, C) where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$ and is written as $(F, A)\tilde{\cup}(G, B) = (H, C)$.

Definition 2.8 [1]

Let (F, A) and (G, B) be two fuzzy soft sets in a soft class (U, E) with $A \cap B \neq \phi$. Then Intersection of two fuzzy soft sets (F, A) and (G, B) in a soft class (U, E) is a fuzzy soft set (H, C) where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$. We write $(F, A)\tilde{\cap}(G, B) = (H, C)$, where $\cap$ is the operation intersection of two fuzzy sets.

Definition 2.9 [6]

The complement of a fuzzy soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \to \tilde{P}(U)$ is a mapping given by $F^c(\alpha) = [F(\alpha)]^c$, $\forall \alpha \in A$.

Definition 2.10[5]

If (F, A) and (G, B) be two fuzzy soft sets, then "(F, A) AND (G, B)" is a fuzzy soft set denoted by $(F,\ A) \wedge (G,\ B)$

and is defined by $(F,\ A) \wedge (G,\ B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$, $\forall \alpha \in A$ and $\forall \beta \in B$, where $\cap$ is the operation intersection of two fuzzy sets.

Definition 2.11[5]

If (F, A) and (G, B) be two fuzzy soft sets, then "(F, A) OR (G, B)" is a fuzzy soft set denoted by $(F, A) \vee (G, B)$ and is defined by $(F, A) \vee (G, B) = (K, A \times B)$, where $K(\alpha, \beta) = F(\alpha) \cup G(\beta)$, $\forall \alpha \in A$ and $\forall \beta \in B$, where $\cup$ is the operation union of two fuzzy sets.

3. FUZZY SOFT MATRIX THEORY

Definition 3.1 [4]

Let $U=\{c_1,c_2,c_3,..............c_m\}$ be the universal set and E be the set of parameters given by $E=\{e_1,e_2,e_3,................e_n\}$. Let $A\subseteq E$ and (F,A) be a fuzzy soft set in the fuzzy soft class (U,E). Then we would represent the fuzzy soft set (F,A) in matrix form as

$A_{m\times n}=[a_{ij}]_{m\times n}$ or simply by $A=[a_{ij}]$, i = 1,2,3…………..m ; j = 1,2,3……………..n ,

where $a_{ij}=\begin{cases}\mu_j(c_i) & \text{if } e_j\in A\\ 0 & \text{if } e_j\notin A\end{cases}$

Here $\mu_j(c_i)$ represents the membership of c_i in the fuzzy set $F(e_j)$. We would identify a fuzzy soft set with its fuzzy soft matrix and use these two concepts interchangeable. The set of all $m\times n$ fuzzy soft matrices over U would be denoted by $FSM_{m\times n}$.

Example 3.1

Let $U=\{c_1,c_2,c_3,c_4\}$ be the universal set and E be the set of parameters given by $E=\{e_1,e_2,e_3,e_4,e_5\}$.

Let $P=\{e_1,e_2,e_4\}\subseteq E$ and (F,P) is the fuzzy soft set

$$(F,P) = \{F(e_1)=\{(c_1,0.7),(c_2,0.6),(c_3,0.7),(c_4,0.5)\},\ F(e_2)=\{(c_1,0.8),(c_2,0.6),(c_3,0.1),(c_4,0.5)\},$$
$$F(e_4)=\{(c_1,0.1),(c_2,0.4),(c_3,0.7),(c_4,0.3)\}\}$$

The fuzzy soft matrix representing this fuzzy soft set would be represented in our notation as

$$A=\begin{bmatrix}0.7 & 0.8 & 0.0 & 0.1 & 0.0\\ 0.6 & 0.6 & 0.0 & 0.4 & 0.0\\ 0.7 & 0.1 & 0.0 & 0.7 & 0.0\\ 0.5 & 0.5 & 0.0 & 0.3 & 0.0\end{bmatrix}_{4\times 5}$$

Definition 3.2 (Fuzzy Soft Matrix Sum)

Let $A=[a_{ij}], B=[b_{ij}]\in FSM_{m\times n}$. Then Fuzzy soft matrix sum of A,B is defined by

$A_{m\times n}\oplus B_{m\times n}=C_{m\times n}=[c_{ij}]_{m\times n}$, where $c_{ij}=\max(a_{ij},b_{ij})$ for all i and j .

