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A NOTE ON FUZZY SOFT PARAOPEN SETS AND MAPS IN FUZZY SOFT TOPOLOGICAL SPACES

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Abstract. We introduce a new class of sets called fuzzy soft paraopen sets and paraclosed sets in fuzzy soft topological spaces and discussed some of their basic properties. We introduce and study a new class of maps such as fuzzy soft paracontinuous map, fuzzy soft para irresolute, fuzzy soft minimal paracontinuous, fuzzy soft maximal paracontinuous maps in fuzzy soft topological spaces and relation between them are investigated by suitable examples.

Keywords: fuzzy soft paraopen; fuzzy soft paraclosed; fuzzy soft paracontinuous; fuzzy soft $*$-paracontinuous; fuzzy soft para irresolute.

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1. Introduction

To overcome uncertainty in science and mathematics many theories have been proposed. There are several classical methods available to solve problems in sociology, environment, engineering, economics etc. L.A.Zadeh[1] in 1965, introduced the concept of fuzzy set theory

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which act as an appropriate framework to represent vague concepts by allowing partial membership. In 1968, C.L.Chang[2] initiated the new theory of fuzzy topology, which generalizes the basic notion of classical topology is helpful to solve the problems in practical life. In 1999, Molodtsov[3] established a new concept namely soft set theory which act as a mathematical tool to deal with uncertainty while modeling problems in sciences. A soft set is a collection of approximate description of an objects and it is free from the parameterization inadequacy syndrome. Soft topological spaces were initiated in the year 2011, by Shabir and Naz[4].Fuzzy soft set is a combination of fuzzy sets and soft sets, in which soft set is defined over fuzzy set is introduced by P.K.Maji[5] et al in 2001. Fuzzy soft sets are useful in solving the various uncertainties arising in the fields of engineering, social sciences, economics, environment, medical science etc. Fuzzy soft sets are very useful structures in solving many problems arising in real life.In 2011, Tanay et al.[6] introduced fuzzy soft topological spaces. In the year 2001 and 2003, F.Nakaoka and N.Oda [7,8], introduced and studied the properties of minimal open (resp. minimal closed) sets and maximal open (resp. maximal closed set) sets, which are subclasses of open and closed sets. In 2011, S.S.Benchalli, Basavaraj M. Ittanagi, R.S. Wali[10], introduced a new class of maps called maximal continuous, minimal continuous, maximal(minimal) irresolute maps in topological spaces and studied their relations with various type of continuous functions. In 2017, Chetana C and K.Naganagouda [15] introduced and studied soft minimal (maximal) continuous maps. Fuzzy soft minimal open and maximal open sets, fuzzy soft minimal (maximal) continuous maps, are introduced and investigated by Shakila.K and Selvi.R. In this paper, we introduce a new class of sets called fuzzy soft paraopen sets and paraclosed sets in fuzzy soft topological spaces and a new class of maps such as fuzzy soft paracontinuous map, fuzzy soft para irresolute, fuzzy soft minimal paracontinuous, fuzzy soft maximal paracontinuous maps in fuzzy soft topological spaces and relation between them are investigated by suitable examples.

2. PRELIMINARIES

Definition 2.1. [5] : Let $A \subset E$ and $F(X)$ be the set of all fuzzy sets in X. Then a pair (f,A) is called a fuzzy soft set over X, denoted by f_A, where $f : A \to F(X)$ is a function.

From the definition, it is clear that $f(a)$ is a fuzzy set in U, for each $a \in A$, and we will denote the membership function of $f(a)$ by $f_a : X \to [0,1]$.

Definition 2.2. [6] : Let τ be a collection of fuzzy soft sets over a universe X with a fixed parameter set E, then (f_E, τ) is called fuzzy soft topology if

(i) $\tilde{0}_E, \tilde{1}_E \in \tau$.

(ii) Union of any members of τ is a member of τ.

(iii) Intersection of any two members of τ is a member of τ.

Each member of τ is called fuzzy soft open set i.e. A fuzzy soft set f_A over X is fuzzy soft open if and only if $f_A \in \tau$. A fuzzy soft set f_A over X is called fuzzy soft closed set if the complement of f_A is fuzzy soft open set.

Definition 2.3. [17] : Let $(X, \tilde{\tau}, E)$ and $(Y, \tilde{\sigma}, K)$ be fuzzy soft topological spaces. Let $\rho : X \to Y$ and $\psi : E \to K$ be the two mappings and $g = (\rho, \psi)$ is said to be fuzzy soft continuous if the inverse image of every fuzzy soft open set in $(Y, \tilde{\sigma}, K)$ is fuzzy soft open in $(X, \tilde{\tau}, E)$ that is $g^{-1}(\tilde{\mu}) \in \tilde{\tau}$ for all $\tilde{\mu} \in \tilde{\sigma}$.

Definition 2.4. [8] : A proper nonempty open subset M of X is said to be a maximal open set if any open set which contains M is X or M.

Definition 2.5. [9] : A proper nonempty closed subset N of a topological space X is said to be maximal closed set if any closed set which contains N is X or N.

Definition 2.6. [7] : A proper non empty open subset M of X is said to be a minimal open set if any open set which is contained in M is ϕ or M.

Definition 2.7. [9] : A proper non empty closed subset N of X is said to be a minimal closed set if any closed set which is contained in N is ϕ or N.

Theorem 2.8. [9] : *Let X be a topological space and $F \subset X$. F is a minimal closed set if and only if $X - F$ is a maximal open set.*

Theorem 2.9. [8] : *Let X be a topological space and $F \subset X$. F is a maximal closed set if and only if $X - F$ is a minimal open set.*

Definition 2.10. [11] : Any open subset U of a topological space X is said to be a paraopen set if it is neither minimal open nor maximal open set.

Definition 2.11. [11] : Any closed subset F of a topological space X is said to be a paraclosed set if and only if its complement $(X-F)$ is paraopen set.

Definition 2.12. [16] : A proper nonempty fuzzy soft open subset f_E of X is said to be a fuzzy soft maximal open set if any fuzzy soft open set which contains f_E is X or f_E.

Definition 2.13. [16] : A proper nonempty fuzzy soft open subset f_E of X is said to be a fuzzy soft minimal open set if and only if any fuzzy soft open set which is contained in f_E is either ϕ or f_E.

3. Fuzzy soft Paraopen sets and their properties

Definition 3.1. A fuzzy soft subset U_E of a fuzzy soft topological space (X,E,τ) is said to be a fuzzy soft paraopen set if it is neither fuzzy soft minimal open nor fuzzy soft maximal open set. The family of all fuzzy soft paraopen sets in a fuzzy soft topological space (X,E,τ) is denoted by $FSP_aO(X)$. A closed subset V_E of a fuzzy soft topological space (X,E,τ) is said to be a fuzzy soft paraclosed set iff its complement $(X-V_E)$ is a fuzzy soft paraopen set. The family of all fuzzy soft paraclosed sets in a fuzzy soft topological space (X,E,τ) is denoted by $FSP_aC(X,E,\tau)$.

Note: Every fuzzy soft paraopen set is a fuzzy soft open set and fuzzy soft paraclosed set is a fuzzy soft closed set but converse is not true, is shown by the following example.

