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NUMERICAL SOLUTION OF UNSTEADY TWO DIMENSIONAL DECELERATING FLOW OVER A WEDGE WITH TEMPERATURE DEPENDENT VISCOSITY

C.K. AJAY*, A.H. SRINIVASA

Department of Mathematics, Maharaja Institute of Technology Mysore, Belawadi-571477, India.

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Abstract: The present work brings into focus the numerical solution of unsteady two dimensional decelerating laminar boundary layer flow and heat transfer of an incompressible fluid above a moving wedge in the existence of variable viscosity. Suitable transformation is used to form a system of coupled nonlinear partial differential equations for governing both the flow and heat transfer. These equations have been solved numerically by using an implicit finite difference method in combination with quasilinearization strategy. The obtained numerical results have been presented graphically in terms of local nusselt number, skin friction, temperature distribution, and velocity distribution for different values of variable viscosity parameter (ε) along with Prandtl number (Pr). The boundary layer isolates itself from its surface, past which the dual results exist up to a critical value of an unsteady parameter (λ).

Keywords: decelerating flow; heat transfer; skin friction; variable viscosity.

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*Corresponding author

E-mail address: ajayarkesh@gmail.com

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1. INTRODUCTION

The boundary layer theory is important in many fields of engineering and real world setbacks. The main application of this theory is all about measuring the skin friction drag. Its main focus is on the way the friction drag acts on a body while it is moving through a fluid, for instance the flow over an airplane wing or past an entire ship. This led to Falkner-Skan [1] developing a model known as wedge flow based on the Prandtl boundary layer theory. Following it, a great deal of work has been done over the most recent couple of years by many investigators [2-13] on the Falkner-Skan problem which is considered a classic. It is carried out by employing various numerical and analytical methods for various types of flow including heat transfer conditions.

Of the previously mentioned investigations, the properties of the fluids were assumed to be constant. In many technical applications in the field of engineering, however, this assumption cannot be conformed to. Assumption of variable viscosity becomes necessary in considering such problems. It is accepted knowledge that there may be a major change in the physical properties of the fluid, whenever the temperature changes. (Take for example the fact that viscosity of water is seen to decrease by about 24% whenever the temperature is seen to increase from 10^0 to 50^0 c).

Herwing & Wickern [14] made the principal endeavor to solve the Falkner-Skan problem by having the variable viscosity also taking into account temperature. Hossain et.al [15] chose to study the fluid flow having variable viscosity moving past a permeable a uniform surface heat flux. Rudrakonta Deka et.al [16] have studied the impact of variable viscosity on flow past a porous wedge having suction or injection. Pantokratorns et.al [17] presented the Falkner-Skan flow having a variable viscosity and steady wall temperature.

The effect of unsteadiness is not considered in all the above published works. In the present study, the unsteady decelerating flow above a moving wedge having variable viscosity shall be analyzed.

2. MATHEMATICAL ANALYSIS

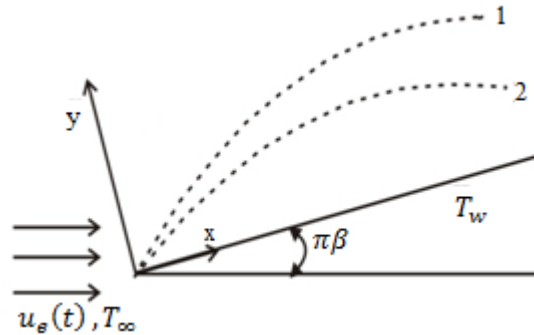


Figure 1 Coordinate system and Flow configuration for Falkner-Skan wedge flow, here edge of thermal and momentum boundary layers represents as 1 & 2, respectively.

Figure 1, shows unsteady incompressible laminar boundary layer flow above a moving wedge in a two dimensional manner. In this, the measurement of x is taken along the wedge surface with y being normal to it. Let u_e be the free stream velocity, which introduces the unsteadiness of the flow field and this is seen to vary inversely with time. The free stream temperature (T_∞) is lesser than the wall temperature (T_w) which is uniform and constant. With the exception of fluid viscosity (μ) which is assumed to be an inverse linear function of the temperature (T) the fluid is accepted to have constant physical properties.

