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## SEVERAL RESULT ON SD-PRIME CORDIAL GRAPHS

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**Abstract.** A bijection  $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$  induces an edge labeling  $f^* : E(G) \rightarrow \{0, 1\}$  such that for any edge  $uv$  in  $G$ ,  $f^*(uv) = 1$  if  $\gcd(S, D) = 1$  and  $f^*(uv) = 0$  otherwise, where  $S = f(u) + f(v)$  and  $D = |f(u) - f(v)|$ . The labeling  $f$  is called SD-prime cordial labeling if  $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ . We say that  $G$  is SD-prime cordial graph if it admits SD-prime cordial labeling. In this paper, we prove that certain classes of zero-divisor graphs of commutative rings are SD-prime cordial graphs.

**Keywords:** zero-divisor graphs; SD-prime cordial labeling.

**2010 AMS Subject Classification:** 05C78.

### 1. INTRODUCTION

Let  $G = (V(G), E(G))$  be a simple, finite and undirected graph with vertex set  $V(G)$  and edge set  $E(G)$ . All notation not defined in this paper can be found in [4]. Graph labeling is currently an emerging area in the research of graph theory. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. The concept of graceful labeling was introduced by Rosa [10] in 1960's. For an excellent survey of graph labeling, we refer the reader to Gallian [3]. Lau et al. have introduced the concepts SD-prime cordial labeling in

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[5]. Further results on SD-prime cordial graphs were discussed in [6, 7, 8, 9]. The idea of a graph associated to zero-divisors of a commutative ring was introduced by I. Beck [2]. Later it was modified by D. F. Anderson and P. S. Livingston [1]. Let  $R$  be a commutative ring with non-zero identity,  $Z(R)$  its set of all zero-divisors in  $R$  and  $Z^*(R) = Z(R) \setminus \{0\}$ . The zero-divisor graph of  $R$  is the simple undirected graph  $\Gamma(R)$  with vertex set  $Z^*(R)$  and two distinct vertices  $x$  and  $y$  are adjacent if and only if  $xy = 0$ . The concept of the coloring of zero-divisor graphs of commutative ring was introduced by I. Beck [2]. Motivated by this, T. Tamizh Chelvam et al. [11] were introduced concept of the labeling of zero-divisor graphs of commutative ring. Also, they have proved that certain classes of zero-divisor graphs of commutative rings are sum cordial graphs. In this paper, we prove that certain classes of zero-divisor graphs are SD-prime cordial graphs.

## 2. PRELIMINARIES

Given a bijection  $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$ , we associate 2 integers  $S = f(u) + f(v)$  and  $D = |f(u) - f(v)|$  with every edge  $uv$  in  $E$ .

**Definition 2.1.** [5] A bijection  $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$  induces an edge labeling  $f^* : E(G) \rightarrow \{0, 1\}$  such that for any edge  $uv$  in  $G$ ,  $f^*(uv) = 1$  if  $\gcd(S, D) = 1$ , and  $f^*(uv) = 0$  otherwise. The labeling  $f$  is called SD-prime cordial labeling if  $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ . We say that  $G$  is SD-prime cordial graph if it admits SD-prime cordial labeling.

**Definition 2.2.** The join of two graphs  $G_1$  and  $G_2$  is denoted by  $G_1 + G_2$  and whose vertex set is  $V(G_1 + G_2) = V(G_1) \cup V(G_2)$  and edge set is  $E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2)\}$ .

**Definition 2.3.** A bipartite graph is a graph whose vertex set  $V(G)$  can be partitioned into two subsets  $V_1$  and  $V_2$  such that every edge of  $G$  has one end in  $V_1$  and the other end in  $V_2$ .  $(V_1, V_2)$  is called a bipartition of  $G$ .

**Definition 2.4.** A complete bipartite graph is a bipartite graph with bipartition  $(V_1, V_2)$  such that every vertex of  $V_1$  is joined to all the vertices of  $V_2$ . It is denoted by  $K_{m,n}$ , where  $|V_1| = m$  and  $|V_2| = n$ . A star graph is a complete bipartite graph  $K_{1,n}$ .