Example 3.2. Let $X=\{p,q,r\}, E=\{e_1,e_2\}$ be with
$\tau=\{\phi, X, [(e_1,(p_{0.5},q_{0.3},r_{0.2})),(e_2,(p_{0.3},q_{0.5},r_{0.2}))]$,
$[(e_1,(p_1,q_0,r_{0.5})),(e_2,(p_{0.5},q_{0.3},r_1))]$,
$[(e_1,(p_{0.5},q_0,r_{0.2})),(e_2,(p_{0.3},q_{0.3},r_{0.2}))]$,
$[(e_1,(p_1,q_{0.3},r_{0.5})),(e_2,(p_{0.5},q_{0.5},r_1))]\}$.
Then, $FSMiO(X,E,\tau)=\{[(e_1,(p_{0.5},q_0,r_{0.2})),(e_2,(p_{0.3},q_{0.3},r_{0.2}))]\}$,
$FSMaO(X,E,\tau)=\{[(e_1,(p_1,q_{0.3},r_{0.5})),(e_2,(p_{0.5},q_{0.5},r_1))]\}$,
$FSMiC(X,E,\tau)=\{[(e_1,(p_{0.5},q_1,r_{0.8})),(e_2,(p_{0.7},q_{0.7},r_{0.8}))]\}$,

$FSMaC(X,E,\tau) = \{[(e_1,(p_0,q_{0.7},r_{0.5})),(e_2,(p_{0.5},q_{0.5},r_0))]\}$,
$FSPaO(X,E,\tau) = \{[(e_1,(p_{0.5},q_{0.3},r_{0.2})),(e_2,(p_{0.3},q_{0.5},r_{0.2}))]$,
$[(e_1,(p_1,q_0,r_{0.5})),(e_2,(p_{0.5},q_{0.3},r_1))]\}$,
$FSPaC(X,E,\tau) = \{[(e_1,(p_{0.5},q_{0.7},r_{0.8})),(e_2,(p_{0.7},q_{0.5},r_{0.8}))]$,
$[(e_1,(p_0,q_1,r_{0.5})),(e_2,(p_{0.5},q_{0.7},r_0))]\}$.
Here, $\{[(e_1,(p_{0.5},q_0,r_{0.2})),(e_2,(p_{0.3},q_{0.3},r_{0.2}))]\}$ is a fuzzy soft open set but not a fuzzy soft paraopen set and $\{[(e_1,(p_{0.5},q_1,r_{0.8})),(e_2,(p_{0.7},q_{0.7},r_{0.8}))]\}$ is a fuzzy soft closed set but not a fuzzy soft paraclosed set.

Remark 3.3. Union and intersection of a fuzzy soft paraopen (resp. fuzzy soft paraclosed) sets need not be a fuzzy soft paraopen (resp. fuzzy soft paraclosed) set.

Example 3.4. In example 3.2, $\{[(e_1,(p_{0.5},q_{0.3},r_{0.2})),(e_2,(p_{0.3},q_{0.5},r_{0.2}))]\}$, $\{[(e_1,(p_1,q_0,r_{0.5})),(e_2,(p_{0.5},q_{0.3},r_1))]\}$ are fuzzy soft paraopen sets but its union $\{[(e_1,(p_1,q_{0.3},r_{0.5})),(e_2,(p_{0.5},q_{0.5},r_1))]\}$ and intersection $\{[(e_1,(p_{0.5},q_0,r_{0.2})),(e_2,(p_{0.3},q_{0.3},r_{0.2}))]\}$ which are not fuzzy soft paraopen sets.

Theorem 3.5. *Let (X,E,τ) be a fuzzy soft topological space with a nonempty fuzzy soft paraopen subset K_E of (X,E,τ). Then there exists a fuzzy soft minimal open set M_E such that $M_E \subseteq K_E$.*

Theorem 3.6. *Let (X,E,τ) be a fuzzy soft topological space with a proper fuzzy soft paraopen subset $K_E \subseteq (X,E,\tau)$, then there exists a fuzzy soft maximal open set N_E such that $K_E \subseteq N_E$.*

Theorem 3.7. *Let (X,E,τ) be a fuzzy soft topological space, then the following holds*

(i) If K_E is a fuzzy soft paraopen set and M_E is a fuzzy soft minimal open set then $K_E \bigcap M_E = \phi$ or $M_E \subset K_E$.

(ii) If K_E is a fuzzy soft paraopen set and N_E is a fuzzy soft maximal open set then $K_E \bigcup N_E = X$ or $K_E \subset N_E$.

(iii) If K_E and L_E are the fuzzy soft paraopen sets in (X,E,τ) then their intersection is either fuzzy soft paraopen set or fuzzy soft minimal open set.

Proof. (i) Let K_E be a fuzzy soft paraopen and M_E be a fuzzy soft minimal open set in (X,E,τ). If $K_E \bigcap M_E = \phi$, then the proof is obvious. Suppose $K_E \bigcap M_E \neq \phi, K_E \bigcap M_E$ is an open set and $K_E \bigcap M_E \subset M_E$. Therefore, $M_E \subset K_E$.

(ii) Let K_E be a fuzzy soft paraopen and N_E be a fuzzy soft maximal open set in (X,E,τ). If $K_E \bigcup N_E = \phi$, then the proof is obvious. Suppose $K_E \bigcup N_E \neq \phi, K_E \bigcup N_E$ is an open set and $N_E \subset K_E \bigcup N_E$. Since N_E is maximal open set $K_E \bigcup N_E = N_E \Rightarrow K_E \subset N_E$.

(iii) Let K_E and L_E be fuzzy soft paraopen sets in (X,E,τ). If $K_E \bigcap L_E$ is a fuzzy soft paraopen set, then the proof is obvious. Suppose $K_E \bigcap L_E$ is not a fuzzy soft paraopen set, then by definition $K_E \bigcap L_E$ is a fuzzy soft minimal open or fuzzy soft maximal open set. If $K_E \bigcap L_E$ is a fuzzy soft minimal open set then there is nothing to prove. Suppose $K_E \bigcap L_E$ is a fuzzy soft maximal open set, then $K_E \bigcap L_E \subset K_E$ and $K_E \bigcap L_E \subset L_E$ which is a contradiction to the fact that K_E and L_E are fuzzy soft paraopen sets. Hence, $K_E \bigcap L_E$ is a fuzzy soft minimal open set.

□

Theorem 3.8. *Let (X,E,τ) be a fuzzy soft topological space with a nonempty fuzzy soft paraclosed subset L_E of (X,E,τ) then the following holds*

(i) L_E is a fuzzy soft paraclosed set iff it is neither a fuzzy soft maximal closed set nor a fuzzy soft minimal closed set.

(ii) Then there exists a fuzzy soft minimal closed set N_E such that $N_E \subset L_E$.

(iii) Then there exists a fuzzy soft maximal closed set M_E such that $L_E \subset M_E$.

Proof. (i) The proof follows from the definition and by the fact that the complement of fuzzy soft minimal open set is fuzzy soft maximal closed set and the complement of fuzzy soft maximal open set is fuzzy soft minimal closed set.

(ii) By the definition of fuzzy soft minimal closed set, we have $N_E \subset L_E$.

(iii) $L_E \subset M_E$ is obvious by the definition of fuzzy soft maximal closed set.

□

Theorem 3.9. *Let (X,E,τ) be a fuzzy soft topological space, then the following holds* *(i) If L_E is a fuzzy soft paraclosed set and M_E is a fuzzy soft minimal closed set then $L_E \bigcap N_E = \phi$ or $N_E \subset L_E$.*

(ii) If L_E is a fuzzy soft paraclosed set and M_E is a fuzzy soft maximal closed set then $L_E \bigcup M_E = X$ or $L_E \subset M_E$.

(iii) If K_E and L_E are the fuzzy soft paraclosed sets in (X,E,τ) then their intersection is either fuzzy soft paraclosed set or fuzzy soft minimal closed set.

Proof. (i) Let L_E be a fuzzy soft paraclosed and N_E be a fuzzy soft minimal closed sets in (X,E,τ). Then, $(X-L_E)$ is a fuzzy soft paraopen and $(X-N_E)$ is a fuzzy soft minimal open sets in (X,E,τ). Then by theorem 3.7(ii), $(X-L_E)\bigcup(X-N_E)=X$ or $(X-L_E) \subset (X-N_E)$ which implies $X-(L_E \bigcap N_E)=X$ or $N_E \subset L_E$. Therefore, $L_E \bigcap N_E = \phi$ or $N_E \subset L_E$.