According to the aforesaid assumption, the unsteady forced convection boundary layer flow equations over a moving wedge are,

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho} \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) \quad (2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

The boundary conditions are précised by

$$\begin{aligned} \text{at } y=0: & \quad u = v = 0 \text{ and } T = T_w \\ \text{as } y \rightarrow \infty: & \quad u \rightarrow U(x) = u_\infty (x/L)^m \text{ and } T = T_\infty \\ \text{at } x=0: & \quad u = u_\infty \text{ and } T = T_\infty \end{aligned} \quad (4)$$

In the present study, the viscosity of the form is shown using a semi-empirical formula

$$\frac{\mu}{\mu_{\infty}} = \frac{1}{1 + \gamma(T - T_{\infty})}$$

Which is created by Ling and Dybbs [18] and has been embraced, where γ is a constant and μ_{∞} is the viscosity of the ambient fluid.

Further,

$$\begin{aligned} u &= \frac{\partial \psi}{\partial y}; \quad v = -\frac{\partial \psi}{\partial x}; \\ f(\eta) &= \sqrt{\frac{1+m}{2}} \frac{L^m}{\nu u_{\infty}} \left(\frac{\psi}{x^{(1+m)/2}} \right) \\ \eta &= \sqrt{\frac{1+m}{2}} \frac{u_{\infty}}{\nu L^m} \left(\frac{y}{x^{(1-m)/2}} \right); \quad G(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}} \end{aligned} \quad (5)$$

Substituting the above transformations from equation (1) to (4), we acquire:

$$(1 + \varepsilon G)F'' - \varepsilon G'F' + (1 + \varepsilon G)^2 \left[\left(\frac{2m}{1+m} \right) (1 - F^2) + \lambda \left(\frac{2}{1+m} \right) \left(1 - F - \frac{\eta}{2} F' \right) + fF' \right] = 0 \quad (6)$$

$$\text{Pr}^{-1} G'' + fG' - \lambda \eta \left(\frac{1}{m+1} \right) G' = 0 \quad (7)$$

Where

$$\begin{aligned} \frac{u}{u_e} &= f' = F; \quad f = \int_0^{\eta} F d\eta \\ v &= -\sqrt{\frac{2}{1+m}} \sqrt{\frac{\nu u_{\infty}}{L^m}} (1 - \lambda t^*)^{-1/2} x^{(m-1)/2} \left[\frac{m+1}{2} f + \eta f' - \frac{m-1}{2} \right] \\ \text{Pr} &= \frac{\nu}{\alpha} \end{aligned} \quad (8)$$

It can next be seen that, in (6) and (7), the parameter m is connected with the apex angle $\pi\beta$ by the relation $m = \beta/(2 - \beta)$ or $\beta = 2m/(m+1)$.

The transformed boundary conditions are:

$$F = 0; \quad G = 1 \quad \text{at } \eta = 0$$

$$F = 1; \quad G = 0 \quad \text{as } \eta \rightarrow \infty \quad (9)$$

Here, $\varepsilon = (T_w - T_\infty)\gamma$ is named as the variation of viscosity parameter; ψ - dimensional stream function and f - dimensionless stream function; m - Falkner- Skan wedge power law parameter; F - dimensionless velocity and G - dimensionless temperature of the fluid; λ - unsteady parameter; Pr - Prandtl number; η - similarity variable. Here prime (') means derivative with respect to η .

Respectively, Skin friction and heat transfer coefficients as nusselt number, can be communicated, as

$$C_f (\text{Re}_L)^{1/2} = \frac{\tau_w}{\frac{1}{2} \rho u_e^2} = \frac{2 \sqrt{\frac{1+m}{2}} (F')_{\eta=0}}{1 + \varepsilon G}$$

$$Nu (\text{Re}_L)^{-1/2} = -k \frac{\left(\frac{\partial T}{\partial y} \right)_{y=0}}{(T_w - T_\infty)} = -\sqrt{\frac{1+m}{2}} (G')_{\eta=0} \quad (10)$$

Here the wall shear stress τ_w is given by $\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}$ where μ is dynamic viscosity, κ is

thermal conductivity and $\text{Re}_L = \frac{u_\infty L}{\nu}$ called the local Reynolds number.