**Definition 2.5.** The complement  $\bar{G}$  of the graph  $G$  is the graph with vertex set  $V(G)$  and two vertices are adjacent in  $\bar{G}$  if and only if they are not adjacent in  $G$ .

**Definition 2.6.** The Cartesian product  $G_1 \times G_2$  of two graphs is defined to be the graph with vertex set  $V_1 \times V_2$  and two vertices  $u = (u_1, u_2)$  and  $v = (v_1, v_2)$  are adjacent in  $G_1 \times G_2$  if either  $u_1 = v_1$  and  $u_2$  is adjacent to  $v_2$  or  $u_2 = v_2$  and  $u_1$  is adjacent to  $v_1$ .

### 3. MAIN RESULTS

**Theorem 3.1.** Let  $p$  be a prime number with  $p > 2$  and  $\Gamma(\mathbb{Z}_{2p})$  be the zero-divisor graph of the commutative ring  $\mathbb{Z}_{2p}$ . Then  $\Gamma(\mathbb{Z}_{2p})$  is SD-prime cordial graph.

*Proof.* Let  $p$  be a prime number and  $p > 2$ . Then the vertex set of  $\Gamma(\mathbb{Z}_{2p})$  is  $Z^*(\mathbb{Z}_{2p}) = \{2, 4, \dots, 2(p-1), p\} = \{v_1, \dots, v_{p-1}, v_p\}$  and the edge set of  $\Gamma(\mathbb{Z}_{2p})$  is  $E(\Gamma(\mathbb{Z}_{2p})) = \{v_i v_p : 1 \leq i \leq p-1\}$ . Therefore,  $|Z^*(\mathbb{Z}_{2p})| = p$  and  $|E(\Gamma(\mathbb{Z}_{2p}))| = p-1$ . We define  $f : V(\Gamma(\mathbb{Z}_{2p})) \rightarrow \{1, 2, 3, \dots, p\}$  by  $f(v_i) = i+1$  for  $1 \leq i \leq p-1$  and  $f(v_p) = 1$ . Here we have  $e_{f^*}(0) = e_{f^*}(1) = \frac{p-1}{2}$ . Hence  $\Gamma(\mathbb{Z}_{2p})$  is SD-prime cordial graph for  $p$  is a prime number and  $p > 2$ .  $\square$

**Theorem 3.2.** Let  $p$  be a prime number with  $p \geq 2$  and  $\Gamma(\mathbb{Z}_{3p})$  be the zero-divisor graph of the commutative ring  $\mathbb{Z}_{3p}$ . Then  $\Gamma(\mathbb{Z}_{3p})$  is SD-prime cordial graph.

*Proof.* If  $p = 2$  then  $\Gamma(\mathbb{Z}_6)$ . Therefore,  $Z^*(\mathbb{Z}_6) = \{2, 3, 4\}$  and  $\Gamma(\mathbb{Z}_6)$  being a path on three vertices is obviously SD-prime cordial graph.

If  $p = 3$  then  $\Gamma(\mathbb{Z}_9)$ . Therefore,  $Z^*(\mathbb{Z}_9) = \{3, 6\}$  and  $\Gamma(\mathbb{Z}_9)$  being a path on two vertices is obviously SD-prime cordial graph.

Let  $\Gamma(\mathbb{Z}_{3p})$  be a zero-divisor graph of  $\mathbb{Z}_{3p}$ , where  $p$  is a prime number and  $p > 3$ . Then the vertex set of  $\Gamma(\mathbb{Z}_{3p})$  is  $Z^*(\mathbb{Z}_{3p}) = \{p, 2p\} \cup \{3, 6, \dots, 3(p-1)\} = \{u_1, u_2\} \cup \{v_1, \dots, v_{p-1}\}$  and the edge set of  $\Gamma(\mathbb{Z}_{3p})$  is  $E(\Gamma(\mathbb{Z}_{3p})) = \{u_1 v_i, u_2 v_i : 1 \leq i \leq p-1\}$ . Therefore,  $|Z^*(\mathbb{Z}_{3p})| = p+1$  and  $|E(\Gamma(\mathbb{Z}_{3p}))| = 2p-2$ . We define the labeling  $f : V(\Gamma(\mathbb{Z}_{3p})) \rightarrow \{1, 2, 3, \dots, p+1\}$  as follows:  $f(u_1) = 1$ ,  $f(u_2) = 2$  and  $f(v_i) = i+2$  for  $1 \leq i \leq p-1$ . Here we have  $e_{f^*}(0) = p-1$  and  $e_{f^*}(1) = p-1$ . Hence  $\Gamma(\mathbb{Z}_{3p})$  is SD-prime cordial graph.  $\square$