(ii) Let L_E be a fuzzy soft paraclosed and M_E be a fuzzy soft maximal closed sets in (X,E,τ). Then, $(X-L_E)$ is a fuzzy soft paraopen and $(X-M_E)$ is a fuzzy soft maximal open sets in (X,E,τ). Then by theorem 3.7(i), $(X-L_E)\bigcap(X-M_E)=\phi$ or $(X-M_E) \subset (X-L_E)$ which implies $X-(L_E \bigcup M_E)=\phi$ or $L_E \subset M_E$. Therefore, $L_E \bigcup M_E = X$ or $L_E \subset M_E$.

(iii) Let K_E and L_E be fuzzy soft paraclosed sets in (X,E,τ). If $K_E \bigcap L_E$ is a fuzzy soft paraclosed set, then the proof is obvious. Suppose $K_E \bigcap L_E$ is not a fuzzy soft paraopen set, then by definition 3.1, $K_E \bigcap L_E$ is a fuzzy soft minimal closed or fuzzy soft maximal closed set.If $K_E \bigcap L_E$ is a fuzzy soft minimal closed set then there is nothing to prove. Suppose $K_E \bigcap L_E$ is a fuzzy soft maximal closed set, then $K_E \subset K_E \bigcap L_E$ and $L_E \subset K_E \bigcap L_E$ which is a contradiction to the fact that K_E and L_E are fuzzy soft paraclosed sets. Hence, $K_E \bigcap L_E$ is a fuzzy soft minimal closed set.

□

4. FUZZY SOFT PARA CONTINUOUS MAPS

Definition 4.1. Let (X,E,τ) and (Y,E,μ) be fuzzy soft topological spaces, then a map $\mathscr{F}: (X,E,\tau) \to (Y,E,\mu)$ is called fuzzy soft para continuous if $\mathscr{F}^{-1}(K_E)$ is a fuzzy soft open set in (X,E,τ) for every fuzzy soft paraopen set K_E in (Y,E,μ). It is represented as FSp_a-continuous.

Definition 4.2. Let (X,E,τ) and (Y,E,μ) be fuzzy soft topological spaces, then a map $\mathscr{F}:(X,E,\tau)\to(Y,E,\mu)$ is called fuzzy soft $*$-para continuous if $\mathscr{F}^{-1}(K_E)$ is a fuzzy soft paraopen set in (X,E,τ) for every fuzzy soft open set K_E in (Y,E,μ). It is represented as $FS*-p_a$-continuous.

Definition 4.3. Let (X,E,τ) and (Y,E,μ) be fuzzy soft topological spaces, then a map $\mathscr{F}:(X,E,\tau)\to(Y,E,\mu)$ is called fuzzy soft para irresolute if $\mathscr{F}^{-1}(K_E)$ is a fuzzy soft paraopen set in (X,E,τ) for every fuzzy soft paraopen set K_E in (Y,E,μ). It is represented as FSp_a-irresolute.

Definition 4.4. Let (X,E,τ) and (Y,E,μ) be fuzzy soft topological spaces, then a map $\mathscr{F}:(X,E,\tau)\to(Y,E,\mu)$ is called fuzzy soft minimal para continuous if $\mathscr{F}^{-1}(M_E)$ is a fuzzy soft paraopen set in (X,E,τ) for every fuzzy soft minimal open set M_E in (Y,E,μ). It is represented as $FSmin$-p_a-continuous.

Definition 4.5. Let (X,E,τ) and (Y,E,μ) be fuzzy soft topological spaces, then a map $\mathscr{F}:(X,E,\tau)\to(Y,E,\mu)$ is called fuzzy soft maximal para continuous if $\mathscr{F}^{-1}(M_E)$ is a fuzzy soft paraopen set in (X,E,τ) for every fuzzy soft maximal open set M_E in (Y,E,μ). It is represented as $FSmax$-p_a-continuous.

Theorem 4.6. *(i) Every fuzzy soft continuous map is a fuzzy soft para continuous map.*

(ii) Every fuzzy soft$*$-para continuous map is fuzzy soft continuous map.

(iii) Every fuzzy soft$*$-para continuous map is fuzzy soft para continuous map.

(iv) Every fuzzy soft para irresolute map is fuzzy soft para continuous map.

(v) Every fuzzy soft$*$-para continuous map is fuzzy soft para irresolute map.

Proof. (i) Let $\mathscr{F}:(X,E,\tau)\to(Y,E,\mu)$ be a fuzzy soft continuous map. Let K_E be any fuzzy soft paraopen set in (Y,E,μ). Since every fuzzy soft paraopen set is a fuzzy soft open set, K_E is a fuzzy soft open set in (Y,E,μ). Since $\mathscr{F}$ is fuzzy soft continuous, $\mathscr{F}^{-1}(K_E)$ is a fuzzy soft open set in (X,E,τ). Therefore, $\mathscr{F}$ is a fuzzy soft paracontinuous.

(ii) Let $\mathscr{F}:(X,E,\tau)\to(Y,E,\mu)$ be a fuzzy soft$*$-paracontinuous map. Let K_E be any fuzzy soft open set in (Y,E,μ). Since $\mathscr{F}$ is fuzzy soft $*$-paracontinuous, $\mathscr{F}^{-1}(K_E)$ is a fuzzy soft paraopen set in (X,E,τ). Since every fuzzy soft paraopen set is a fuzzy soft open set,$\mathscr{F}^{-1}(K_E)$ is a fuzzy soft open set in (Y,E,μ). Therefore, $\mathscr{F}$ is a fuzzy soft continuous map.

(iii) The proof follows from (i) and (ii).

(iv) Let $\mathscr{F}:(X,E,\tau)\rightarrow(Y,E,\mu)$ be a fuzzy soft parairresolute map. Let K_E be any fuzzy soft paraopen set in (Y,E,μ). Since $\mathscr{F}$ is fuzzy soft parairresolute,$\mathscr{F}^{-1}(K_E)$ is a fuzzy soft paraopen set in (X,E,τ) and since every fuzzy soft paraopen set is a fuzzy soft open set,$\mathscr{F}^{-1}(K_E)$ is a fuzzy soft open set in (X,E,τ). Hence, $\mathscr{F}$ is a fuzzy soft para continuous map.

(v) Let $\mathscr{F}:(X,E,\tau)\rightarrow(Y,E,\mu)$ be a fuzzy soft $*$-paracontinuous map. Let K_E be any fuzzy soft paraopen set in (Y,E,μ). Since every fuzzy soft paraopen set is a fuzzy soft open set,K_E is a fuzzy soft open set in (Y,E,μ). $\mathscr{F}$ is fuzzy soft $*$-paracontinuous, which implies $\mathscr{F}^{-1}(K_E)$ is a fuzzy soft paraopen set in (X,E,τ). Hence, $\mathscr{F}$ is a fuzzy soft parairresolute map.

□

Remark 4.7. (i) Fuzzy soft paracontinuous map $\nRightarrow$ fuzzy soft continuous map.