3. RESULTS AND DISCUSSIONS

The coupled nonlinear partial differential Equations (6) and (7) are solved alongside the boundary conditions (9) by utilizing an implicit finite difference method. This is agreed out in combination with a quasilinearization technique. Since the technique is described in Inouye and Tate [20], in the interests of brevity, the description of the same has been omitted here. The numerical computations as shown in graphical representations of this paper have been carried out for different values of temperature dependent viscosity ($0 < \varepsilon < 1.0$), unsteady parameter (λ) and Falkner-Skan parameter (m). To validate the accuracy of our numerical technique, we have

compared skin friction (F'_w) and heat transfer (G'_w) parameters with those given by Watanabe [7] ranging from $0 \leq m \leq 1.0$ [as shown in Table 1] taking $Pr = 0.72$ for decelerating flow ($\lambda < 0$).

Table 1 Comparison of steady state ($\lambda = 0.0$) results for the range of m ($0 \leq m \leq 1.0$) when $\varepsilon = 0.0$ with those given by Watanabe [7].

m	F'_w		G'_w	
	Watanabe[7]	Present	Watanabe[7]	Present
0.0	0.46960	0.46961	0.41512	0.41511
0.014	0.50461	0.50460	0.42051	0.42050
0.0425	0.56898	0.56899	0.42984	0.42988
0.0909	0.65498	0.65499	0.44125	0.44124
0.1429	0.73200	0.73201	0.45042	0.45041
0.2	0.80213	0.80214	0.45826	0.45827
0.3333	0.92765	0.92768	0.47083	0.47084
1.0	1.23258	1.23259	0.49571	0.49570

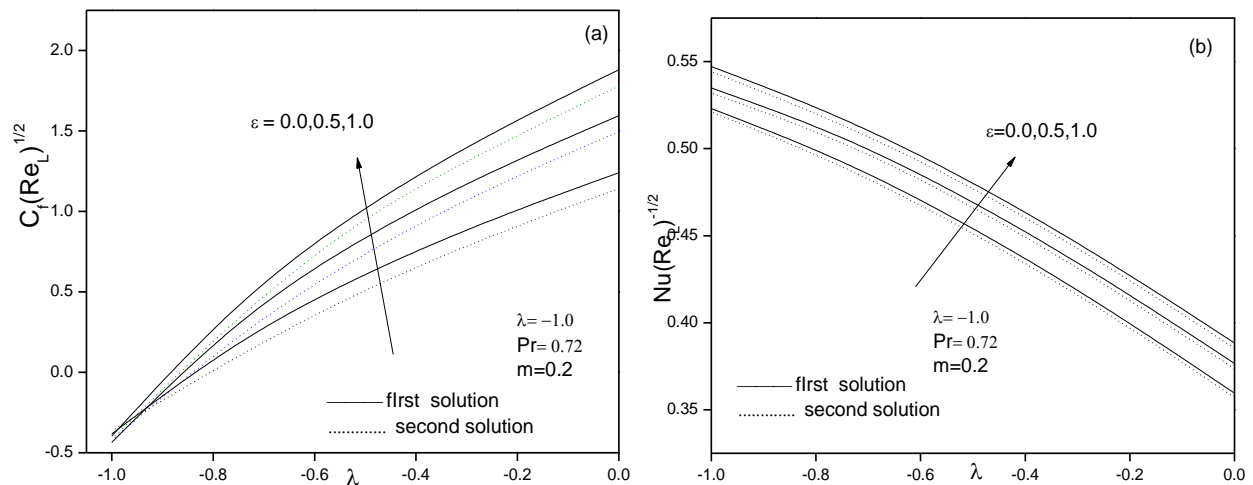


Figure 2 (a) skin friction & (b) heat transfer coefficients for various values of ε .

The impact of variable viscosity (ε) on skin friction $[C_f(\text{Re}_L)^{1/2}]$ and heat transfer $[N_u(\text{Re}_L)^{-1/2}]$ coefficients when $m = 0.2$ (60°), $\text{Pr} = (0.72)$ for $\lambda < 0$ (decelerating flow) is as shown in Figure 2. It is noticed that as variable viscosity increases, both $[C_f(\text{Re}_L)^{1/2}]$ and $[N_u(\text{Re}_L)^{-1/2}]$ are seen to increase quantitatively. The percentage of increase in $[C_f(\text{Re}_L)^{1/2}]$ is about 51.65% and in $[N_u(\text{Re}_L)^{-1/2}]$ is found to be 2.91% in the range $-1.0 \leq \lambda \leq 0.0$. It is intriguing to see the presence of dual solutions for both $[C_f(\text{Re}_L)^{1/2}]$ and $[N_u(\text{Re}_L)^{-1/2}]$, in the vary of λ ($\lambda_c < \lambda < 0$), & there is no result for $\lambda < \lambda_c$, here λ_c is a critical value of λ . Hence, the result exists up to a critical $\lambda = \lambda_c < 0$, further than, the boundary layer isolates from the wedge surface and the result depend on the boundary layer approximation is beyond the realm of imagination. Based on our calculation, the estimations of λ_c are -1.05 and -1.0 , correspondingly for $[C_f(\text{Re}_L)^{1/2}]$ and $[N_u(\text{Re}_L)^{-1/2}]$.

