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STRONG FORMS OF $\psi\widehat{g}$-CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

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Abstract. In this paper, we introduce a new class of functions called $\psi\widehat{g}$-irresolute functions,Strongly $\psi\widehat{g}$-continuous functions and Perfectly $\psi\widehat{g}$-continuous functions in topological space and study some of their properties and relations among them.

Keywords: $\psi\widehat{g}$-closed(open)sets,$\psi\widehat{g}$-continuous functions,$\psi\widehat{g}$-open functions,$\psi\widehat{g}$-closed functions.

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1. Introduction

Levine[3]introduced generalized closed (briefly g-closed) sets and studied their basic properties.Veera Kumar [14] introduced ψ-closed sets in topological spaces.Recently Ramya and Parvathi[6] have introduced and investigated $\psi\widehat{g}$-closed sets. Ramya and Parvathi[8] have introduced and studied $\psi\widehat{g}$-continuous functions in a topological spaces. In this paper, we introduce new class of functions namely $\psi\widehat{g}$-irresolute functions, strongly $\psi\widehat{g}$-continuous functions and perfectly $\psi\widehat{g}$-continuous functions.

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2. Preliminaries

Throughout this paper (X,τ) and (Y,σ) and (Z,η) represent non-empty topological spaces on which no speration axiams are assumed,unless otherwise mentioned.
Now, we recollect some notations and definitions which are used in this paper.

Definition 2.1. A subset A of a topological space(X,τ) is called
(i)semi-open if $A \subseteq cl(int)(A))$
(ii)α-open if $\alpha \subseteq int(cl(int(A)))$
(iii)regular open if $A \subseteq int(cl(A))$
(iv)pre-open if $A \subseteq int(cl(A))$.
The complements of the above sets are called semi-closed,α-closed,regular closed and pre closed sets respectively.

Definition 2.2. A subset A of a topological space(X,τ) is called
(i)a generalized closed (briefly g-closed) set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).
(ii) a semi-generalized closed (briefly sg-closed) set if $scl(A) \subseteq U$ whenever$A \subseteq U$ and U is semi-open in (X,τ).
(iii)a generalized semi-closed (briefly gs-closed) set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).
(iv) a $\widehat{g}$-closed (briefly $\widehat{g}$-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X,τ).
(v) a ψ-closed if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open in (X,τ).
(vi)a ψ-generalized closed set (briefly ψg-closed) if $\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).
(vii)a $\psi\widehat{g}$-closed set (briefly $\psi\widehat{g}$-closed) if $\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\widehat{g}$-open in (X,τ).
The complements of the above sets are called their respective open sets.

Definition 2.3. A function $f : (X,\tau) \to (Y,\sigma)$ is called
(i)semi-continuous if $f^{-1}(V)$ is open in (X,τ) for every open set V in (Y,σ).

(ii)g-continuous if $f^{-1}(V)$ is g-open in (X,τ) for every open set V in (Y,σ).

(iii)sg-continuous if $f^{-1}(V)$ is sg-open in (X,τ) for every open set V in (Y,σ).

(iv)gs-continuous if $f^{-1}(V)$ is gs-open in (X,τ) for every open set V in (Y,σ).

(v)$\widehat{g}$-continuous if $f^{-1}(V)$ is $\widehat{g}$-open in (X,τ) for every open set V in (Y,σ).

(vi)ψ-continuous if $f^{-1}(V)$ is ψ-open in (X,τ) for every open set V in (Y,σ).

(vii)ψg-continuous if $f^{-1}(V)$ is ψg-open in (X,τ) for every open set V in (Y,σ).

(viii)$\psi\widehat{g}$-continuous if $f^{-1}(V)$ is $\psi\widehat{g}$-open in (X,τ) for every open set V in (Y,σ).

3. $\psi\widehat{g}$-irresolute functions

Definition 3.1. A function $f : X \to Y$ from a topological space (X,τ) into a topological space (Y,σ) is called $\psi\widehat{g}$-irresolute if the inverse image of every $\psi\widehat{g}$-closed set in Y is $\psi\widehat{g}$-closed set in X.

Theorem 3.2. *A function $f : X \to Y$ is $\psi\widehat{g}$-irresolute if and only if the inverse image of every $\psi\widehat{g}$-open in Y is $\psi\widehat{g}$-open in X.*

Proof.

