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EFFECT OF VARIABLE VISCOSITY ON THE PERISTALTIC FLOW OF A NEWTONIAN FLUID IN AN ASYMMETRIC CHANNEL UNDER THE EFFECT OF A MAGNETIC FIELD

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Abstract: In this paper, effects of MHD on peristaltic flow of a Newtonian fluid with variable viscosity in an asymmetric channel under the assumptions of long wavelength and low Reynolds number assumptions is investigated. The expressions for the velocity, pressure gradient and pressure rise per one wavelength are obtained by a regular perturbation technique. The effects of viscosity parameter α, Hartmann number M, wave amplitudes a, b and phase shift θ on pumping characteristics are discussed in detail.

Keywords: Asymmetric channel; Hartmann number; peristaltic flow; variable vicsosity.

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1. Introduction

The word peristalsis stems from the Greek word peristalitikos, which means clasping and compressing. It is used to describe a progressive wave of contraction along

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a channel or tube whose cross-sectional area consequently varies in physiology, it has been found to be involved in many biological organs, e.g. in transport of spermatozoa in the ductus efferent us of the male reproductive tracts and in the cervical canal, in the movement of ovum in the fallopian tubes and in the vasomotion of small blood vessels as well as blood flow in arteries. Some worms use peristalsis as a means of locomotion. Roller and Finger pumps using viscous fluid also operate on this principle. The mechanism of peristaltic transport has been exploited for industrial applications like sanitary fluid transport, blood pumps in heart lung machine and transport of corrosive fluid where the contact of the fluid with the machinery parts is prohibited.

There are many fluids whose behaviour cannot be described by the Navier-Stokes model with constant viscosity. Also the inadequacy of the classical Navier-stokes theory of Newtonian fluids in predicting the behaviours of some fluids, especially those with high molecular weight, leads to the developments of non-Newtonian fluid mechanics. The governing equations for such fluids are of higher order, much more complicated and subtle than the Newtonian fluid. Peristaltic transport of a power-law fluid with variable consistency has been studied by Shukla and Gupta [11]. Srivastava et al. [12] have studied the peristaltic transport of a fluid with variable viscosity through a non-uniform tube. Abd El Hakeem et al. [2] have investigated the effect of endoscope and fluid with variable viscosity on peristaltic motion. Abd El Hakeem et al. [1] have investigated the peristaltic flow of a fluid with variable viscosity under the effect of magnetic held.

The magnetic hydrodynamic flow of blood in a channel having walls that execute peristaltic waves using long wave length approximation has been discussed by Agrawal and Anwaruddin [3]. Peristaltic flow of Johnson-Segalman fluid under effect of a magnetic field was studied by Elshahed and Haroun [4]. Nonlinear peristaltic transport of MHD flow through a porous medium was studied by Mekheimer and Al-Arabi [7]. Mekheimer [8] have studied the peristaltic transport of blood under effect of a magnetic field in non uniform channels.

Eytan and Elad [5] have presented a Mathematical model of wall-induced peristaltic fluid flow in a two dimensional channel with wave trains having a phase difference moving independently on the upper and lower walls to simulate intra-uterine fluid motion in a sagittal cross-section of the uterus. They have obtained a time dependent flow solution in a fixed by using lubrication approach. Mishra and Ramachandra Rao [9] discussed the peristaltic motion of viscous fluid in a two dimensional asymmetric channel under long wave length assumption. Ramachandra Rao and Mishra [10] also analyzed the curvature effects on peristalsis in an asymmetric channel. Effect of variable viscosity on the peristaltic transport of a Newtonian fluid in an asymmetric channel has been studied Hayat and Ali [6].

In view of these, we investigated the MHD peristaltic flow of a Newtonian fluid with variable viscosity in an asymmetric channel under the assumptions of long wavelength and low Reynolds number assumptions. The expressions for the velocity, pressure gradient and pressure rise per one wavelength are obtained by a regular perturbation technique. The effects of viscosity parameter α, Hartmann number M, wave amplitudes a,b and phase shift θ on pumping characteristics are discussed in detail.