Example 3.2

Let $U = \{c_1, c_2, c_3, c_4\}$ be the set of four cars under consideration and

$E = \{e_1(\text{Costly}), e_2(\text{Beautiful}), e_3(\text{Fuel Efficient}), e_4(\text{ModernTechnology}), e_5(\text{Luxurious})\}$ be the set of parameters and $A = \{e_1, e_2, e_3\} \subseteq E$. We consider

$(F, A) = \{F(e_1) = \{ (c_1, 0.7), (c_2, 0.1), (c_3, 0.2), (c_4, 0.6)\},$

$F(e_2) = \{ (c_1, 0.3), (c_2, 0.8), (c_3, 0.4), (c_4, 0.5)\},$

$F(e_3) = \{ (c_1, 0.1), (c_2, 0.2), (c_3, 0.7), (c_4, 0.3)\}\}$ as the fuzzy soft set representing the ‘attractiveness of a car’ according to Mr. X.

Let $B = \{e_3, e_4\} \subseteq E$ and

$(G, B) = \{G(e_3) = \{ (c_1, 0.2), (c_2, 0.5), (c_3, 0.1), (c_4, 0.7)\},$

$G(e_4) = \{ (c_1, 0.8), (c_2, 0.4), (c_3, 0.1), (c_4, 0.6)\}\}$ be the fuzzy soft set representing a ‘good car’ according to the same person Mr. X. These two fuzzy soft sets would be represented by the fuzzy soft matrices

$$P = \begin{bmatrix} 0.7 & 0.3 & 0.1 & 0 \\ 0.1 & 0.8 & 0.2 & 0 \\ 0.2 & 0.4 & 0.7 & 0 \\ 0.6 & 0.5 & 0.3 & 0 \end{bmatrix}_{4\times 4} \text{ and } Q = \begin{bmatrix} 0 & 0 & 0.2 & 0.8 \\ 0 & 0 & 0.5 & 0.4 \\ 0 & 0 & 0.1 & 0.1 \\ 0 & 0 & 0.7 & 0.6 \end{bmatrix}_{4\times 4} \text{ respectively.}$$

Then $A_{4\times 4} \oplus B_{4\times 4} = C_{4\times 4}$

$$= \begin{bmatrix} \max\{0.7,0\} & \max\{0.3,0\} & \max\{0.1,0.2\} & \max\{0,0.8\} \\ \max\{0.1,0\} & \max\{0.8,0\} & \max\{0.2,0.5\} & \max\{0,0.6\} \\ \max\{0.2,0\} & \max\{0.4,0\} & \max\{0.7,0.1\} & \max\{0,0.1\} \\ \max\{0.6,0\} & \max\{0.5,0\} & \max\{0.3,0.7\} & \max\{0,0.4\} \end{bmatrix}_{4\times 4}$$

$$= \begin{bmatrix} 0.7 & 0.3 & 0.2 & 0.8 \\ 0.1 & 0.8 & 0.5 & 0.6 \\ 0.2 & 0.4 & 0.7 & 0.1 \\ 0.6 & 0.5 & 0.7 & 0.4 \end{bmatrix}_{4\times 4}$$

Proposition 3.1

Let $A, B \in FSM_{m\times n}$ .Then

(i) $A \oplus \tilde{0} = A$

(ii) $A \oplus \tilde{U} = \tilde{U}$

(iii) $A \oplus B = B \oplus A$

(iv) $(A \oplus B) \oplus C = A \oplus (B \oplus C)$

Proof:

Let $A = \lfloor a_{ij} \rfloor_{m\times n}$, $B = \lfloor b_{ij} \rfloor_{m\times n}$, $C = \lfloor c_{ij} \rfloor_{m\times n}$ be three fuzzy soft matrices .