Assume that f is $\psi\widehat{g}$-irresolute.Let A be any $\psi\widehat{g}$-open set in Y.Then A^c is $\psi\widehat{g}$-closed in Y.Since f is $\psi\widehat{g}$-irresolute,$f^{-1}(A^c)$is $\psi\widehat{g}$-closed in X .But $f^{-1}(A^c) = X - f^{-1}(A)$ and so $f^{-1}(A)$ is $\psi\widehat{g}$-open in X.Hence the inverse image of every $\psi\widehat{g}$-open in Y is $\psi\widehat{g}$-open in X. Conversely,assume that the inverse image of every $\psi\widehat{g}$-open set in Y is $\psi\widehat{g}$-open in X.Let A be any $\psi\widehat{g}$-closed in Y.Then A^c $\psi\widehat{g}$-open in Y.By assumption,$f^{-1}(A^c)$ is $\psi\widehat{g}$-open in X.But $f^{-1}(A^c) = X - f^{-1}(A)$ and so $f^{-1}(A)$ is $\psi\widehat{g}$-closed in X.Therefore f is $\psi\widehat{g}$-irresolute.

Theorem 3.3. *A function $f : X \to Y$ is $\psi\widehat{g}$-irresolute if and only if it is $\psi\widehat{g}$-continuous*

Proof. Assume that f is $\psi\widehat{g}$-irresolute.Let F be any closed set in Y.By Theorem 3.2[6]every closed set is $\psi\widehat{g}$-closed,F is $\psi\widehat{g}$-closed in Y.Since f is $\psi\widehat{g}$-irresolute,$f^{-1}(F)$ is $\psi\widehat{g}$-closed in X.Therefore f is $\psi\widehat{g}$-continuous.

Conversely,assume that f is $\psi\widehat{g}$-continuous.Let F be any closed set in Y.By Theorem 3.2[6] every closed set is $\psi\widehat{g}$-closed ,F is $\psi\widehat{g}$-closed in Y.Since f is $\psi\widehat{g}$-continuous,$f^{-1}(F)$ is $\psi\widehat{g}$-closed in X.Therefore f is $\psi\widehat{g}$-irresolute.

Theorem 3.4. *Let X,Y and Z be any topological spaces.For any $\psi\widehat{g}$-irresolute map $f : X \to Y$ and any $\psi\widehat{g}$-continuous map $g : Y \to Z$,the composition $g \circ f : X \to Z$ is $\psi\widehat{g}$-continuous.*

Proof. Let F be any closed set in Z.Since g is $\psi\widehat{g}$-continuous,$g^{-1}(F)$ is $\psi\widehat{g}$ -closed in Y.Since f is $\psi\widehat{g}$-irresolute,$f^{-1}(g^{-1}(F))$ is $\psi\widehat{g}$-closed in X.But $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ .Therefore $g \circ f$ is $\psi\widehat{g}$-continuous.

4. Strongly $\psi\widehat{g}$-continuous and Perfectly $\psi\widehat{g}$-continuous

Definition 4.1. A function $f : X \to Y$ from a topological space (X, τ) into a topological space (Y, σ) is said to be strongly $\psi\widehat{g}$-cotinuous if the inverse image of every $\psi\widehat{g}$-open set in Y is open set in X.

Theorem 4.2. *If a function $f : X \to Y$ from a topological space (X, τ) into a topological space (Y, σ) is strongly $\psi\widehat{g}$-continuous,then it is $\psi\widehat{g}$-continuous.*

Proof. Assume that f is strongly $\psi\widehat{g}$-continuous .Let G be any closed set in Y.By Theorem 3.2[6]every closed set is $\psi\widehat{g}$-closed in Y,G is $\psi\widehat{g}$-closed in Y.Since f is strongly $\psi\widehat{g}$-continuous,$f^{-1}(G)$ is closed in X.Therefore f is $\psi\widehat{g}$-continuous.

The converse need not be true as seen from the following example

Example 4.3. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{c\}\}$ and $\sigma = \{X, \emptyset, \{c\}, \{a, b\}\}$.Let $f : (X, \tau) \to (Y, \sigma)$ be an identity map.Then f is $\psi\widehat{g}$-continuous .But f is not strongly $\psi\widehat{g}$-continuous,since for the $\psi\widehat{g}$ closed set $V = c$ in Y,$f^{-1}V = c$ is not closed in X.