2. Mathematical formulation

We consider the peristaltic flow of an incompressible viscous Newtonian fluid with variable viscosity in a two-dimensional asymmetric channel under the effect of a magnetic field. The channel asymmetry is produced by the propagation of waves on the channel walls traveling with same speed c but with different amplitudes and phases. A rectangular co-ordinate system (X,Y) is chosen such that X-axis lies along the centre line of the channel in the direction of wave propagation and Y-axis transverse to it, as shown in Fig. 1. A uniform magnetic field B_0 is applied in the transverse direction to the flow. The electrical conductivity of the fluid is assumed to be small so that the magnetic Reynolds number is small and therefore the induced magnetic field is neglected. The external electric field is zero and the electric field due to polarization of charges is also negligible. Also heat due to Joule dissipation is neglected.

The channel walls are defined by

$$Y = H_1(X,t) = d + a_1 \cos\frac{2\pi}{\lambda}(X - ct) \quad \text{(upper wall)} \tag{2.1}$$

$$Y = H_2(X,t) = -d - a_2 \cos\left(\frac{2\pi}{\lambda}(X - ct) + \theta\right) \quad \text{(lower wall)} \tag{2.2}$$

where a_1, a_2 are the amplitudes of the waves, X is the wave length, $2d$ is the width of the channel, θ is the phase difference which varies in the range $0 \le \theta \le \pi$, $\theta = 0$ corresponds to a symmetric channel with waves out of phase and $\theta = \pi$ defines the waves with in phase and further a_1, a_2 and θ satisfies the condition $a_1^2 + a_2^2 + 2a_1a_2 \cos\theta \le (2d)^2$.

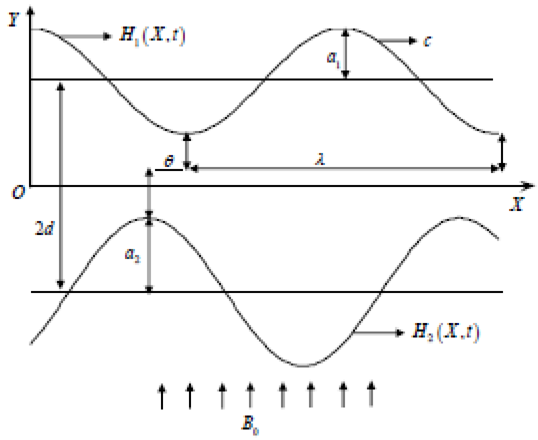

Fig. 1. The Physical Model

We shall carryout this investigation in a co-ordinate system moving with wave speed *c*, in which the boundary shape is stationary. The co-ordinates and velocities in the laboratory frame (X,Y) and the wave frame (x, y) are related by

$$x = X - ct, y = Y, u = U - c, v = V, p(x) = P(X,t) \tag{2.3}$$

where $(u,\ v)$ and $(U,\ V)$ are the velocity components, p and P are pressures in the wave and fixed frames of reference, respectively.

The equations governing the flow field in a wave frame are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.4}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(M(y)\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(M(y)\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right)-\sigma B_0^2(u+c) \tag{2.5}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(M(y)\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right)+\frac{\partial}{\partial x}\left(M(y)\frac{\partial v}{\partial y}\right) \tag{2.6}$$

where ρ is the density, B_0 magnetic field strength and σ - electrical conductivity. The dimensional boundary conditions are

$$u=-c \quad \text{at} \quad y=H_1,H_2 \tag{2.7}$$

Introducing the following non-dimensional quantities

$$\bar{x}=\frac{x}{\lambda},\bar{y}=\frac{y}{a},\delta=\frac{a}{\lambda},\bar{u}=\frac{u}{c}, \qquad \bar{v}=\frac{v}{\delta c},\bar{p}=\frac{pa^2}{\mu_0 c\lambda},\bar{t}=\frac{ct}{\lambda},$$

$$h_1=\frac{H_1}{d},h_2=\frac{H_2}{d},a=\frac{a_1}{d},b=\frac{a_2}{d}$$

where μ_0 is the viscosity constant, δ is the wave number and a and b are amplitude ratios, in the equations (2.1), (2.2) and (2.4) – (2.6) dropping the bars, we obtain