Figure 3 demonstrates the significant velocity and temperature distributions of the subsequent first and second solutions, when $\lambda = -1.0$. The second solution profile confirms the presence of dual solutions for decelerated flow. As variable viscosity increases it is unmistakably observed that both the thicknesses of thermal and momentum boundary layers are found to be diminishing. Further, these distributions fulfill the far field boundary conditions being an asymptote, which bolster the acquired numerical outcomes. It is commented that, the first outcomes are steady and physically feasible, where as the second solutions are most certainly not. Solutions such as these, though lacking physical significance are however seen to possess mathematical significance [19].

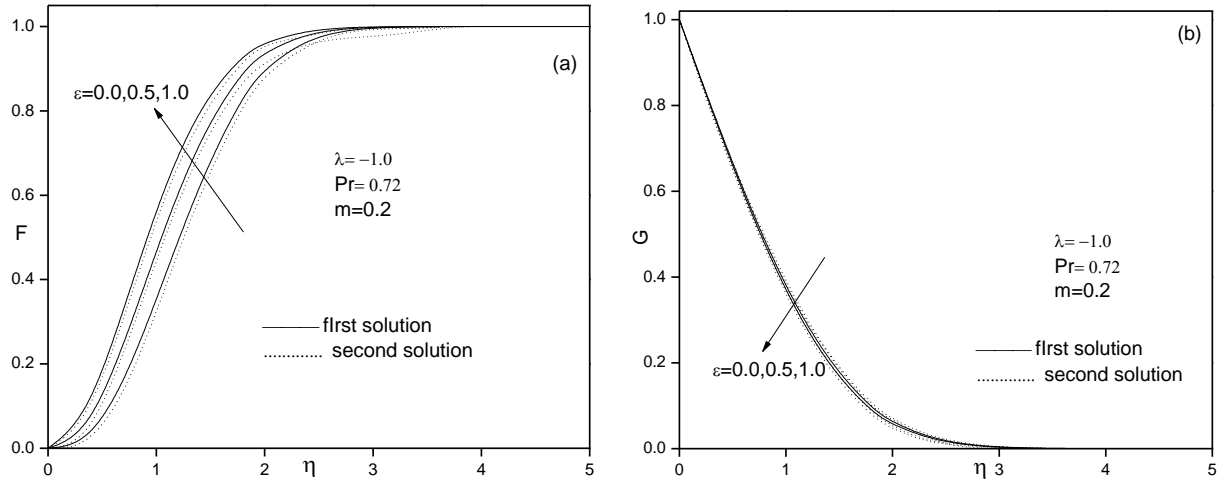


Figure 3 (a) Velocity (F) and (b) Temperature (G) distributions for various values of ϵ .