Theorem 4.4. *A function $f : X \to Y$ from a topological space (X, τ) into a topological space (Y, σ) strongly $\psi\widehat{g}$-continuous if and only if the inverse image of every $\psi\widehat{g}$-closed set in Y is closed in X*

Proof. Assume that f is strongly $\psi\widehat{g}$-continuous.Let F be any $\psi\widehat{g}$-closed set in Y.Then F^c is $\psi\widehat{g}$-open in Y.Since f is strongly $\psi\widehat{g}$-continuous,$f^{-1}(F^c)$ is open in X.But $f^{-1}(F^c) = X - f^{-1}(F)$and o $f^{-1}(F)$ is closed in X.
Conveersely assume that the inverse image of every $\psi\widehat{g}$-closed set in Y is closed in X.Let G be any $\psi\widehat{g}$-open set in Y.Then G^c is $\psi\widehat{g}$-closed set in Y.By assumption,$f^{-1}(G^c)$ s closed in X.But $f^{-1}(G^c) = X - f^{-1}(G)$ and so $f^{-1}(G)$is open in X.Therefore f is strongly $\psi\widehat{g}$-continuous.

Theorem 4.5.
If a function $f : X \to Y$ is strongly $\psi\widehat{g}$-continuous and a map $g : Y \to Z$ is $\psi\widehat{g}$-continuous,then the composition $g \circ f : X \to Z$ is strongly $\psi\widehat{g}$-continuous.

Proof. Let G be any closed set in Z.Since g is $\psi\widehat{g}$-continuous,$g^{-1}(G)$ is $\psi\widehat{g}$-closed in Y.Since f is strongly $\psi\widehat{g}$-continuous $f^{-1}(g^{-1}(G))$ is closed in X.But $(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$.Therefore $g \circ f$ is strongly $\psi\widehat{g}$-continuous.

Theorem 4.6. *If a function $f : X \to Y$ is strongly $\psi\widehat{g}$-continuous and a map $g : Y \to Z$ is $\psi\widehat{g}$-continuous,then the composition $g \circ f : X \to Z$ is $\psi\widehat{g}$-continuous.*

Proof. Let G be any closed set in Z.Since g is $\psi\widehat{g}$-continuous,$g^{-1}(G)$ is $\psi\widehat{g}$-closed in Y.Since f is strongly $\psi\widehat{g}$-continuous $f^{-1}(g^{-1}(G))$ is closed in X.By Theorem 3.2[6]every closed set is $\psi\widehat{g}$-closed,$f^{-1}(g^{-1}(G))$ is $\psi\widehat{g}$-closed. But$f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$.Therefore $g \circ f$ is $\psi\widehat{g}$-continuous.

Theorem 4.7. *If a function$f : X \to Y$ from a topological spaces(X, τ) into a topological spaces (Y, σ) is continuous then it is strongly $\psi\widehat{g}$-continuous but not conversely.*

Proof. Let $f : X \to Y$ be continuous .Let F be a closed set in Y.Since f is continuous ,$f^{-1}(F)$is closed in X .By Theorem 3.2[6] every closed set is $\psi\widehat{g}$-closed,$f^{-1}(F)$ is $\psi\widehat{g}$-closed.Hence f is $\psi\widehat{g}$-closed.

Converse of the above theorem need not be true as seen from the following example.

Example 4.8. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{a\}, \{b, c\}\}$ and $\sigma = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}\}$.Let $f : (X, \tau) \to (Y, \sigma)$ be an identity map.Then f is strongly $\psi\widehat{g}$-continuous .But f is not continuous,since for the $\psi\widehat{g}$ closed set $V = b$ in Y,$f^{-1}V = b$ is not closed in X.

Definition 4.9. A function $f : X \to Y$ from a topological space (X, τ) into a topological space (Y, σ) is said to be perfectly $\psi\widehat{g}$-continuous if the inverse image of every $\psi\widehat{g}$-closed set in Y is both open and closed in X.