**Theorem 3.3.** Let  $p$  be a prime number with  $p \geq 2$  and  $\Gamma(\mathbb{Z}_{4p})$  be the zero-divisor graph of the commutative ring  $\mathbb{Z}_{4p}$ . Then  $\Gamma(\mathbb{Z}_{4p})$  is SD-prime cordial graph.

*Proof.* If  $p = 2$  then  $\Gamma(\mathbb{Z}_8)$ . Therefore,  $Z^*(\mathbb{Z}_8) = \{2, 4, 6\}$  and  $\Gamma(\mathbb{Z}_8)$  being a path on three vertices is obviously SD-prime cordial graph.

Let  $\Gamma(\mathbb{Z}_{4p})$  be a zero-divisor graph of  $\mathbb{Z}_{4p}$ , where  $p$  is a prime number and  $p \geq 3$ . Here the vertex set of  $\Gamma(\mathbb{Z}_{4p})$  is partitioned into two sets  $V_1$  and  $V_2$ , where  $V_1 = \{p, 2p, 3p\} = \{u_1, u_2, u_3\}$  and  $V_2 = \{2, 4, \dots, 2(p-1), 2(p+1), \dots, 2(2p-1)\} = \{v_1, v_2, \dots, v_{p-1}, v_{p+1}, \dots, v_{2p-1}\}$  and the edge set of  $\Gamma(\mathbb{Z}_{4p})$  is  $E(\Gamma(\mathbb{Z}_{4p})) = \{u_1v_2, u_1v_4, \dots, u_1v_{p-1}, u_1v_{p+1}, \dots, u_1v_{2p-2}, u_2v_1, u_2v_2, u_2v_3, \dots, u_2v_{p-1}, u_2v_{p+1}, \dots, u_2v_{2p-1}, u_3v_2, u_3v_4, \dots, u_3v_{p-1}, u_3v_{p+1}, \dots, u_3v_{2p-2}\}$ . Therefore,  $|Z^*(\mathbb{Z}_{4p})| = 2p + 1$  and  $|E(\Gamma(\mathbb{Z}_{4p}))| = 4p - 4$ . We define  $f : V(\Gamma(\mathbb{Z}_{4p})) \rightarrow \{1, 2, 3, \dots, 2p + 1\}$  is as follows:  $f(u_1) = 1$ ;  $f(u_2) = 2$ ;  $f(u_3) = t$ , where  $t$  be a largest prime number  $\leq 2p + 1$ ;  $f(v_j) = j + 2$  for  $1 \leq j \leq p - 1$ ;  $f(v_j) = j + 1$  for  $p + 1 \leq j \leq t - 2$  and

$$f(v_j) = \begin{cases} j + 3 & \text{if } j \text{ is even and } t - 2 < j \leq 2p - 1 \\ j + 1 & \text{if } j \text{ is odd and } t - 2 < j \leq 2p - 1. \end{cases}$$

Here  $e_{f^*}(0) = 2p - 2$  and  $e_{f^*}(1) = 2p - 2$ . Therefore  $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ . Hence  $\Gamma(\mathbb{Z}_{4p})$  is SD-prime cordial graph.  $\square$

**Theorem 3.4.** Let  $p$  be a prime number with  $p > 2$  and  $\Gamma(\mathbb{Z}_{2p}) + \Gamma(\mathbb{Z}_4)$  be the zero-divisor graph of the commutative ring  $\mathbb{Z}_{2p} + \mathbb{Z}_4$ . Then  $\Gamma(\mathbb{Z}_{2p}) + \Gamma(\mathbb{Z}_4)$  is SD-prime cordial graph.