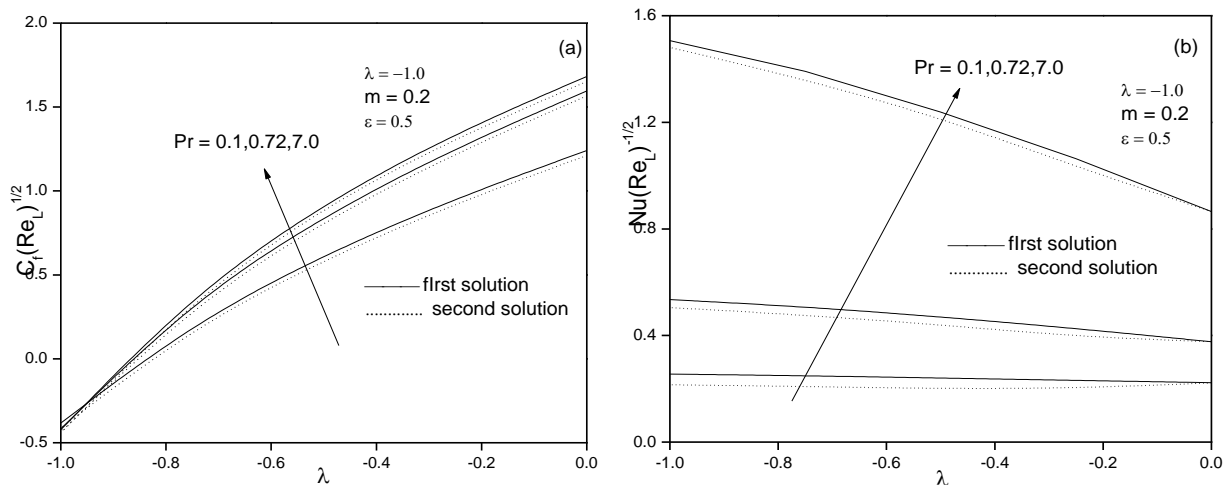


Figure 4 (a) skin friction and (b) heat transfer coefficients for various values of Prandtl numbers

Figure 4 displays the variation of $[C_f(Re_L)^{1/2}]$ and $[Nu(Re_L)^{-1/2}]$ for various values of Prandtl number ($Pr = 0.1, 0.72, 7.0$) for various unsteady parameters $\lambda < 0$, corresponding to wedge angle $m = 0.2$ (60°) and variable viscosity $\epsilon = 0.5$. It is clear that both $[C_f(Re_L)^{1/2}]$ and $[Nu(Re_L)^{-1/2}]$ increase with the expansion of Prandtl number. The level of enhance of skin friction is about 33.73% and heat transfer is around 86.15% for an increase of ϵ in the range $-1.0 \leq \lambda \leq 0.0$.

UNSTEADY TWO DIMENSIONAL DECELERATING FLOW OVER A WEDGE

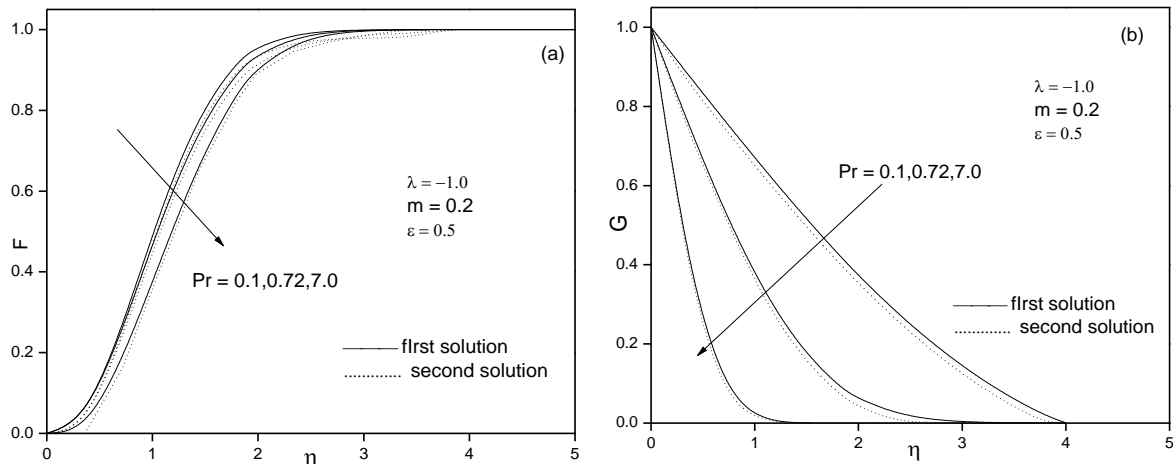


Figure 5. (a) Velocity (F) and (b) Temperature (G) distributions for various values of Prandtl numbers

Figure 5 depicts the effects of different Prandtl numbers (Pr) on velocity [F] and temperature [G] distributions. It is seen that both the velocity and temperature profile decrease with an increase of Prandtl number. The cause for such a manner is that the fluid which has higher Prandtl number has a generally low thermal conductivity which restricts conduction. This outcome shows the decrease of thickness the thermal boundary layer and subsequently a diminishing in the temperature distributions.

4. CONCLUSIONS

In the present study, the impact of variable viscosity on the unsteady decelerating flow of an incompressible fluid over a moving wedge has been investigated.