Example: Let $X=Y=\{p,q,r\},E=\{e_1,e_2\}$ be with

$\tau=\{\phi,X,[(e_1,(p_{0.5},q_{0.3},r_{0.3})),(e_2,(p_{0.5},q_1,r_{0.5}))],$

$[(e_1,(p_{0.5},q_{0.2},r_{0.2})),(e_2,(p_{0.5},q_0,r_0))],$

$[(e_1,(p_{0.5},q_{0.3},r_{0.2})),(e_2,(p_{0.5},q_1,r_0))],$

$[(e_1,(p_{0.5},q_{0.2},r_{0.3})),(e_2,(p_{0.5},q_0,r_{0.5}))]\}.$

$\mu=\{\phi,Y,[(e_1,(p_{0.3},q_{0.2},r_{0.2})),(e_2,(p_0,q_0,r_0))],$

$[(e_1,(p_{0.6},q_{0.5},r_{0.5})),(e_2,(p_{0.7},q_1,r_{0.5}))],$

$[(e_1,(p_{0.5},q_{0.3},r_{0.3})),(e_2,(p_{0.5},q_1,r_{0.5}))],$

$[(e_1,(p_{0.5},q_{0.2},r_{0.2})),(e_2,(p_{0.5},q_0,r_0))]\}.$

$\mathscr{F}:(f_E,\tau)\rightarrow(g_E,\mu)$ be an identity map. Here,$\mathscr{F}$ is fuzzy soft paracontinuous but it is not a fuzzy soft continuous map, since the fuzzy soft open set

$[(e_1,(p_{0.3},q_{0.2},r_{0.2})),(e_2,(p_0,q_0,r_0))],$

$[(e_1,(p_{0.6},q_{0.5},r_{0.5})),(e_2,(p_{0.7},q_1,r_{0.5}))]$ in (Y,E,μ) is not a fuzzy soft open set in (X,E,τ).

(ii) Fuzzy soft continuous map $\nRightarrow$ fuzzy soft *-para continuous map.

Example: Let $X=Y=\{p,q,r\}, E=\{e_1,e_2\}$ be with

$\tau=\{\phi,X,[(e_1,(p_{0.3},q_{0.5},r_{0.2})),(e_2,(p_{0.2},q_{0.4},r_{0.6}))],$

$[(e_1,(p_{0.7},q_{0.8},r_{0.6})),(e_2,(p_{0.6},q_{0.9},r_{0.8}))],$

$[(e_1,(p_{0.4},q_{0.6},r_{0.4})),(e_2,(p_{0.3},q_{0.5},r_{0.7}))],$

$[(e_1,(p_{0.6},q_{0.7},r_{0.5})),(e_2,(p_{0.5},q_{0.7},r_{0.7}))]\}.$

$\mu=\{\phi,Y,[(e_1,(p_{0.3},q_{0.5},r_{0.2})),(e_2,(p_{0.2},q_{0.4},r_{0.6}))],$

$[(e_1,(p_{0.7},q_{0.8},r_{0.6})),(e_2,(p_{0.6},q_{0.9},r_{0.8}))],$

$[(e_1,(p_{0.4},q_{0.6},r_{0.4})),(e_2,(p_{0.3},q_{0.5},r_{0.7}))],$

$[(e_1,(p_{0.6},q_{0.7},r_{0.5})),(e_2,(p_{0.5},q_{0.7},r_{0.7}))]\}.$

$\mathscr{F}:(f_E,\tau)\to(g_E,\mu)$ be an identity map. Then, $\mathscr{F}$ is fuzzy soft continuous map but it is not a fuzzy soft $*$-para continuous map. Since the fuzzy soft open set $\{[(e_1,(p_{0.3},q_{0.5},r_{0.2})),(e_2,(p_{0.2},q_{0.4},r_{0.6}))]\}$ in (Y,E,μ) is not a fuzzy soft paraopen set in (X,E,τ).

(iii) Fuzzy soft para continuous map $\nRightarrow$ fuzzy soft $*$-para continuous map.

Example: In above example, $\mathscr{F}$ is fuzzy soft paracontinuous map but it is not a fuzzy soft $*$-para continuous map.

(iv) Fuzzy soft para continuous map $\nRightarrow$ fuzzy soft para irresolute map.

Example: Let $X=Y=\{p,q,r\}, E=\{e_1,e_2\}$ be with

$\tau=\{\phi,X,[(e_1,(p_{0.5},q_{0.6},r_{0.4})),(e_2,(p_{0.4},q_{0.5},r_{0.5}))],$

$[(e_1,(p_{0.6},q_{0.6},r_{0.8})),(e_2,(p_{0.8},q_{0.5},r_{0.7}))],$

$[(e_1,(p_{0.6},q_{0.7},r_{0.8})),(e_2,(p_{0.8},q_{0.9},r_{0.7}))],$

$[(e_1,(p_{0.5},q_{0.7},r_{0.4})),(e_2,(p_{0.4},q_{0.9},r_{0.5}))]\}.$

$\mu=\{\phi,Y,[(e_1,(p_{0.4},q_{0.5},r_{0.2})),(e_2,(p_{0.3},q_{0.2},r_{0.4}))],$

$[(e_1,(p_{0.6},q_{0.7},r_{0.8})),(e_2,(p_{0.8},q_{0.9},r_{0.7}))],$

$[(e_1,(p_{0.8},q_{0.7},r_{0.9})),(e_2,(p_{0.9},q_1,r_{0.7}))],$

$[(e_1,(p_{0.5},q_{0.6},r_{0.4})),(e_2,(p_{0.4},q_{0.5},r_{0.5}))]\}.$

$\mathscr{F} : (f_E, \tau) \to (g_E, \mu)$ be an identity map. Here, $\mathscr{F}$ is fuzzy soft paracontinuous but it is not a fuzzy soft parairresolute map, since the fuzzy soft paraopen set $\{[(e_1, (p_{0.6}, q_{0.7}, r_{0.8})), (e_2, (p_{0.8}, q_{0.9}, r_{0.7}))]$, $[(e_1, (p_{0.5}, q_{0.6}, r_{0.4})), (e_2, (p_{0.4}, q_{0.5}, r_{0.5}))]\}$ in (Y, E, μ) is not a fuzzy soft paraopen set in (X, E, τ).

(v) Fuzzy soft para irresolute map $\not\Rightarrow$ fuzzy soft $*$-para continuous map. Example: In 4.7, example(ii), $\mathscr{F}$ is fuzzy soft parairresolute map but it is not a $*$-para continuous map.

Theorem 4.8. *Every fuzzy soft minimal paracontinuous map is fuzzy soft minimal continuous but converse is not true.*

Proof. Let $\mathscr{F} : (X, E, \tau) \to (Y, E, \mu)$ be a fuzzy soft minimal paracontinuous map. Let N_E be any fuzzy soft minimal open set in (Y, E, μ). Since $\mathscr{F}$ is fuzzy soft minimal paracontinuous, $\mathscr{F}^{-1}(N_E)$ is a fuzzy soft paraopen set in (X, E, τ). Since every fuzzy soft paraopen set is a fuzzy soft open set, $\mathscr{F}^{-1}(N_E)$ is a fuzzy soft open set in (X, E, τ). Hence, $\mathscr{F}$ is a fuzzy soft minimal continuous. $\square$

Example 4.9. In 4.7 example(ii), $\mathscr{F}$ is fuzzy soft minimal continuous but it is not a fuzzy soft minimal paracontinuous, since the fuzzy soft minimal open set $[(e_1, (p_{0.3}, q_{0.5}, r_{0.2})), (e_2, (p_{0.2}, q_{0.4}, r_{0.6}))]$ in (Y, E, μ) is not a fuzzy soft paraopen set in (X, E, τ).

Remark 4.10. Fuzzy soft minimal para continuous and fuzzy soft para continuous (resp. fuzzy soft continuous) maps are independent of each other.