$$\left.\begin{aligned} h_1&=1+a\cos 2\pi x\\ h_2&=-1-b\cos(2\pi x+\theta)\end{aligned}\right\} \tag{2.8}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.9}$$

$$\operatorname{Re}\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial x}+2\delta^2\frac{\partial}{\partial x}\left(\mu(y)\frac{\partial u}{\partial x}\right)$$
$$+\frac{\partial}{\partial y}\left(\mu(y)\left(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right)-M^2(u+1) \tag{2.10}$$

$$\operatorname{Re}\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\delta^2\frac{\partial}{\partial x}\left(\mu(y)\left(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right)+2\delta^2\frac{\partial}{\partial y}\left(\mu(y)\frac{\partial v}{\partial y}\right) \tag{2.11}$$

where $\operatorname{Re}=\frac{\rho dc}{\mu_0}$ is the Reynolds number, $M=B_0 d\sqrt{\frac{\sigma}{\mu_0}}$ is the Hartmann number and under the assumptions of low Reynolds number ($\operatorname{Re}\to 0$) and long wave length ($\delta << 1$), the equations (2.10) and (2.11) become

$$O = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\frac{\mu(y)}{1+\lambda_1}\frac{\partial u}{\partial y}\right) - M^2(u+1) \quad (2.12)$$

$$O = \frac{\partial p}{\partial y} \quad (2.13)$$

The corresponding dimensionless boundary conditions are

$$u = -1 \ \text{ at } \ y = h_1, h_2 \quad (2.14)$$

From Eq. (2.13) we conclude that p is only function of x alone. Therefore, the Eq. (2.12) can be rewritten as

$$\frac{dp}{dx} = \frac{1}{1+\lambda_1}\frac{\partial}{\partial y}\left(\mu(y)\frac{\partial u}{\partial y}\right) - M^2(u+1) \quad (2.15)$$

The non-dimensional viscosity here is of the following form

$$\mu(y) = 1 - \alpha y \ \text{ or } \ \mu(y) = e^{-\alpha y} \ \text{ for } \ \alpha << 1 \quad (2.16)$$

where α is the viscosity parameter.

The dimensionless volume flow rate q in the wave frame of reference is given by

$$q = \int_{h_2}^{h_1} u dy \quad (2.17)$$

The instantaneous flux $Q(x,t)$ in the laboratory frame is

$$Q(x,t) = \int_{h_2}^{h_1} (u+1)\ dy = \int_{h_2}^{h_1} u\ dy + \int_{h_2}^{h_1} 1\ dy = q + h_1 - h_2 \quad (2.18)$$

The time averaged volume flow rate over one period $T\left(=\frac{\lambda}{c}\right)$ of the peristaltic wave is

$$\overline{Q} = \frac{1}{T}\int_0^T Q(x,t)dt = \int_0^1 (q + h_1 - h_2)dx = q + 2 \quad (2.19)$$

3. Solution

We seek for a regular perturbation solution in terms of a small parameter α as follows

$$u = u_0 + \alpha u_1 + o(\alpha^2) \tag{3.1}$$

$$\frac{dp}{dx} = \frac{dp_0}{dx} + \alpha \frac{dp_1}{dx} + o(\alpha^2) \tag{3.2}$$

$$q = q_0 + \alpha q_1 + o(\alpha^2) \tag{3.3}$$

Substituting the equations (3.1) and (3.2) into the equations (2.14) and (2.15) and using Eq. (2.16), we get

3.1 The system of order zero

$$\frac{dp_0}{dx} = \frac{\partial^2 u_0}{\partial y^2} + M^2(u_0 + 1) \tag{3.4}$$

with the dimensionless boundary conditions

$$u_0 = -1 \text{ at } y = h_1, h_2 \tag{3.5}$$

3.2 The system of order one

$$\frac{\partial^2 u_1}{\partial y^2} - M^2 u_1 = \frac{dp_1}{dx} + y\frac{\partial^2 u_0}{\partial y^2} + \frac{\partial u_0}{\partial y} \tag{3.6}$$

with the corresponding dimensionless boundary conditions

$$u_1 = 0 \text{ at } y = h_1, h_2 \tag{3.7}$$

3.3 Solution of order zero

Solving Eq. (3.4) together with the boundary conditions Eq. (3.5), we get

$$u_0 = \frac{1}{M^2}\frac{dp_0}{dx}\left(c_1 \cosh My + c_2 \sinh My - 1\right) - 1 \tag{3.8}$$

where $c_1 = \dfrac{\sinh Mh_2 - \sinh Mh_1}{\sinh M(h_2 - h_1)}$ and $c_2 = \dfrac{\cosh Mh_1 - \cosh Mh_2}{\sinh M(h_2 - h_1)}$