- The skin friction and heat transfer coefficient increases with an enhance of variable viscosity parameter ($\varepsilon = 0, 0.5, 1.0$) and the temperature distribution decreases but the opposite trend in velocity distribution for the fixed Prandtl number ($Pr = 0.72$) and the wedge angle ($m = 0.2$) was observed.

- Increasing the Prandtl number with the fixed variable viscosity ($\varepsilon = 0.5$) and wedge angle ($m = 0.2$) leads to an increase in both the coefficients of skin friction and heat transfer, where as both temperature and velocity profiles decreases.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

- [1] V.M. Falkner, S.W. Skan, Some approximate solutions of the boundary layer equations, *Philos. Mag.* 12 (1931), 865-896.
- [2] D.R. Hartree, On an equation occurring in Falkner and Skan's approximate treatment of the equations of the boundary layer, *Proc. Cambridge Philos. Soc.* 33 (1937), 223-239.
- [3] K. Stewartson, Further solutions the Falkner-Skan equation, *Proc. Cambridge Phil. Soc.*, 50 (1954), 454-465.
- [4] J.C.Y. Koh, J.P. Harnett, Skin friction and heat transfer for incompressible laminar flow over a porous wedge with suction and variable wall temperature, *Int. J. Heat Mass Transfer*, 2 (1961), 185-198.
- [5] K. K. Chen, P. A. Libby, Boundary layers with small departure from the Falkner-Skan profile, *J. Fluid Mech.* 33 (1968), 273-282.
- [6] H. T. Lin, L. K. Lin, Similarity solutions for laminar forced convection heat transfer from wedges to fluids of any Prandtl number, *Int. J. Heat Transfer*, 30 (1987), 1111-1118.
- [7] T. Watanabe, Thermal boundary layer over a wedge with uniform suction and injection in forced flow, *Acta Mech.* 83 (1990), 119-126.
- [8] A. Asaithambi, A finite -difference method for the Falkner- Skan equation, *Appl. Math. Comput.* 92 (1998), 135-141.

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- [9] A.T. Eswara, A parametric differentiated finite-difference study of Falkner-Skan problem, *Bull. Cal. Math. Soc.* 90 (1998), 191-196.
- [10] N.A. Yacob, A. Ishak, I. Pop, Falkner–Skan problem for a static or moving wedge in nano fluids. *Int. J. Therm. Sci.* 50 (2011), 133-139.
- [11] M. Turkyilmazoglu, Slip flow and heat transfer over a specific wedge: an exactly solvable Falkner-Skan equation. *J. Eng. Math.* 92(1) (2015), 73-81.
- [12] C.S.K. Raju, N. Sandeep, A comparative study on heat and mass transfer of the Blasius and Falkner-Skan flow of a bio-convective Casson fluid past a wedge, *Eur. Phys. J. Plus.* 131 (2016) 405.
- [13] G. Ashwini, A.T. Eswara, Unsteady MHD decelerating flow over a wedge with heat generation/absorption, *Int. J. Math. Comput. Sci.* 1 (2015), 303-309.
- [14] H. Herwig, G. Wickern, The effect of variable properties on laminar boundary layer flow, *Wärme- Und Stoffübertragung.* 20 (1986), 47–57.
- [15] Md.A. Hossain, Md.S. Munir, D.A.S. Rees, Flow of viscous incompressible fluid with temperature dependent viscosity and thermal conductivity past a permeable wedge with uniform surface heat flux, *Int. J. Therm. Sci.* 39 (2000), 635–644.
- [16] R.K. Deka, M. Basumatary, Effect of variable viscosity on flow past a porous wedge with suction or injection: new results, *Afr. Mat.* 26 (2015), 1263–1279.
- [17] A. Pantokratoras, The Falkner – Skan flow with constant wall temperature and variable viscosity, *Int. J. Therm. Sci.* 45 (2006), 378 – 389.
- [18] X. Ling, A. Dybbs, Force convection over a flat plate submersed in a porous medium: variable viscosity case, Conference: ASME winter meeting, Boston, MA, USA, 13 Dec 1987; Technical Paper 87-WA/HT-23.
- [19] H. Schlichting, K. Gersten, *Boundary Layer Theory.* Springer-Verlag, Berlin, 2000.
- [20] K. Inouye, A. Tate, Finite-Difference Version of Quasi-Linearization Applied to Boundary-Layer Equations, *AIAA J.* 12 (1974) 558–560.