Example 4.11. Let $X = Y = \{p, q, r\}, E = \{e_1, e_2\}$ be with
$\tau = \{\phi, X, [(e_1, (p_{0.5}, q_{0.1}, r_{0.2})), (e_2, (p_{0.3}, q_{0.3}, r_{0.2}))]$,
$[(e_1, (p_{0.5}, q_0, r_{0.1})), (e_2, (p_{0.3}, q_{0.2}, r_{0.1}))]$,
$[(e_1, (p_{0.7}, q_{0.1}, r_{0.2})), (e_2, (p_{0.8}, q_{0.3}, r_{0.2}))]$,
$[(e_1, (p_{0.7}, q_0, r_{0.1})), (e_2, (p_{0.8}, q_{0.2}, r_{0.1}))]\}$.
$\mu = \{\phi, Y, [(e_1, (p_{0.5}, q_{0.3}, r_{0.2})), (e_2, (p_{0.3}, q_{0.5}, r_{0.2}))]$,
$[(e_1, (p_{0.9}, q_{0.1}, r_{0.5})), (e_2, (p_{0.5}, q_{0.3}, r_{0.9}))]$,

$[(e_1,(p_{0.5},q_{0.1},r_{0.2})),(e_2,(p_{0.3},q_{0.3},r_{0.2}))]$,
$[(e_1,(p_{0.9},q_{0.3},r_{0.5})),(e_2,(p_{0.5},q_{0.5},r_{0.9}))]\}$,
$\mathscr{F}:(f_E,\tau)\rightarrow(g_E,\mu)$ be an identity map. Here, $\mathscr{F}$ is a fuzzy soft minimal paracontinuous but it is not a fuzzy soft paracontinuous. In 4.7 example(ii),$\mathscr{F}$ is fuzzy soft paracontinuous but it is not a fuzzy soft minimal paracontinuous.

Theorem 4.12. *Every fuzzy soft maximal para continuous map is fuzzy soft maximal continuous but not conversly.*

Proof. Let $\mathscr{F}:(X,E,\tau)\rightarrow(Y,E,\mu)$ be a fuzzy soft maximal paracontinuous map. Let M_E be any fuzzy soft maximal open set in (Y,E,μ). Since $\mathscr{F}$ is fuzzy soft maximal paracontinuous,$\mathscr{F}^{-1}(M_E)$ is a fuzzy soft paraopen set in (X,E,τ). Since every fuzzy soft paraopen set is a fuzzy soft open set, $\mathscr{F}^{-1}(M_E)$ is a fuzzy soft open set in (X,E,τ). Hence, $\mathscr{F}$ is a fuzzy soft maximal continuous. $\square$

Example 4.13. In 4.7 example(ii),$\mathscr{F}$ is fuzzy soft maximal continuous but it is not a fuzzy soft maximal paracontinuous.

Remark 4.14. Fuzzy soft maximal para continuous and fuzzy soft para continuous (resp. fuzzy soft continuous) maps are independent of each other.

Example 4.15. Let $X=Y=\{p,q,r\},E=\{e_1,e_2\}$ be with
$\tau=\{\phi,X,[(e_1,(p_{0.9},q_{0.3},r_{0.5})),(e_2,(p_{0.5},q_{0.5},r_{0.9}))]$,
$[(e_1,(p_1,q_{0.7},r_{0.5})),(e_2,(p_{0.8},q_{0.6},r_{0.9}))]$,
$[(e_1,(p_{0.9},q_{0.3},r_{0.4})),(e_2,(p_{0.5},q_{0.5},r_{0.2}))]$,
$[(e_1,(p_1,q_{0.7},r_{0.4})),(e_2,(p_{0.8},q_{0.6},r_{0.9}))]\}$.
$\mu=\{\phi,Y,[(e_1,(p_{0.5},q_{0.3},r_{0.2})),(e_2,(p_{0.3},q_{0.5},r_{0.2}))]$,
$[(e_1,(p_{0.9},q_{0.1},r_{0.5})),(e_2,(p_{0.5},q_{0.3},r_{0.9}))]$,
$[(e_1,(p_{0.5},q_{0.1},r_{0.2})),(e_2,(p_{0.3},q_{0.3},r_{0.2}))]$,
$[(e_1,(p_{0.9},q_{0.3},r_{0.5})),(e_2,(p_{0.5},q_{0.5},r_{0.9}))]\}$
$\mathscr{F}:(f_E,\tau)\rightarrow(g_E,\mu)$ be an identity map. Here, $\mathscr{F}$ is a fuzzy soft maximal paracontinuous but it is not a fuzzy soft paracontinuous. In 4.7 example(ii),$\mathscr{F}$ is fuzzy soft paracontinuous but it is not a fuzzy soft maximal paracontinuous.

Remark 4.16. Fuzzy soft minimal para continuous and fuzzy soft maximal para continuous maps are independent of each other.

Example 4.17. In example 4.11,$\mathscr{F}$ is fuzzy soft minimal paracontinuous but it is not a fuzzy soft maximal paracontinuous. In example 4.15,$\mathscr{F}$ is fuzzy soft maximal paracontinuous but it is not a fuzzy soft minimal paracontinuous.

Theorem 4.18. *Let (X,E,τ) and (Y,E,μ) be fuzzy soft topological spaces, then a map $\mathscr{F}$: $(X,E,\tau) \to (Y,E,\mu)$ is fuzzy soft para continuous if and only if the inverse image of each fuzzy soft paraclosedset in (Y,E,μ) is a fuzzy soft closed set in (X,E,τ).*

Proof. The proof follows from the definition and fact that the complement of fuzzy soft paraopen set is fuzzy soft paraclosed set. □

Remark 4.19. The composition of fuzzy soft para continuous maps need not be fuzzy soft para continuous map.

Example 4.20. Let $X = Y = Z = \{p,q,r\}, E = \{e_1,e_2\}$ be with
$\tau = \{\phi, X, [(e_1,(p_{0.4},q_{0.3},r_{0.8})),(e_2,(p_{0.5},q_{0.4},r_{0.5}))],$
$[(e_1,(p_{0.6},q_{0.4},r_{0.9})),(e_2,(p_{0.7},q_{0.5},r_{0.6}))],$
$[(e_1,(p_{0.6},q_{0.3},r_{0.8})),(e_2,(p_{0.7},q_{0.4},r_{0.5}))],$
$[(e_1,(p_{0.4},q_{0.4},r_{0.9})),(e_2,(p_{0.5},q_{0.5},r_{0.6}))]\}.$
$\mu = \{\phi, Y, [(e_1,(p_{0.3},q_{0.2},r_{0.6})),(e_2,(p_{0.4},q_{0.3},r_{0.5}))],$
$[(e_1,(p_{0.7},q_{0.4},r_{0.9})),(e_2,(p_{0.8},q_{0.5},r_{0.6}))],$
$[(e_1,(p_{0.4},q_{0.3},r_{0.8})),(e_2,(p_{0.5},q_{0.4},r_{0.5}))],$
$[(e_1,(p_{0.6},q_{0.4},r_{0.9})),(e_2,(p_{0.7},q_{0.5},r_{0.6}))]\}.$
$\eta = \{\phi, Z, [(e_1,(p_{0.2},q_{0},r_{0.4})),(e_2,(p_{0.3},q_{0.1},r_{0.5}))],$
$[(e_1,(p_{0.8},q_{0.6},r_{1})),(e_2,(p_{1},q_{0.9},r_{0.7}))],$
$[(e_1,(p_{0.3},q_{0.2},r_{0.6})),(e_2,(p_{0.4},q_{0.3},r_{0.5}))],$
$[(e_1,(p_{0.7},q_{0.4},r_{0.9})),(e_2,(p_{0.8},q_{0.5},r_{0.6}))]\}.$
$\mathscr{F} : (f_E,\tau) \to (g_E,\mu)$ and $\mathscr{G} : (g_E,\mu) \to (h_E,\eta)$ be an identity maps. Then, $\mathscr{F}$ and $\mathscr{G}$ are fuzzy soft continuous maps but $\mathscr{G} \circ \mathscr{F} : (f_E,\tau) \to (h_E,\eta)$ is

not a fuzzy soft paracontinuous map. Since the fuzzy soft paraopen set $[(e_1,(p_{0.3},q_{0.2},r_{0.6})),(e_2,(p_{0.4},q_{0.3},r_{0.5}))],[(e_1,(p_{0.7},q_{0.4},r_{0.9})),(e_2,(p_{0.8},q_{0.5},r_{0.6}))]$ in (Z,E,η) is not a fuzzy soft open set in (X,E,τ).