$(i)\ A \oplus \tilde{0} = \lfloor \max(a_{ij}, 0) \rfloor_{m\times n} = \lfloor a_{ij} \rfloor_{m\times n} = A$

$(ii)\ A \oplus \tilde{U} = \lfloor \max(a_{ij}, 1) \rfloor_{m\times n} = [1]_{m\times n} = \tilde{U}$

$(iii)\ A \oplus B = \lfloor \max(a_{ij}, b_{ij}) \rfloor_{m\times n}$

$= \lfloor \max(b_{ij}, a_{ij}) \rfloor_{m\times n}$

$= B \oplus A$

$(iv)\ (A \oplus B) \oplus C = \lfloor \max(a_{ij}, b_{ij}) \rfloor_{m\times n} \oplus \lfloor c_{ij} \rfloor_{m\times n}$

$= \lfloor \max(a_{ij}, b_{ij}, c_{ij}) \rfloor_{m\times n}$

$A \oplus (B \oplus C) = \lfloor a_{ij} \rfloor_{m\times n} \oplus \lfloor \max(b_{ij}, c_{ij}) \rfloor_{m\times n}$

$= \lfloor \max(a_{ij}, b_{ij}, c_{ij}) \rfloor_{m\times n}$

Hence $(A \oplus B) \oplus C = A \oplus (B \oplus C)$.

Definition 3.3 (Fuzzy Soft Matrix Difference)

Let $A = \lfloor a_{ij} \rfloor, B = \lfloor b_{ij} \rfloor \in FSM_{m\times n}$. Then Fuzzy soft matrix difference of A, B is defined by

$A_{m\times n} \Theta B_{m\times n} = C_{m\times n} = [c_{ij}]_{m\times n}$, where $c_{ij} = \min(a_{ij}, 1 - b_{ij})$ for all i and j .

Example 3.3

For the two fuzzy soft matrices A and B given in **Example 3.2**, we have

$$A_{4\times 4}\Theta B_{4\times 4} = C_{4\times 4}$$

$$= \begin{bmatrix} \min\{0.7,1-0\} & \min\{0.3,1-0\} & \min\{0.1,1-0.2\} & \min\{0,1-0.8\} \\ \min\{0.1,1-0\} & \min\{0.8,1-0\} & \min\{0.2,1-0.5\} & \min\{0,1-0.6\} \\ \min\{0.2,1-0\} & \min\{0.4,1-0\} & \min\{0.7,1-0.1\} & \min\{0,1-0.1\} \\ \min\{0.6,1-0\} & \min\{0.5,1-0\} & \min\{0.3,1-0.7\} & \min\{0,1-0.4\} \end{bmatrix}_{4\times 4}$$

$$= \begin{bmatrix} \min\{0.7,1\} & \min\{0.3,1\} & \min\{0.1,0.8\} & \min\{0,0.2\} \\ \min\{0.1,1\} & \min\{0.8,1\} & \min\{0.2,0.5\} & \min\{0,0.4\} \\ \min\{0.2,1\} & \min\{0.4,1\} & \min\{0.7,0.9\} & \min\{0,0.9\} \\ \min\{0.6,1\} & \min\{0.5,1\} & \min\{0.3,0.3\} & \min\{0,0.6\} \end{bmatrix}_{4\times 4}$$

$$= \begin{bmatrix} 0.7 & 0.3 & 0.1 & 0 \\ 0.1 & 0.8 & 0.2 & 0 \\ 0.2 & 0.4 & 0.7 & 0 \\ 0.6 & 0.5 & 0.3 & 0 \end{bmatrix}_{4\times 4}$$

Definition 3.4 (Scalar Product of a Fuzzy Soft Matrix)

Let $A = [a_{ij}]_{m\times n}$, $a_{ij} = \mu_j(c_i)$; where $\mu_j(c_i)$ represents the fuzzy membership respectively of c_i .