*Proof.* Let  $G = \Gamma(\mathbb{Z}_{2p}) + \Gamma(\mathbb{Z}_4)$ . Let  $V(G) = \{2, 4, \dots, p - 1, p\} \cup \{x : x = 2 \in \mathbb{Z}_4\} = \{u_1, \dots, u_p, x\}$  and  $E(G) = \{u_iu_p, u_ix, u_px : 1 \leq i \leq p - 1\}$ . Therefore,  $|V(G)| = p + 1$  and  $|E(G)| = 2p - 1$ . Define  $f : V(G) \rightarrow \{1, 2, 3, \dots, p + 1\}$  by  $f(u_p) = 1$ ,  $f(x) = 2$  and  $f(u_i) = i + 2$  for  $1 \leq i \leq p - 1$ . Clearly  $e_{f^*}(0) = p - 1$  and  $e_{f^*}(1) = p$  and  $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ . Hence  $G$  is SD-prime cordial graph.  $\square$

**Theorem 3.5.** Let  $p$  be a prime number with  $p > 2$  and  $\Gamma(\mathbb{Z}_{2p}) + \Gamma(\mathbb{Z}_9)$  be the zero-divisor graph of the commutative ring  $\mathbb{Z}_{2p} + \mathbb{Z}_9$ . Then  $\Gamma(\mathbb{Z}_{2p}) + \Gamma(\mathbb{Z}_9)$  is SD-prime cordial graph.

*Proof.* Let  $G = \Gamma(\mathbb{Z}_{2p}) + \Gamma(\mathbb{Z}_9)$ . Let  $V(G) = \{2, 4, \dots, 2(p-1), p\} \cup \{x, y : x = 3, y = 6 \in \mathbb{Z}_9\} = \{u_1, \dots, u_p, x, y\}$  and  $E(G) = \{u_iu_p, u_ix, u_iy, u_px, u_py, xy : 1 \leq i \leq p - 1\}$ . Therefore,  $|V(G)| = p + 2$  and  $|E(G)| = 3p$ . We define the vertex labeling  $f : V(G) \rightarrow \{1, 2, 3, \dots, p + 2\}$  is as follows:  $f(x) = 1$ ;  $f(y) = 2$ ;  $f(u_p) = t$ , where  $t$  be a largest prime number  $\leq p + 2$ ;  $f(u_i) = i + 2$  for  $1 \leq i \leq t - 3$  and

$$f(u_i) = \begin{cases} i+4 & \text{if } i \text{ is odd and } t-3 < i \leq p-1 \\ i+2 & \text{if } i \text{ is even and } t-3 < i \leq p-1. \end{cases}$$

Clearly,  $e_{f^*}(0) = \frac{3p-1}{2}$  and  $e_{f^*}(1) = \frac{3p+1}{2}$  and  $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ . Hence  $\Gamma(\mathbb{Z}_{2p}) + \Gamma(\mathbb{Z}_9)$  is SD-prime cordial graph.  $\square$

**Corollary 3.6.** *Let  $p$  be a prime number with  $p > 2$  and  $\overline{\Gamma(\mathbb{Z}_{p^2})} + \Gamma(\mathbb{Z}_4)$  be the zero-divisor graph of the commutative ring  $\overline{\mathbb{Z}_{p^2}} + \mathbb{Z}_4$ . Then  $\overline{\Gamma(\mathbb{Z}_{p^2})} + \Gamma(\mathbb{Z}_4)$  is SD-prime cordial graph.*

*Proof.* Since the graph  $\overline{\Gamma(\mathbb{Z}_{p^2})} + \Gamma(\mathbb{Z}_4) \cong \Gamma(\mathbb{Z}_{2p})$ , by Theorem 3.1,  $\overline{\Gamma(\mathbb{Z}_{p^2})} + \Gamma(\mathbb{Z}_4)$  is SD-prime cordial graph.  $\square$

**Theorem 3.7.** *Let  $p$  be a prime number with  $p > 2$  and  $\overline{\Gamma(\mathbb{Z}_{p^2})} + \Gamma(\mathbb{Z}_6)$  be the zero-divisor graph of the commutative ring  $\overline{\mathbb{Z}_{p^2}} + \mathbb{Z}_6$ . Then  $\overline{\Gamma(\mathbb{Z}_{p^2})} + \Gamma(\mathbb{Z}_6)$  is SD-prime cordial graph.*