The volume flow rate q_0 in the wave frame of reference is given by

$$q_0 = \int_{h_2}^{h_1} u_0 dy$$

$$= \frac{1}{M^3}\frac{dp_0}{dx}\frac{\left(2-2\cosh M(h_1-h_2)-M(h_1-h_2)\sinh M(h_1-h_2)\right)}{\sinh M(h_2-h_1)} - \left(h_1-h_2\right) \quad (3.9)$$

From Eq. (3.9), we have

$$\frac{dp_0}{dx} = \frac{\left(q_0+h_1-h_2\right)M^3\sinh M(h_2-h_1)}{\left(2-2\cosh M(h_1-h_2)-M(h_1-h_2)\sinh M(h_2-h_1)\right)} \quad (3.10)$$

3.2 Solution of order one

Substituting Eq. (3.8) in the Eq. (3.6) and solving it by using the boundary conditions Eq. (3.7), we obtain

$$u_1 = \frac{1}{M^2}\frac{dp_1}{dx}\left(c_1\sinh My + c_2\sinh My - 1\right) + \frac{y^2}{4M}\frac{dp_0}{dx}\left(c_1\sinh My + c_2\cosh My\right)$$

$$-\frac{1}{4M}\frac{dp_0}{dx}\frac{\left(1-\cosh M(h_1-h_2)\right)}{\sinh^2 M(h_2-h_1)}\left((h_1^2\sinh Mh_2 + h_2^2\sinh Mh_1\right)\cosh My$$

$$+\frac{1}{4M}\frac{dp_0}{dx}\frac{\left(1-\cosh M(h_1-h_2)\right)}{\sinh^2 M(h_2-h_1)}\left((h_1^2\cosh Mh_2 + h_2^2\cosh Mh_1\right)\sinh My \quad (3.11)$$

The volume flow rate q_1 in the wave frame of reference is given by

$$q_1 = \int_{h_2}^{h_1} u_1 dy = \frac{1}{M^3}\frac{dp_1}{dx}\frac{\left(2-2\cosh M(h_1-h_2)-M(h_1-h_2)\sinh M(h_2-h_1)\right)}{\sinh M(h_2-h_1)}$$

$$+\frac{(h_2^2-h_1^2)}{4M^2}\frac{dp_0}{dx}\frac{\left(1-\cosh M(h_1-h_2)\right)^2}{\sinh^2 M(h_2-h_1)} \quad (3.12)$$

From Eq. (3.12), we have

$$\frac{dp_1}{dx} = \frac{q_1 M^3\sinh M(h_2-h_1)}{\left(2-2\cosh M(h_1-h_2)-M(h_1-h_2)\sinh M(h_2-h_1)\right)}$$

$$-\frac{(h_2^2-h_1^2)}{4}\frac{\partial p_0}{\partial x}\frac{\left(1-\cosh M(h_1-h_2\right)^2}{\sinh M(h_2-h_1)}k \quad (3.13)$$

where $k = \dfrac{M}{\left(2-2\cosh M(h_1-h_2)-M(h_1-h_2)\sinh M(h_2-h_1)\right)}$

Substituting from equations (3.10) and (3.13) into Eq. (3.2), we get

$$\frac{dp}{dx}=\frac{M^3\left(q+h_1-h_2\right)\sinh M(h_2-h_1)}{\left(2-2\cosh M(h_1-h_2)-M(h_1-h_2)\sinh M(h_2-h_1)\right)}$$

$$-\alpha\frac{(h_2^2-h_1^2)M^4\left(1-\cosh M(h_1-h_2)\right)^2\left(q_0+h_1-h_2\right)}{\left(2-2\cosh M(h_1-h_2)-M(h_1-h_2)\sinh M(h_2-h_1)\right)^2}k \qquad (3.14)$$