Theorem 4.21. *If $\mathscr{F}:(X,E,\tau)\to(Y,E,\mu)$ is fuzzy soft continuous and $\mathscr{G}:(Y,E,\mu)\to(Z,E,\eta)$ is fuzzy soft paracontinuous maps, then $\mathscr{G}\circ\mathscr{F}:(X,E,\tau)\to(Z,E,\eta)$ is a fuzzy soft paracontinuous map.*

Proof. Let K_E be any fuzzy soft paraopen set in (Z,E,η). Since $\mathscr{G}$ is fuzzy soft paracontinuous,$\mathscr{G}^{-1}(K_E)$ is a fuzzy soft open set in (Y,E,μ). Since $\mathscr{F}$ is fuzzy soft continuous,$\mathscr{F}^{-1}(\mathscr{G}^{-1}(K_E))=(\mathscr{G}\circ\mathscr{F})^{-1}(K_E)$ is a fuzzy soft open set in (X,E,τ). Hence,$\mathscr{G}\circ\mathscr{F}$ is a fuzzy soft paracontinuous map. □

Theorem 4.22. *Let (X,E,τ) and (Y,E,μ) be fuzzy soft topological spaces, then a map $u:(X,E,\tau)\to(Y,E,\mu)$ is a fuzzy soft $*$-para continuous if and only if the inverse image of each fuzzy soft closed set in (Y,E,μ) is a fuzzy soft paraclosedset in (X,E,τ).*

Proof. The proof follows from the definition and fact that the complement of fuzzy soft paraopen set is fuzzy soft paraclosed set. □

Theorem 4.23. *If $\mathscr{F}:(X,E,\tau)\to(Y,E,\mu)$ and $\mathscr{G}:(Y,E,\mu)\to(Z,E,\eta)$ are fuzzy soft $*$-para continuous maps, then $\mathscr{G}\circ\mathscr{F}:(X,E,\tau)\to(Z,E,\eta)$ is a fuzzy soft $*$-para continuous.*

Proof. Let K_E be any fuzzy soft open set in (Z,E,η). Since $\mathscr{G}$ is fuzzy soft $*$-paracontinuous,$\mathscr{G}^{-1}(K_E)$ is a fuzzy soft paraopen set in (Y,E,μ). Since every fuzzy soft paraopen set is a fuzzy soft open set,$\mathscr{G}^{-1}(K_E)$ is a fuzzy soft open set in (Y,E,μ). Since $\mathscr{F}$ is fuzzy soft $*$-paracontinuous, $\mathscr{F}^{-1}(\mathscr{G}^{-1}(K_E))=(\mathscr{G}\circ\mathscr{F}^{-1}(K_E)$ is a fuzzy soft paraopen set in (X,E,τ). Hence,$\mathscr{G}\circ\mathscr{F}$ is a fuzzy soft $*$-paracontinuous. □

Theorem 4.24. *If $\mathscr{F}:(X,E,\tau)\to(Y,E,\mu)$ is fuzzy soft para continuous and $\mathscr{G}:(Y,E,\mu)\to(Z,E,\eta)$ is fuzzy soft $*$-para continuous maps. Then $\mathscr{G}\circ\mathscr{F}:(X,E,\tau)\to(Z,E,\eta)$ is a fuzzy soft para continuous (resp. fuzzy soft continuous) map.*

Proof. Let K_E be any fuzzy soft paraopen (resp. fuzzy soft open) set in (Z,E,η). Since every fuzzy soft paraopen set is a fuzzy soft open set, K_E is a fuzzy soft open set in (Z,E,η) and $\mathscr{G}$ is fuzzy soft $*$-paracontinuous implies $\mathscr{G}^{-1}(K_E)$ is a fuzzy soft paraopen set in (Y,E,μ). Since $\mathscr{F}$ is fuzzy soft $*$-paracontinuous, $\mathscr{F}^{-1}(\mathscr{G}^{-1}(K_E)) = (\mathscr{G}\circ\mathscr{F})^{-1}(K_E)$ is a fuzzy soft paraopen set in (X,E,τ). Hence, $\mathscr{G}\circ\mathscr{F}$ is a fuzzy soft $*$-paracontinuous (resp. fuzzy soft continuous) map. □

Theorem 4.25. *Let (X,E,τ) and (Y,E,μ) be fuzzy soft topological spaces. A map $\mathscr{F}$: $(X,E,\tau)\to(Y,E,\mu)$ is a fuzzy soft para irresolute if and only if the inverse image of each fuzzy soft paraclosed set in (Y,E,μ) is a fuzzy soft paraclosedset in (X,E,τ).*

Proof. The proof follows from the definition and fact that the complement of fuzzy soft paraopen set is fuzzy soft paraclosed set. □

Theorem 4.26. *If $\mathscr{F}:(X,E,\tau)\to(Y,E,\mu)$ is fuzzy soft paracontinuous and $\mathscr{G}:(Y,E,\mu)\to(Z,E,\eta)$ is fuzzy soft para irresolute maps, then $\mathscr{G}\circ\mathscr{F}:(X,E,\tau)\to(Z,E,\eta)$ is a fuzzy soft paracontinuous map.*

Proof. Let K_E be any fuzzy soft paraopen set in (Z,E,η). Since $\mathscr{G}$ is fuzzy soft para irresolute, $\mathscr{G}^{-1}(K_E)$ is a fuzzy soft paraopen set in (Y,E,μ). Since $\mathscr{F}$ is fuzzy soft paracontinuous, $\mathscr{F}^{-1}(\mathscr{G}^{-1}(K_E)) = (\mathscr{G}\circ\mathscr{F})^{-1}(K_E)$ is a fuzzy soft open set in (X,E,τ). Hence, $\mathscr{G}\circ\mathscr{F}$ is a fuzzy soft paracontinuous map. □

Theorem 4.27. *If $\mathscr{F}:(X,E,\tau)\to(Y,E,\mu)$ and $\mathscr{G}:(Y,E,\mu)\to(Z,E,\eta)$ are fuzzy soft para irresolute maps, then $\mathscr{G}\circ\mathscr{F}:(X,E,\tau)\to(Z,E,\eta)$ is a fuzzy soft para irresolute map.*

Proof. Let K_E be any fuzzy soft paraopen set in (Z,E,η). Since $\mathscr{G}$ is fuzzy soft para irresolute, $\mathscr{G}^{-1}(K_E)$ is a fuzzy soft paraopen set in (Y,E,μ). Since $\mathscr{F}$ is fuzzy soft para irresolute, $\mathscr{F}^{-1}(\mathscr{G}^{-1}(K_E)) = (\mathscr{G}\circ\mathscr{F})^{-1}(K_E)$ is a fuzzy soft para open set in (X,E,τ). Hence, $\mathscr{G}\circ\mathscr{F}$ is a fuzzy soft para irresolute map. □

Theorem 4.28. *If $\mathscr{F}:(X,E,\tau)\to(Y,E,\mu)$ is fuzzy soft$*$-para continuous and $\mathscr{G}:(Y,E,\mu)\to(Z,E,\eta)$ is fuzzy soft para irresolute maps, then $\mathscr{G}\circ\mathscr{F}:(X,E,\tau)\to(Z,E,\eta)$ is a fuzzy soft para irresolute map.*