Theorem 4.10. *If a fuction $f : X \to Y$ from a topological space (X, τ)into a topological space(Y, σ) is perfectly $\psi\widehat{g}$-continuous then it is strongly $\psi\widehat{g}$-continuous.*

Proof. Assume that f is perfectly $\psi\widehat{g}$-continuous .Let G be any $\psi\widehat{g}$-closed set in Y.Since f is perfectly $\psi\widehat{g}$-continuous ,$f^{-1}(G)$ is closed in X.Therefore f is strongly $\psi\widehat{g}$-continuous.

Converse of the above theorem need not be true as seen from the following example.

Example 4.11. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{a\}\}$ and $\sigma = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$.Let $f : (X, \tau) \to (Y, \sigma)$ be an identity map.Then f is strongly $\psi\widehat{g}$-continuous .But f is not perfectly $\psi\widehat{g}$-continuous,since for the $\psi\widehat{g}$-closed set $V = b, c$ in Y,$f^{-1}V = b, c$ is not closed in X.

Theorem 4.12. *A function $f : X \to Y$ from a topological space(x, τ) into a topological space (Y, σ) is perfectly $\psi\widehat{g}$-continuous if and only if the inverse image of every $\psi\widehat{g}$-closed set in Y is both open and closed in X.*

Proof. Assume that f is perfectly $\psi\widehat{g}$-continuous .Let F be any $\psi\widehat{g}$-closed set in Y.Then F^c is $\psi\widehat{g}$-open in Y.Since f is perfectly$\psi\widehat{g}$-continuous ,$f^{-1}(F^c)$ is both open and closed in X.But $f^{-1}(F^c) = X - F^{-1}(F^c)$ and so $f^{-1}(F)$ is both open and closed in X.
Conversely assume that the inverse image of every $\psi\widehat{g}$-closed set in Y is both open and closed in X.Let G be any $\psi\widehat{g}$-open in Y.By assumption $f^{-1}(G^c) = X - f^{-1}(G)$ and so $f^{-1}(G)$ is both open and closed in X.Therefore f is perfectly $\psi\widehat{g}$-continuous.

Theorem 4.13. *If a function $f : X \to Y$ from a topological space (X, τ) into a topological space (Y, σ) is strongly $\psi\widehat{g}$-continuous then it is $\psi\widehat{g}$-irresolute.*

Proof. Let $f : X \to Y$ be strongly $\psi\widehat{g}$-continuous function .Let F be a $\psi\widehat{g}$-closed in Y.Since f is strongly $\psi\widehat{g}$-continuous ,$f^{-1}(F)$ is closed in X.By Theorem 3.2[6] every closed set is $\psi\widehat{g}$-closed,$f^{-1}(F)$ is $\psi\widehat{g}$-closed in X.Hence f is $\psi\widehat{g}$-irresolute.
Converse of the above theorem need not be true as seen from the following example.

Example 4.14 Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{c\}\}$ and $\sigma = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$.Let $f : (X, \tau) \to (Y, \sigma)$ be an identity map.Then f is $\psi\widehat{g}$-irresolute.But f is not strongly $\psi\widehat{g}$-continuous,since for the $\psi\widehat{g}$-closed set $V = c$ in Y,$f^{-1}V = c$ is not closed in X.

Theorem 4.15. *If a function $f : X \to Y$ from a topological space (X, τ) into a topological space (Y, σ) is perfectly $\psi\widehat{g}$-continuous,then it is $\psi\widehat{g}$-irresolute*

Proof. Let $f : X \to Y$ be perfectly $\psi\widehat{g}$-continuous function.Let F be a $\psi\widehat{g}$-closed set in Y.Since f is perfectly $\psi\widehat{g}$-continuous ,$f^{-1}(F)$ is both o closed set is bpth open and closed in X.By Theorem 3.2[6]every closed set is $\psi\widehat{g}$-closed ,$f^{-1}(F)$ is $\psi\widehat{g}$-closed in X.Hence f is

$\psi\widehat{g}$-irresolute.

Converse of the above theorem need not be true as seen from the following example.

Example 4.16. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{X, \emptyset, \{b\}, \{a, b\}, \{b, c\}\}$.Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = b, f(b) = c, f(c) = a$.Then f is $\psi\widehat{g}$-irresolute .But f is not perfectly $\psi\widehat{g}$-continuous,since for the $\psi\widehat{g}$ closed set $V = c$ in Y,$f^{-1}V = c$ is open in X but not closed in X.

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