Using $q_0=q-\alpha q_1$ and neglecting $o(\alpha^2)$ terms, Eq. (3.14), we get

$$\frac{dp}{dx}=\frac{M^3\left(q+h_1-h_2\right)\sinh M(h_2-h_1)}{\left(2-2\cosh M(h_1-h_2)-M(h_1-h_2)\sinh M(h_2-h_1)\right)}$$

$$-\alpha\frac{(h_2^2-h_1^2)M^4\left(q+h_1-h_2\right)\left(1-\cosh M(h_1-h_2)\right)^2}{\left(2-2\cosh M(h_1-h_2)-M(h_1-h_2)\sinh M(h_2-h_1)\right)^2} \qquad (3.15)$$

The pressure rise Δp per one wave length is given as

$$\Delta p=\int_0^1\frac{dp}{dx}dx \qquad (3.16)$$

4. Results and discussions

Fig. 2 shows the variation of pressure rise with time averaged flux $\overline{Q}$ for different values of viscosity parameter α with $a=0.5, b=0.7, M=1$ and $\theta=\frac{\pi}{4}$ It is observe that in the pumping region $(\Delta p>o)$, the $\overline{Q}$ decreases with an increase in α.

The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of phase shift θ with $M=1, a=0.5, b=0.7$ and $\alpha=0.1$ is illustrated in Fig. 3 It is found that, in the pumping region, the $\overline{Q}$ decreases with an increase in θ.

Fig. 4 depicts the variation of pressure vise Δp with time averaged flux $\overline{Q}$ for different values of Hartman number M with $a=0.5, b=0.7, \alpha=0.1$ and $\theta=\frac{\pi}{4}$.

It is observed that any two pumping curves intersect at a point in first quadrant. To the left of this point of intersection the $\overline{Q}$ increases with increasing M and to the right of this point of intersection $\overline{Q}$ decreases with M. As $M \to 0$ and $r \to 0$ results agree with those results obtained by Mishra and Ramachandra Rao [9].

The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of a with $b = 0.5, M = 1, \alpha = 0.1$ and $\theta = \frac{\pi}{4}$ is presented in Fig. 5. It is observed that in the pumping region the $\overline{Q}$ increases with an increase in a.

Fig. 6 illustrates the variation of Δp with $\overline{Q}$ for different values of b with $a = 0.4, M = 1, \alpha = 0.1$ and $\theta = \frac{\pi}{4}$. It is observed that in the pumping region the $\overline{Q}$ increases with increasing b.

5. Conclusions

In this paper, we investigated the MHD peristaltic flow of a Newtonian fluid with variable viscosity in an asymmetric channel under the assumptions of long wavelength and low Reynolds number assumptions. It is found that, in the pumping region the $\overline{Q}$ increases with increasing M, a and b, while it decreases with increasing α and θ.