*Proof.* Let  $G = \overline{\Gamma(\mathbb{Z}_{p^2})} + \Gamma(\mathbb{Z}_6)$ . Let  $V(G) = \{u_1, \dots, u_{p-1}\} \cup \{x, y, z : x = 2, y = 3, z = 4 \in \mathbb{Z}_6\} = \{u_1, \dots, u_{p-1}, x, y, z\}$  and  $E(G) = \{u_i x, u_i y, u_i z, xy, yz : 1 \leq i \leq p-1\}$ . Therefore,  $|V(G)| = p+2$  and  $|E(G)| = 3p-1$ . Define the vertex labeling  $f : V(G) \rightarrow \{1, 2, \dots, p+2\}$  by  $f(x) = 1$ ;  $f(z) = 2$ ;  $f(y) = t$ , where  $t$  be largest prime number  $\leq p+2$ ;  $f(u_i) = i+2$  for  $1 \leq i \leq t-3$  and

$$f(u_i) = \begin{cases} i+4 & \text{if } i \text{ is odd and } t-3 < i \leq p-1 \\ i+2 & \text{if } i \text{ is even and } t-3 < i \leq p-1. \end{cases}$$

Here we have  $e_{f^*}(0) = \frac{3p-1}{2}$  and  $e_{f^*}(1) = \frac{3p+1}{2}$ . Thus  $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ . Hence  $G$  is SD-prime cordial graph.  $\square$

**Theorem 3.8.** *Let  $p$  be a prime number with  $p > 2$  and  $\overline{\Gamma(\mathbb{Z}_{p^2})} + \Gamma(\mathbb{Z}_9)$  be the zero-divisor graph of the commutative ring  $\overline{\mathbb{Z}_{p^2}} + \mathbb{Z}_9$ . Then  $\overline{\Gamma(\mathbb{Z}_{p^2})} + \Gamma(\mathbb{Z}_9)$  is SD-prime cordial graph.*

*Proof.* Let  $G = \overline{\Gamma(\mathbb{Z}_{p^2})} + \Gamma(\mathbb{Z}_9)$ . Let  $V(G) = \{u_1, \dots, u_{p-1}\} \cup \{x, y : x = 3, y = 6 \in \mathbb{Z}_9\} = \{u_1, \dots, u_{p-1}, x, y\}$  and  $E(G) = \{u_i x, u_i y, xy : 1 \leq i \leq p-1\}$ . Therefore,  $|V(G)| = p+1$  and  $|E(G)| = 2p-1$ . Define the vertex labeling  $f : V(G) \rightarrow \{1, 2, \dots, p+1\}$  by  $f(x) = 1$ ,  $f(y) = 2$  and  $f(u_i) = i+2$  for  $1 \leq i \leq p-1$ . Clearly  $e_{f^*}(0) = p-1$  and  $e_{f^*}(1) = p$ . Therefore  $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ . Hence  $G$  is SD-prime cordial graph.  $\square$

**Theorem 3.9.** *Let  $p$  be a prime number with  $p > 2$  and  $\overline{\Gamma(\mathbb{Z}_{p^2})} + \overline{\Gamma(\mathbb{Z}_9)}$  be the zero-divisor graph of the commutative ring  $\overline{\mathbb{Z}_{p^2}} + \overline{\mathbb{Z}_9}$ . Then  $\overline{\Gamma(\mathbb{Z}_{p^2})} + \overline{\Gamma(\mathbb{Z}_9)}$  is SD-prime cordial graph.*

*Proof.* Let  $G = \overline{\Gamma(\mathbb{Z}_{p^2})} + \overline{\Gamma(\mathbb{Z}_9)}$ . Let  $V(G) = \{u_1, \dots, u_{p-1}\} \cup \{x, y : x = 3, y = 6 \in \overline{\mathbb{Z}_9}\} = \{u_1, \dots, u_{p-1}, x, y\}$  and  $E(G) = \{u_i x, u_i y : 1 \leq i \leq p-1\}$ . Therefore,  $|V(G)| = p+1$  and  $|E(G)| = 2p-2$ . The labeling pattern is analogous to that of the Theorem 3.8. Clearly,  $e_{f^*}(0) = p-1$  and  $e_{f^*}(1) = p-1$ . Thus  $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ . Hence  $G$  is SD-prime cordial graph.  $\square$