Proof. Let K_E be any fuzzy soft paraopen set in (Z,E,η). Since $\mathscr{G}$ is fuzzy soft para irresolute, $\mathscr{G}^{-1}(K_E)$ is a fuzzy soft paraopen set in (Y,E,μ). Since every fuzzy soft paraopen set is a fuzzy soft open set, $\mathscr{G}^{-1}(K_E)$ is a fuzzy soft open set in (Y,E,μ). Since $\mathscr{F}$ is fuzzy soft $*$-para continuous, $\mathscr{F}^{-1}(\mathscr{G}^{-1}(K_E)) = (\mathscr{G}\circ\mathscr{F}^{-1}(K_E)$ is a fuzzy soft paraopen set in (X,E,τ). Hence, $\mathscr{G}\circ\mathscr{F}$ is a fuzzy soft para irresolute map. □

Theorem 4.29. *If $\mathscr{F}:(X,E,\tau)\to(Y,E,\mu)$ is fuzzy soft para irresolute and $\mathscr{G}:(Y,E,\mu)\to(Z,E,\eta)$ is fuzzy soft $*$-para continuous maps, then $\mathscr{G}\circ\mathscr{F}:(X,E,\tau)\to(Z,E,\eta)$ is a fuzzy soft para irresolute map.*

Proof. Let K_E be any fuzzy soft paraopen set in (Z,E,η). Since every fuzzy soft paraopen set is a fuzzy soft open set (Z,E,η). Since $\mathscr{G}$ is fuzzy soft $*$-para continuous, $\mathscr{G}^{-1}(K_E)$ is a fuzzy soft paraopen set in (Y,E,μ). Since $\mathscr{F}$ is fuzzy soft para irresolute, $\mathscr{F}^{-1}(\mathscr{G}^{-1}(K_E)) = (\mathscr{G}\circ\mathscr{F})^{-1}(K_E)$ is a fuzzy soft paraopen set in (X,E,τ). Hence, $\mathscr{G}\circ\mathscr{F}$ is a fuzzy soft para irresolute map. □

Theorem 4.30. *Let (X,E,τ) and (Y,E,μ) be fuzzy soft topological spaces. A map $\mathscr{F}:(X,E,\tau)\to(Y,E,\mu)$ is a fuzzy soft minimal para continuous if and only if the inverse image of each fuzzy soft maximal closed set in (Y,E,μ) is a fuzzy soft paraclosedset in (X,E,τ).*

Proof. The proof follows from the definition and fact that the complement of fuzzy soft minimal open set is fuzzy soft maximal closed set and the complement of fuzzy soft paraopen set is fuzzy soft paraclosed set. □

Remark 4.31. The composition of fuzzy soft minimal para continuous maps need not be fuzzy soft minimal para continuous map.

Example 4.32. Let $X=Y=Z=\{p,q,r\}, E=\{e_1,e_2\}$ be with
$\tau=\{\phi,X,[(e_1,(p_{0.6},q_{0.4},r_{0.7})),(e_2,(p_{0.5},q_{0.6},r_{0.3}))],$
$[(e_1,(p_{0.4},q_{0.3},r_{0.5})),(e_2,(p_{0.3},q_{0.5},r_{0.2}))],$
$[(e_1,(p_{0.3},q_{0.2},r_{0})),(e_2,(p_{0.1},q_{0.4},r_{0.2}))]\}.$
$\mu=\{\phi,Y,[(e_1,(p_{0.4},q_{0.3},r_{0.5})),(e_2,(p_{0.3},q_{0.5},r_{0.2}))],$
$[(e_1,(p_{0.9},q_{0.7},r_{0.8})),(e_2,(p_{0.6},q_{0.7},r_{0.8}))],$

$[(e_1,(p_{0.6},q_{0.4},r_{0.7})),(e_2,(p_{0.5},q_{0.6},r_{0.3}))]\}$.

$\eta=\{\phi,Z,[(e_1,(p_{0.6},q_{0.4},r_{0.7})),(e_2,(p_{0.5},q_{0.6},r_{0.3}))]$,

$[(e_1,(p_{0.9},q_{0.6},r_{0.8})),(e_2,(p_{0.7},q_{0.8},r_{0.5}))]$,

$[(e_1,(p_{0.6},q_{0.6},r_{0.7})),(e_2,(p_{0.5},q_{0.8},r_{0.3}))],[(e_1,(p_{0.9},q_{0.4},r_{0.8})),(e_2,(p_{0.7},q_{0.6},r_{0.5}))]\}$.

$\mathscr{F}:(f_E,\tau)\rightarrow(g_E,\mu)$ and $\mathscr{G}:(g_E,\mu)\rightarrow(h_E,\eta)$ be an identity maps. Then, $\mathscr{F}$ and $\mathscr{G}$ are fuzzy soft minimal paracontinuous maps but $\mathscr{G}\circ\mathscr{F}:(f_E,\tau)\rightarrow(h_E,\eta)$ is not a fuzzy soft minimal paracontinuous map. Since the fuzzy soft minimal open set $[(e_1,(p_{0.6},q_{0.4},r_{0.7})),(e_2,(p_{0.5},q_{0.6},r_{0.3}))]$ in (Z,E,η) is not a fuzzy soft paraopen set in (X,E,τ).

Theorem 4.33. *If $\mathscr{F}:(X,E,\tau)\rightarrow(Y,E,\mu)$ is fuzzy soft para irresolute and $\mathscr{G}:(Y,E,\mu)\rightarrow(Z,E,\eta)$ is fuzzy soft minimal para continuous maps, then $\mathscr{G}\circ\mathscr{F}:(X,E,\tau)\rightarrow(Z,E,\eta)$ is a fuzzy soft minimal para continuous.*

Proof. Let N_E be any fuzzy soft minimal open set in (Z,E,η). Since $\mathscr{G}$ is fuzzy soft minimal paracontinuous, $\mathscr{G}^{-1}(N_E)$ is a fuzzy soft paraopen set in (Y,E,μ). Since $\mathscr{F}$ is fuzzy soft parairresolute, $\mathscr{F}^{-1}(\mathscr{G}^{-1}(N_E))=(\mathscr{G}\circ\mathscr{F})^{-1}(N_E)$ is a fuzzy soft paraopen set in (X,E,τ). Hence, $\mathscr{G}\circ\mathscr{F}$ is a fuzzy soft minimal paracontinuous. □

Theorem 4.34. *If $\mathscr{F}:(X,E,\tau)\rightarrow(Y,E,\mu)$ is fuzzy soft para continuous and $\mathscr{G}:(Y,E,\mu)\rightarrow(Z,E,\eta)$ is fuzzy soft minimal para continuous maps, then $\mathscr{G}\circ\mathscr{F}:(X,E,\tau)\rightarrow(Z,E,\eta)$ is a fuzzy soft minimal continuous.*

Proof. Let N_E be any fuzzy soft minimal open set in (Z,E,η). Since $\mathscr{G}$ is fuzzy soft minimal paracontinuous, $\mathscr{G}^{-1}(N_E)$ is a fuzzy soft paraopen set in (Y,E,μ). Since $\mathscr{F}$ is fuzzy soft paracontinuous, $\mathscr{F}^{-1}(\mathscr{G}^{-1}(N_E))=(\mathscr{G}\circ\mathscr{F})^{-1}(N_E)$ is a fuzzy soft open set in (X,E,τ). Hence, $\mathscr{G}\circ\mathscr{F}$ is a fuzzy soft minimal paracontinuous. □

