FIXED POINT THEOREMS FOR INTIMATE MAPPINGS IN METRIC SPACES

VINOD KUMAR[*], MONIKA

Department of Mathematics, Baba Mastnath University, Rohtak, Haryana, India

Abstract: In this paper, fixed-point theorems for intimate mappings is proved by us in complete metric spaces by using four self-mappings with rational inequality, which generalizes the results of P. C. Lohani, V.H. Badshah, G. Jungck, B. Fisher, J. Jachymski, S.M. Kang, Y.P. Kim and B.E. Rhoades.

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1. INTRODUCTION AND PRELIMINARIES

The concept of compatible mapping of type (A) in metric space in 1992 was introduced by Jungck G., Murthy P.P. and Cho Y.J. [6] and the results of various authors were improved by proving common fixed point theorems for these mappings using various contractive conditions. Recently, the concept of compatible mappings of type (A) was generalized by Sahu, Dhagat and Srivastava M. [10], so called intimate mappings in 2001. In fact, newly defined mapping is a generalization of the compatible mappings of type (A).

The most important feature of intimate mapping condition is that all said above mapping conditions require the mappings pairs, which should commute at coincidence point, but for newly defined mapping, conditions such necessity is not required i.e. the mapping pair does not necessarily commute at coincidence points. In this paper, a common fixed point theorem is

[*]Corresponding author

E-mail address: kakoriavinod@gmail.com

presented by us. Fixed point theorems of Fisher B. [2], Jungck G. [4], Lohani P.C. and Badshah V. H. [8] and Sahu, Dhagat and Srivastava M. [10] are improved by us.

The paper is also concentrated on some results for intimate mappings, introduced by Sahu et. al. [10], which generalizes the results of Jungck G. [5], Fisher B. [1], Jachymski J. [3], Kang S.M. and Kim Y.P. [7] , Rhoades B.E. [9] and others.

1.1 Definition [10]. Let S and T be self mappings of metric space (X, d). The pair {S, T} is said to be T-intimate iff

$$\alpha d(TSx_n, Tx_n) \leq \alpha d(SSx_n, Sx_n)$$

where α = limit Supremum or limit Infimum $\{x_n\}$ is a sequence in X such that

$$\lim_{n \to \infty} SX_n = \lim_{n \to \infty} TX_n = t \text{ , for some } t \in X.$$

1.2 Proposition. If the pair (P, Q) is compatible of type (A) then it is both P and Q-intimate.

Proof: Since

$$d(PQx_n, Px_n) \leq d(PQx_n, QQx_n) + d(QQx_n, Qx_n) \text{ , for } n \in N$$

Therefore, $\beta d(PQx_n, Px_n) \leq \beta 0 + \beta d(QQx_n, Qx_n)$,

implies $\beta d(QPx_n, Px_n) \leq d(QQx_n, Qx_n)$,

whenever $\{x_n\}$ is a sequence in metric space X,

such that $\lim_{n \to \infty} Px_n = \lim_{n \to \infty} Qx_n = r$,

for some $r \in X$. Hence, the pair {P, Q} is Q-intimate. Similarly, we can show that the pair {P, Q} is P-intimate but its converse is not true.

1.3 Example. Let X = [0, 1] with d(x,y) = $|x - y|$ and P, Q are self mappings on X defined as follows:

$$Px = \frac{2}{x+2} \text{ and } Qx = \frac{1}{x+1}$$

for all $x \in [0,1]$. Now, the sequence $\{x_n\}$ in X defined by

$$x_n = \frac{1}{n}, n \in N$$

Then, we have

$$\lim_{n \to \infty} Px_n = \lim_{n \to \infty} Qx_n = 1$$

.

Also, $\left|PQx_n - Px_n\right| \to \dfrac{1}{3}$ as $n \to \infty$,

and $\left|QQx_n - Qx_n\right| \to \dfrac{1}{2}$ as $n \to \infty$.

Clearly, we have

$$\lim_{n\to\infty}\left|PQx_n - Px_n\right| < \lim_{n\to\infty}\left|QQx_n - Qx_n\right|.$$

Hence, {P , Q} is P-intimate.

But $\left|PQx_n - QQx_n\right| \to \dfrac{1}{6}$ as $n \to \infty$

Hence, (P, Q) is not compatible of type (A).

 1.4 Proposition. Let P and Q be self mappings of a metric space (X, d). If the pair {P, Q} is   Q-intimate and $\mathrm{Pr} = \mathrm{Qr} = z \in X$ for some $r \in X$. Then $d(Qz, z) \le d(Pz, z)$.

Proof. Suppose $x_n = r$ for all $n \ge 1$.

So $Px_n = Qx_n \to \mathrm{Pr} = Qr = z$.