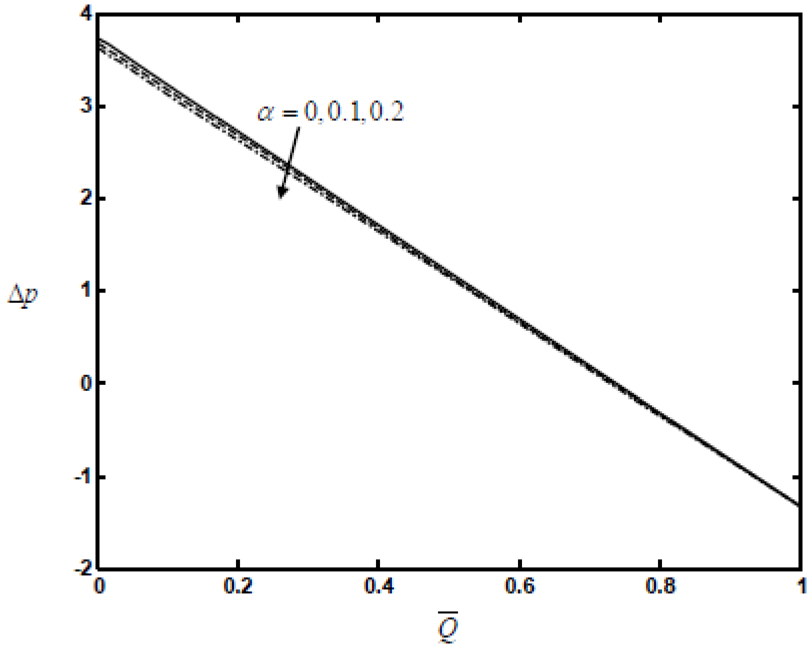

Fig. 2. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of viscosity parameter α with $a = 0.5, b = 0.7, M = 1$ and $\theta = \pi/4$.

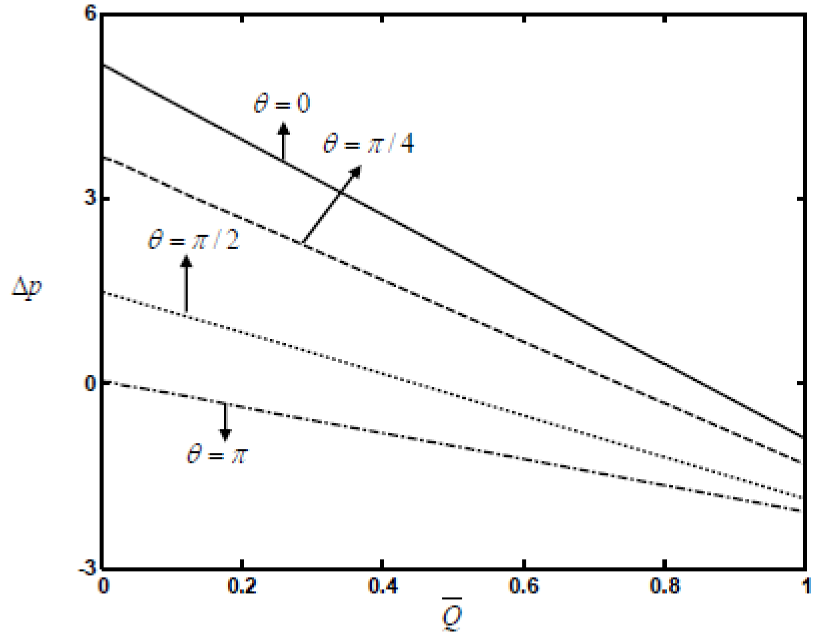

Fig. 3. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of phase shift θ with $a = 0.5, b = 0.7, M = 1$ and $\alpha = 0.1$

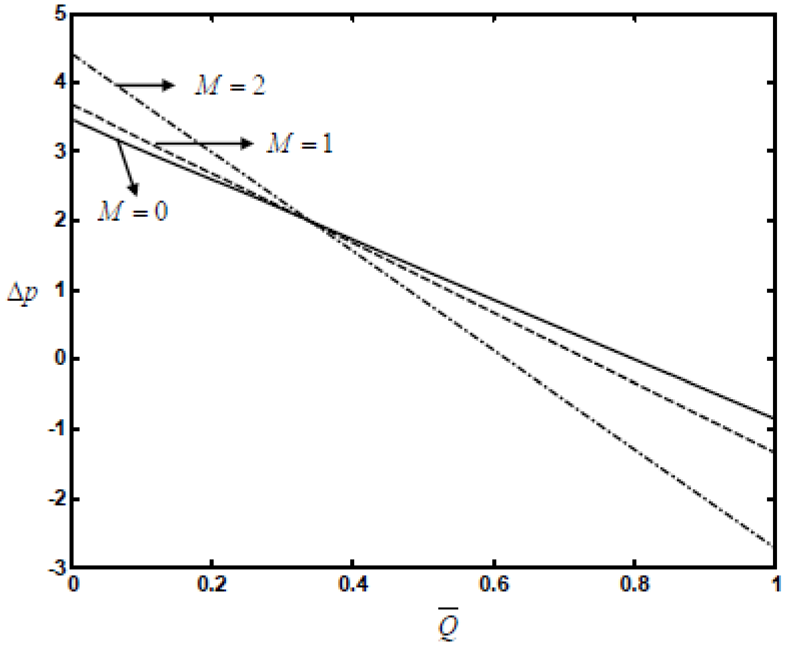

Fig. 4. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of Hartmann number M with $a=0.5, b=0.7, \theta=\pi/4$ and $\alpha=0.1$.