Also let $k \in [0,1]$ be a scalar. Then kA is defined as -

$$kA = k[a_{ij}]_{m\times n}$$

$$= k[\mu_j(c_i)]_{m\times n}$$

$$= \left[\min\left(k, \mu_j(c_i)\right)\right]_{m\times n}$$

Example 3.4

Let $A = \begin{bmatrix} 0.1 & 0.4 & 0.3 \\ 0.2 & 0.9 & 0.2 \\ 0.6 & 0.1 & 1.0 \end{bmatrix}$ be a fuzzy soft matrix representing a fuzzy soft set and k = 0.3. Then

$$kA = 0.3A = \begin{bmatrix} \min(0.1,0.3) & \min(0.4,0.3) & \min(0.3,0.3) \\ \min(0.2,0.3) & \min(0.9,0.3) & \min(0.2,0.3) \\ \min(0.6,0.3) & \min(0.1,0.3) & \min(1.0,0.3) \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 & 0.3 & 0.3 \\ 0.2 & 0.3 & 0.2 \\ 0.3 & 0.1 & 0.3 \end{bmatrix}$$

Definition 3.5 (Fuzzy Soft Matrix Product)

Let us consider a fuzzy soft set over a universe having 'm' elements and the corresponding set of parameters

having 'n' parameters and let A = $[a_{ij}]_{m\times n}$, $a_{ij} = \mu_j(c_i)$; where $\mu_j(c_i)$ represents the fuzzy membership respectively of c_i be the corresponding fuzzy soft matrix.

Also let us consider another fuzzy soft set over a universe having 'n' elements and the corresponding set of parameters having 'p' parameters and let B = $[b_{jk}]_{n\times p}$, $b_{jk} = \chi_k(c_j)$; where $\chi_k(c_j)$ represents the fuzzy membership of c_j be the corresponding fuzzy soft matrix.

We now define $A.B$, the product of A and B as,

$$A.B = [d_{ik}]_{m\times p}$$

$$= \left[\max\min\left(\mu_j(c_i), \chi_k(c_j)\right)\right]_{m\times p}, 1 \le i \le m, 1 \le k \le p \text{ for } j = 1,2,\ldots.n$$

Example 3.5

Let $A = \begin{bmatrix} 0.7 & 0.3 & 0.1 & 0 \\ 0.1 & 0.8 & 0.2 & 0 \\ 0.2 & 0.4 & 0.7 & 0 \\ 0.6 & 0.5 & 0.3 & 0 \end{bmatrix}_{4\times 4}$ and $B = \begin{bmatrix} 0 & 0.3 & 0.8 \\ 0 & 0.4 & 0.6 \\ 0 & 0.1 & 0.7 \\ 0 & 0.2 & 0.3 \end{bmatrix}_{4\times 3}$ be two fuzzy soft matrices.

Then
$$A.B = \begin{bmatrix} \max(0,0,0,0) & \max(0.3,0.3,0.1,0) & \max(0.7,0.3,0.1,0) \\ \max(0,0,0,0) & \max(0.1,0.4,0.1,0) & \max(0.1,0.6,0.2,0) \\ \max(0,0,0,0) & \max(0.2,0.4,0.1,0) & \max(0.2,0.4,0.7,0) \\ \max(0,0,0,0) & \max(0.3,0.4,0.1,0) & \max(0.6,0.5,0.3,0) \end{bmatrix}_{4\times 3}$$

$$= \begin{bmatrix} 0 & 0.3 & 0.7 \\ 0 & 0.4 & 0.6 \\ 0 & 0.4 & 0.7 \\ 0 & 0.4 & 0.6 \end{bmatrix}_{4\times 3}$$

From the above example, it is seen that even if the product $A.B$ is defined, $B.A$ may not be defined. Also when both $A.B$ and $B.A$ are defined they may not be equal. The following example makes it clear.

Example 3.6

Let $A=\begin{bmatrix}0.1 & 0.4 & 0.3\\ 0.2 & 0.9 & 0.2\\ 0.6 & 0.1 & 1.0\end{bmatrix}$ and $B=\begin{bmatrix}0.1 & 0.4 & 0.3\\ 0.5 & 0.6 & 0.6\\ 0.3 & 0.2 & 0.0\end{bmatrix}$ be two fuzzy soft square matrices representing two fuzzy soft sets.