**Theorem 3.10.** *Let  $p$  be a prime number with  $p > 2$  and  $\overline{\Gamma(\mathbb{Z}_{p^2})} + \overline{\Gamma(\mathbb{Z}_6)}$  be the zero-divisor graph of the commutative ring  $\overline{\mathbb{Z}_{p^2}} + \overline{\mathbb{Z}_6}$ . Then  $\overline{\Gamma(\mathbb{Z}_{p^2})} + \overline{\Gamma(\mathbb{Z}_6)}$  is SD-prime cordial graph.*

*Proof.* Let  $G = \overline{\Gamma(\mathbb{Z}_{p^2})} + \overline{\Gamma(\mathbb{Z}_6)}$ . Let  $V(G) = \{u_1, \dots, u_{p-1}\} \cup \{x, y, z : x = 2, y = 3, z = 4 \in \overline{\mathbb{Z}_6}\} = \{u_1, \dots, u_{p-1}, x, y, z\}$  and  $E(G) = \{u_i x, u_i y, u_i z : 1 \leq i \leq p-1\}$ . Therefore,  $|V(G)| = p+2$  and  $|E(G)| = 3p-3$ . The labeling pattern is analogous to that of the Theorem 3.7. Therefore,  $e_{f^*}(0) = \frac{3p-3}{2}$  and  $e_{f^*}(1) = \frac{3p-3}{2}$ . Thus  $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ . Hence  $G$  is SD-prime cordial graph.  $\square$

**Theorem 3.11.** *Let  $p$  be a prime number with  $p > 2$  and  $\Gamma(\mathbb{Z}_{2p}) + \overline{\Gamma(\mathbb{Z}_9)}$  be the zero divisor graph of the commutative ring  $\mathbb{Z}_{2p} + \overline{\mathbb{Z}_9}$ . Then  $\Gamma(\mathbb{Z}_{2p}) + \overline{\Gamma(\mathbb{Z}_9)}$  is SD-prime cordial graph.*

*Proof.* Let  $G = \Gamma(\mathbb{Z}_{2p}) + \overline{\Gamma(\mathbb{Z}_9)}$ . Let  $V(G) = \{2, 4, \dots, 2(p-1), p\} \cup \{x, y : x = 3, y = 6 \in \overline{\mathbb{Z}_9}\} = \{u_1, \dots, u_p, x, y\}$  and  $E(G) = \{u_i u_p, u_i x, u_i y, u_p x, u_p y : 1 \leq i \leq p-1\}$ . Therefore,  $|V(G)| = p+2$  and  $|E(G)| = 3p-1$ . The labeling pattern is same as in the Theorem 3.5. From the labeling, we get  $e_{f^*}(0) = \frac{3p-1}{2}$  and  $e_{f^*}(1) = \frac{3p-1}{2}$  and also  $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ . Hence  $\Gamma(\mathbb{Z}_{2p}) + \overline{\Gamma(\mathbb{Z}_9)}$  is SD-prime cordial graph.  $\square$

**Theorem 3.12.** *Let  $p$  be a prime number with  $p > 2$  and  $\Gamma(\mathbb{Z}_{2p}) \times \Gamma(\mathbb{Z}_4)$  be the zero-divisor graph of the commutative ring  $\mathbb{Z}_{2p} \times \mathbb{Z}_4$ . Then  $\Gamma(\mathbb{Z}_{2p}) \times \Gamma(\mathbb{Z}_4)$  is SD-prime cordial graph.*

*Proof.* Since the graph  $\Gamma(\mathbb{Z}_{2p}) \times \Gamma(\mathbb{Z}_4) \cong \Gamma(\mathbb{Z}_{2p})$ , by Theorem 3.1,  $\Gamma(\mathbb{Z}_{2p}) \times \Gamma(\mathbb{Z}_4)$  is SD-prime cordial graph.  $\square$

**CONFLICT OF INTERESTS**

The author(s) declare that there is no conflict of interests.

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