Theorem 4.35. *If $\mathscr{F}:(X,E,\tau)\rightarrow(Y,E,\mu)$ is fuzzy soft para irresolute and $\mathscr{G}:(Y,E,\mu)\rightarrow(Z,E,\eta)$ is fuzzy soft $*$-para continuous maps, then $\mathscr{G}\circ\mathscr{F}:(X,E,\tau)\rightarrow(Z,E,\eta)$ is a fuzzy soft minimal para continuous map.*

Proof. Let N_E be any fuzzy soft minimal open set in (Z,E,η). Since every fuzzy soft minimal open set is a fuzzy soft open set, N_E is a fuzzy soft open set in (Z,E,η). Since $\mathscr{G}$ is fuzzy soft $*$-paracontinuous, $\mathscr{G}^{-1}(N_E)$ is a fuzzy soft paraopen set in (Y,E,μ). Since $\mathscr{F}$ is fuzzy soft para irresolute, $\mathscr{F}^{-1}(\mathscr{G}^{-1}(N_E)) = (\mathscr{G}\circ\mathscr{F})^{-1}(N_E)$ is a fuzzy soft paraopen set in (X,E,τ). Hence, $\mathscr{G}\circ\mathscr{F}$ is a fuzzy soft minimal paracontinuous map. □

Theorem 4.36. *Let (X,E,τ) and (Y,E,μ) be fuzzy soft topological spaces. A map $\mathscr{F}$: $(X,E,\tau) \to (Y,E,\mu)$ is a fuzzy soft maximal para continuous if and only if the inverse image of each fuzzy soft minimal closed set in (Y,E,μ) is a fuzzy soft paraclosed set in (X,E,τ).*

Proof. The proof follows from the definition and fact that the complement of fuzzy soft maximal open set is fuzzy soft minimal closed set and the complement of fuzzy soft paraopen set is fuzzy soft paraclosed set. □

Remark 4.37. The composition of fuzzy soft maximal para continuous maps need not be a fuzzy soft maximal para continuous map.

Example 4.38. Let $X = Y = Z = \{p,q,r\}, E = \{e_1,e_2\}$ be with
$\tau = \{\phi, X, [(e_1,(p_{0.6},q_{0.7},r_{0.4})),(e_2,(p_{0.8},q_{0.4},r_{0.6}))],$
$[(e_1,(p_{0.7},q_{0.8},r_{0.5})),(e_2,(p_{0.9},q_{0.6},r_{0.7}))],$
$[(e_1,(p_{0.8},q_{1},r_{0.6})),(e_2,(p_{1},q_{0.7},r_{0.9}))]\}.$
$\mu = \{\phi, Y, [(e_1,(p_{0.3},q_{0.5},r_{0})),(e_2,(p_{0.5},q_{0.2},r_{0.3}))],$
$[(e_1,(p_{0.6},q_{0.7},r_{0.4})),(e_2,(p_{0.8},q_{0.4},r_{0.6}))],$
$[(e_1,(p_{0.7},q_{0.8},r_{0.5})),(e_2,(p_{0.9},q_{0.6},r_{0.3}))]\}.$
$\eta = \{\phi, Z, [(e_1,(p_{0.6},q_{0.7},r_{0.4})),(e_2,(p_{0.8},q_{0.4},r_{0.6}))],$
$[(e_1,(p_{0.4},q_{0.3},r_{0.1})),(e_2,(p_{0.5},q_{0.2},r_{0.3}))],$
$[(e_1,(p_{0.6},q_{0.3},r_{0.1})),(e_2,(p_{0.8},q_{0.2},r_{0.3}))],[(e_1,(p_{0.4},q_{0.7},r_{0.4})),(e_2,(p_{0.5},q_{0.4},r_{0.6}))]\}.$
$\mathscr{F} : (f_E,\tau) \to (g_E,\mu)$ and $\mathscr{G} : (g_E,\mu) \to (h_E,\eta)$ be an identity maps. Then, $\mathscr{F}$ and $\mathscr{G}$ are fuzzy soft maximal paracontinuous maps but $\mathscr{G}\circ\mathscr{F} : (f_E,\tau) \to (h_E,\eta)$ is not a fuzzy soft maximal paracontinuous map. Since the fuzzy soft maximal open set $[(e_1,(p_{0.6},q_{0.7},r_{0.4})),(e_2,(p_{0.8},q_{0.4},r_{0.6}))]$ in (Z,E,η) is not a fuzzy soft paraopen set in (X,E,τ).

Theorem 4.39. *If $\mathscr{F}:(X,E,\tau)\rightarrow(Y,E,\mu)$ is fuzzy soft para irresolute and $\mathscr{G}:(Y,E,\mu)\rightarrow(Z,E,\eta)$ is fuzzy soft maximal para continuous maps, then $\mathscr{G}\circ\mathscr{F}:(X,E,\tau)\rightarrow(Z,E,\eta)$ is a fuzzy soft maximal para continuous map.*

Proof. Let M be any fuzzy soft maximal open set in (Z,E,η). Since $\mathscr{G}$ is fuzzy soft maximal paracontinuous, $\mathscr{G}^{-1}(M_E)$ is a fuzzy soft paraopen set in (Y,E,μ). Since $\mathscr{F}$ is fuzzy soft parairresolute, $\mathscr{F}^{-1}(\mathscr{G}^{-1}(M_E))=(\mathscr{G}\circ\mathscr{F}^{-1}(M_E)$ is a fuzzy soft paraopen set in (X,E,τ). Hence, $\mathscr{G}\circ\mathscr{F}$ is a fuzzy soft maximal paracontinuous map. □

Theorem 4.40. *If $\mathscr{F}:(X,E,\tau)\rightarrow(Y,E,\mu)$ is fuzzy soft para continuous and $\mathscr{G}:(Y,E,\mu)\rightarrow(Z,E,\eta)$ is fuzzy soft maximal para continuous maps, then $\mathscr{G}\circ\mathscr{F}:(X,E,\tau)\rightarrow(Z,E,\eta)$ is a fuzzy soft maximal continuous map.*

Proof. Let M_E be any fuzzy soft maximal open set in (Z,E,η). Since $\mathscr{G}$ is fuzzy soft maximal paracontinuous, $\mathscr{G}^{-1}(M_E)$ is a fuzzy soft paraopen set in (Y,E,μ). Since $\mathscr{F}$ is fuzzy soft paracontinuous, $\mathscr{F}^{-1}(\mathscr{G}^{-1}(M_E))=(\mathscr{G}\circ\mathscr{F})^{-1}(M_E)$ is a fuzzy soft open set in (X,E,τ). Hence, $\mathscr{G}\circ\mathscr{F}$ is a fuzzy soft maximal continuous map. □

Theorem 4.41. *If $\mathscr{F}:(X,E,\tau)\rightarrow(Y,E,\mu)$ is fuzzy soft para irresolute and $\mathscr{G}:(Y,E,\mu)\rightarrow(Z,E,\eta)$ is fuzzy soft $*$-para continuous maps, then $\mathscr{G}\circ\mathscr{F}:(X,E,\tau)\rightarrow(Z,E,\eta)$ is a fuzzy soft maximal para continuous map.*

Proof. Let M_E be any fuzzy soft maximal open set in (Z,E,η). Since every fuzzy soft maximal open set is a fuzzy soft open set, M_E is a fuzzy soft open set in (Z,E,η). Since $\mathscr{G}$ is fuzzy soft $*$-para continuous map, $\mathscr{G}^{-1}(M_E)$ is a fuzzy soft paraopen set in (Y,E,μ). Since $\mathscr{F}$ is fuzzy soft para irresolute, $\mathscr{F}^{-1}(\mathscr{G}^{-1}(M_E))=(\mathscr{G}\circ\mathscr{F})^{-1}(M_E)$ is a fuzzy soft paraopen set in (X,E,τ). Hence, $\mathscr{G}\circ\mathscr{F}$ is a fuzzy soft maximal paracontinuous. □

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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