Then $A.B$ $=\begin{bmatrix}0.1 & 0.4 & 0.3\\ 0.2 & 0.9 & 0.2\\ 0.6 & 0.1 & 1.0\end{bmatrix}\cdot\begin{bmatrix}0.1 & 0.4 & 0.3\\ 0.5 & 0.6 & 0.6\\ 0.3 & 0.2 & 0.0\end{bmatrix}$

$$=\begin{bmatrix}0.4 & 0.4 & 0.4\\ 0.5 & 0.6 & 0.6\\ 0.3 & 0.4 & 0.3\end{bmatrix}$$

and $B.A$ $=\begin{bmatrix}0.1 & 0.4 & 0.3\\ 0.5 & 0.6 & 0.6\\ 0.3 & 0.2 & 0.0\end{bmatrix}\cdot\begin{bmatrix}0.1 & 0.4 & 0.3\\ 0.2 & 0.9 & 0.2\\ 0.6 & 0.1 & 1.0\end{bmatrix}$

$$=\begin{bmatrix}0.3 & 0.4 & 0.3\\ 0.6 & 0.6 & 0.6\\ 0.2 & 0.3 & 0.3\end{bmatrix}$$

We see that $A.B \neq B.A$

Example 3.7 (Positive Integral Powers of Fuzzy Soft Matrices)

Let $A=\begin{bmatrix}0.1 & 0.4 & 0.3\\ 0.2 & 0.9 & 0.2\\ 0.6 & 0.1 & 1.0\end{bmatrix}$ be a fuzzy soft square matrix.

Then $A\cdot A$ $=\begin{bmatrix}0.1 & 0.4 & 0.3\\ 0.2 & 0.5 & 0.2\\ 0.6 & 0.1 & 0.3\end{bmatrix}\cdot\begin{bmatrix}0.1 & 0.4 & 0.3\\ 0.2 & 0.5 & 0.2\\ 0.6 & 0.1 & 0.3\end{bmatrix}$

$$=\begin{bmatrix}0.3 & 0.4 & 0.3\\ 0.2 & 0.5 & 0.2\\ 0.3 & 0.4 & 0.3\end{bmatrix}$$

We would write $A.A = A^2 = \begin{bmatrix} 0.3 & 0.4 & 0.3 \\ 0.2 & 0.5 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$

Definition 3.6

Let $A = [a_{ij}]_{m\times n}$, $a_{ij} = \mu_j(c_i)$; where $\mu_j(c_i)$ represents the fuzzy membership respectively of c_i.

We define

$A^T = [a_{ij}^{\ T}]_{n\times m} \in FSM_{n\times m}$, where $a_{ij}^{\ T} = a_{ji}$.

Proposition 3.2

Let $A, B \in FSM_{m\times n}$. Then the following results hold.

(i) $\left(A^T\right)^T = A$

(ii) $(A \oplus B)^T = A^T \oplus B^T$

Proof:

(i) Let $A = [a_{ij}]_{m\times n}$

$= [\mu_j(c_i)]_{m\times n}$

Now, $A^T = \left[\mu_j(c_i)\right]_{m\times n}^{\ T}$

$= [\mu_i(c_j)]_{n\times m}$

$\left(A^T\right)^T = [\mu_i(c_j)]_{n\times m}^{\ T}$

$= [\mu_j(c_i)]_{m\times n}$

$= A$

(ii) Let $A = [a_{ij}]_{m\times n}$

$= [\mu_j(c_i)]_{m\times n}$

and $B = [b_{ij}]_{m\times n}$

$= [\chi_j(c_i)]_{m\times n}$

Now, $A \oplus B = [\max(\mu_j(c_i), \chi_j(c_i))]_{m\times n}$

$$(A \oplus B)^T = [\max(\mu_j(c_i), \chi_j(c_i))]_{m\times n}{}^T$$

$$= [\max(\mu_i(c_j), \chi_i(c_j))]_{n\times m}$$

$$= [\mu_i(c_j)]_{n\times m} \oplus [\chi_i(c_j)]_{n\times m}$$

$$= [\mu_j(c_i)]_{m\times n}{}^T \oplus [\chi_j(c_i)]_{m\times n}{}^T$$

$$= A^T \oplus B^T$$

Definition 3.7 (Fuzzy Soft Symmetric Matrix)

Let $A = [a_{ij}]_{m\times n}$, $a_{ij} = \mu_j(c_i)$; where $\mu_j(c_i)$ represents the fuzzy membership respectively of c_i, be a fuzzy soft square matrix. Then A is said to be a fuzzy soft symmetric matrix if $A^T = A$

Example 3.8

Let $A = \begin{bmatrix} 0.3 & 0.2 & 0.3 \\ 0.2 & 0.5 & 0.2 \\ 0.3 & 0.2 & 0.3 \end{bmatrix}$ be a fuzzy soft square matrix. We see that

$$A^T = \begin{bmatrix} 0.3 & 0.2 & 0.3 \\ 0.2 & 0.5 & 0.2 \\ 0.3 & 0.2 & 0.3 \end{bmatrix} = A$$

By definition, A is a fuzzy soft symmetric matrix.