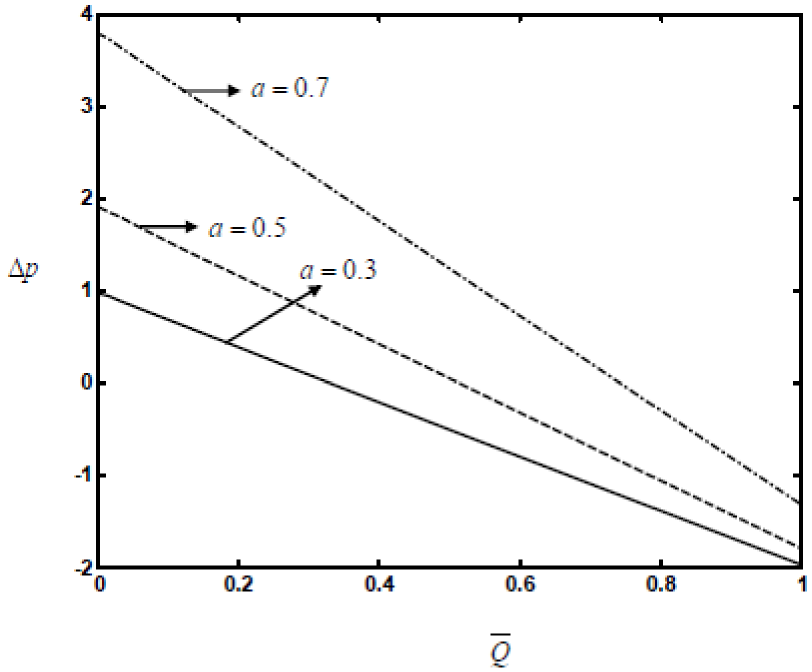

Fig. 5. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of a with $\theta=\pi/4, b=0.5, M=1$ and $\alpha=0.1$.

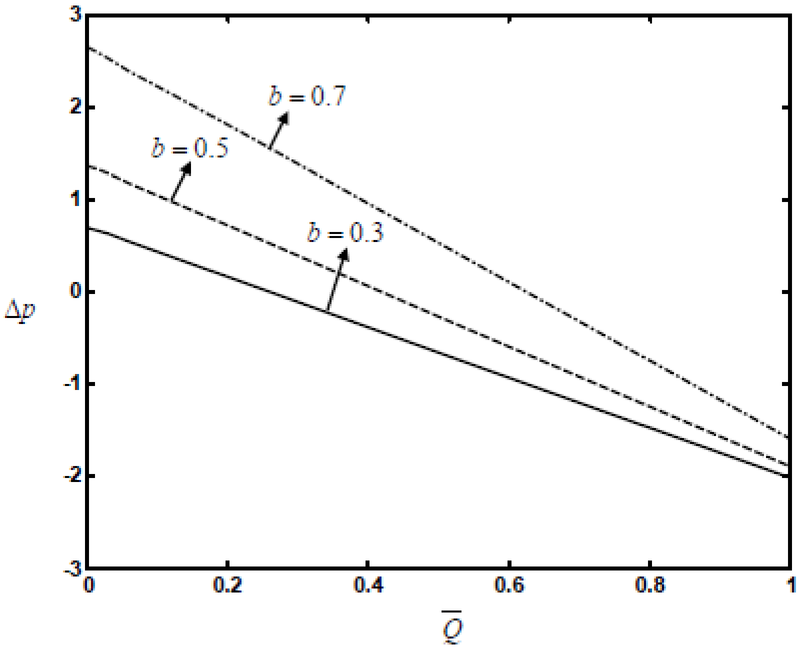

Fig. 6. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of b with $a = 0.4, \theta = \pi/4, M = 1$ and $\alpha = 0.1$

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