Definition 3.8 (Fuzzy Soft Idempotent Matrix)

Let $A = [a_{ij}]_{m\times n}$, $a_{ij} = \mu_j(c_i)$; where $\mu_j(c_i)$ represents the fuzzy membership respectively of c_i, be a fuzzy soft square matrix. Then A is said to be a fuzzy soft idempotent matrix if $A^2 = A$

Example 3.9

Let $A = \begin{bmatrix} 0.3 & 0.4 & 0.2 \\ 0.2 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.3 \end{bmatrix}$ be a fuzzy soft square matrix.

Then $A \cdot A = A^2 = \begin{bmatrix} 0.3 & 0.4 & 0.2 \\ 0.2 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.3 \end{bmatrix} \cdot \begin{bmatrix} 0.3 & 0.4 & 0.2 \\ 0.2 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.3 \end{bmatrix}$

$$= \begin{bmatrix} 0.3 & 0.4 & 0.2 \\ 0.2 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.3 \end{bmatrix}$$

Thus $A.A \quad = A^2 \quad = \begin{bmatrix} 0.3 & 0.4 & 0.2 \\ 0.2 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.3 \end{bmatrix} = A$

It follows that A is a fuzzy soft idempotent matrix.

4. APPLICATION IN DECISION MAKING PROBLEM

Let (F, E) and (G, E) be two fuzzy soft sets representing a good car among the cars $U = \{c_1, c_2, c_3\}$ according to Mr. X and Mrs X. respectively.

Let $E = \{e_1(\text{stylish look}), e_2(\text{latest technology}), e_3(\text{comfortability})\}$ be the set of parameters.

$(F, E) \quad = \{F(e_1) = \{ (c_1, 0.7), (c_2,0.1), (c_3,0.2)\}, F(e_2) = \{ (c_1, 0.3), (c_2,0.8), (c_3,0.4)\},$

$F(e_3) = \{ (c_1, 0.1), (c_2,0.2), (c_3,0.7)\}\}$

$(G, E) \quad = \{G(e_1) = \{ (c_1, 0.3), (c_2,0.5), (c_3,0.3)\}, G(e_2) = \{ (c_1, 0.4), (c_2,0.6), (c_3,0.2)\},$

$G(e_3) = \{ (c_1, 0.4), (c_2,0.3), (c_3,0.6)\}\}$

These two fuzzy soft sets are represented by the following fuzzy matrices respectively.

$$A = \begin{bmatrix} 0.7 & 0.3 & 0.1 \\ 0.1 & 0.8 & 0.2 \\ 0.2 & 0.4 & 0.7 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0.3 & 0.4 & 0.4 \\ 0.5 & 0.6 & 0.3 \\ 0.3 & 0.2 & 0.6 \end{bmatrix}$$

The fuzzy soft sets representing not-good cars of the three cars $U = \{c_1, c_2, c_3\}$ are given by

$(F, E)^c \quad = \{F^c(e_1) = \{ (c_1, 0.3), (c_2,0.9), (c_3,0.8)\}, F^c(e_2) = \{ (c_1, 0.7), (c_2,0.2), (c_3,0.6)\},$

$F^c(e_3) = \{ (c_1, 0.9), (c_2,0.8), (c_3,0.3)\}\}$

$(G, E)^c = \{G^c(e_1) = \{ (c_1, 0.7), (c_2,0.5), (c_3,0.7)\}, G^c(e_2) = \{ (c_1, 0.6), (c_2,0.4),$

$(c_3,0.8)\},$

$G^c(e_3) = \{ (c_1, 0.6), (c_2,0.7), (c_3,0.4)\}\}$

These two fuzzy soft sets are represented by the following fuzzy matrices respectively -

$$\overline{A} = \begin{bmatrix} 0.3 & 0.7 & 0.9 \\ 0.9 & 0.2 & 0.8 \\ 0.8 & 0.6 & 0.3 \end{bmatrix} \quad \text{and} \quad \overline{B} = \begin{bmatrix} 0.7 & 0.6 & 0.6 \\ 0.5 & 0.4 & 0.7 \\ 0.7 & 0.8 & 0.4 \end{bmatrix}$$

Then the fuzzy soft matrix $A \oplus B$ represents the maximum membership of the good cars of the cars $U = \{c_1, c_2, c_3\}$.

$$A \oplus B = \begin{bmatrix} 0.7 & 0.4 & 0.4 \\ 0.5 & 0.8 & 0.3 \\ 0.3 & 0.4 & 0.7 \end{bmatrix}$$

Again the fuzzy soft matrix $\overline{A} \oplus \overline{B}$ represents the maximum membership of the not-good cars of the cars $U = \{c_1, c_2, c_3\}$.

$$\overline{A} \oplus \overline{B} = \begin{bmatrix} 0.7 & 0.7 & 0.9 \\ 0.9 & 0.4 & 0.8 \\ 0.8 & 0.8 & 0.4 \end{bmatrix}$$

We now calculate the score matrix $S_{((A+B),(\overline{A}+\overline{B}))}$ by $S_{(A+B,\overline{A}+\overline{B})} = [\eta_{ij}]_{m\times n}$ where $\eta_{ij} = \left|\delta_{(A+B)ij} - \delta_{(\overline{A}+\overline{B})ij}\right|$.

$$S_{(A+B,\overline{A}+\overline{B})} = \begin{bmatrix} 0.0 & 0.3 & 0.5 \\ 0.4 & 0.4 & 0.5 \\ 0.5 & 0.4 & 0.3 \end{bmatrix}$$

The total score of each car is calculated by Score $(c_i) = \sum \eta_{ij}$

Total score:

$$\text{Score}\,(c_1) = \sum \eta_{1j} = 0.0 + 0.3 + 0.5 = 0.8$$
$$\text{Score}\,(c_2) = \sum \eta_{2j} = 0.4 + 0.4 + 0.5 = 1.3$$
$$\text{Score}\,(c_3) = \sum \eta_{3j} = 0.5 + 0.4 + 0.3 = 1.2$$

We see that the car c_2 has maximum score and thus is the best car.

5. CONCLUSION

In our work, we have put forward some new notions of fuzzy soft matrices. Some related properties have been established with proof and examples. A decision problem has been considered to get the optimal solution with the help of fuzzy soft matrices. We hope that our work would enhance this study on fuzzy soft sets as well as matrices.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

[1] B. Ahmad, A. Kharal On Fuzzy Soft Sets, Adv. Fuzzy Syst. 2009 (2009), Article ID 586507.

[2] D.A. Molodstov, Soft Set Theory - First Result, Computers Math. Appl. 37 (1999), 19-31.

[3] L. A. Zadeh, Fuzzy sets, Inform. Control, 8 (3) (1965), 338-353.

[4] M. Borah, T.J. Neog and D.K. Sut, Fuzzy Soft Matrix Theory and its Decision Making, Int. J. Modern Eng. Res. 2 (2) (2012), 121-127.

[5] P. K. Maji, R. Biswas, A.R.Roy, Fuzzy Soft Sets, J. Fuzzy Math. 9 (3) (2001), 589 - 602.

[6] T.J. Neog, D.K. Sut, Theory of Fuzzy Soft Sets from a New Perspective, Int. J. Latest Trends Comput. 2(3) (2011), 439-450.

[7] T.J. Neog, M. Bora, D.K. Sut, On Fuzzy Soft Matrix Theory, Int. J. Math. Arch. 3 (2) (2012), 491-500.

[8] Y. Yang and C. Ji, Fuzzy Soft Matrices and Their Applications, Int. Conf. Artif. Intell. Comput. Intell. Springer, Berlin, Heidelberg, Part I, LNAI 7002, 618–627, 2011.