Since the pair {P, Q} is Q-intimate, then

$$d(Q\mathrm{Pr}, Qr) = \lim_{n\to\infty} d(QPx_n, Qx_n)$$

$$= \lim_{n\to\infty} d(QQx_n, Px_n)$$

$$= d(P\mathrm{Pr}, \mathrm{Pr})$$

Implies $d(Qz, z) \le d(Pz, z)$.

2. MAIN RESULTS

The following lemma was given by Singh S. P. and Meade B. A. [11] in 1977:

2.1 Lemma [11]. "For every t > 0, $\gamma(t) < t$ if and only if $\lim_{n\to\infty} \gamma^n(t) = 0$, where γ^n denotes the n times composition of γ.

Before presenting our result of this section, we state the following lemma:

2.2 Lemma. Let P, Q, R and S be the four mappings from metric space (X, d) into itself such that

(2.2.1) $P(X) \subset R(X)$ and $Q(X) \subset S(X)$

(2.2.2) $\quad d(Px, Qy) \le \alpha \dfrac{d(Rx, Qy)[1 + d(Sx, Px)]}{[1 + d(Sx, Ry)]}$

$$+ \beta[d(Sx, Px) + d(Ry, Qy)] + \gamma[d(Sx, Qy) + d(Ry, Px)] + \delta d[(Sx, Ry)]$$

for all x,y in X, where $\alpha, \beta, \gamma, \delta \ge 0, 0 \le \alpha + 2\beta + 2\gamma + \delta < 1$.

Then for any arbitrary point x_0 in X by (2.2.1) there exists a point $x_1 \in X$ such that $R(x_1) = Px_0$ and for this point x_1. We can choose a point $x_2 \in X$ such that $Qx_1 \in Sx_2$ and so on. Inductively, we can define a sequence $\{y_n\}$ in X such that

(2.2.3) $\quad y_{2n} = Rx_{2n+1} = Px_{2n}$ and $y_{2n+1} = Sx_{2n+2} = Qx_{2n+1}$, for $n = 0,1,2....$

Then $\lim\limits_{n \to \infty} d(y_n, y_{n+1}) = 0$ and (y_n) is a Cauchy sequence in X.

The following common fixed point theorem is presented by us which generalizes the result of Lohani P. C. and Badshah V.H. [8] on intimate mappings in metric spaces:

2.3 Theorem Let P, Q, R and S be mappings from a metric space (X, d) into itself satisfying (2.2.1), (2.2.2), (2.2.3) and following:

(2.2.4) the pairs (P , S) is S-intimate and (Q , R) is R-intimate.

(2.2.5) S(X) is complete.

Then P, Q, R and S have a unique common fixed point in X.

Proof: We can observe that the sequence {y_n} defined in (2.2.3) is a Cauchy sequence in X. Since S(X) is complete and {Sx_{2n}} is Cauchy. Then it converges to a point a = Sz for some z in X. Then $y_n \to a$ and hence

$\qquad Px_{2n,}\, Sx_{2n}, Qx_{2n+1}, Rx_{2n+1} \to a.$

Now from (2.2.2)

$d(Pz, Qx_{2n+1}) \le \alpha \dfrac{d(Rx_{2n+1}, Qx_{2n+1})[1 + d(Sz, Pz)]}{[1 + d(Sz, Rx_{2n+1})]}$

$$+ \gamma[d(Sz, Qx_{2n+1}) + d(Rx_{n+1}, Pz)] + \delta d[(Sz, Rx_{2n+1})], \text{ for all } x, y \in X.$$

Where

$\qquad \alpha, \beta, \gamma, \delta \ge 0, 0 \le \alpha + 2\beta + 2\gamma + \delta < 1$

Taking limit as n→∞, we obtain

$$d(Pz, a) \le \alpha \dfrac{d(y_{2n+1}, y_{2n})[1 + d(Sz, Pz)]}{[1 + d(Sz, y_{2n+1})]} + \beta[d(a, Pz) + d(a, a)]$$

$$+ \gamma \left[d(a,a) + d(a, Pz) \right] + \delta d \left[(a,a) \right]$$

$$= \alpha 0 + \beta d(a,Pz) + \gamma \left[d(p, Au) \right] + \delta 0$$

$$= (\beta + \gamma) d(Pz,a), \qquad \text{which is contradict.}$$

Thus, pz = z.

Since P(X) ⊂R(X), there exist w ϵ X such that Rw = a. Hence, from (2.2.2), we obtain,

$$d(a , Qw) \leq (\alpha + \beta + \gamma) \, d(a , Qw),$$

which is a contradiction, so Qw = a. Since Pz = Sz = a . The pair {P , S} be S-intimate. Then we get,

$$d(Sa , a) \leq d(Pa , a).$$

Suppose Pa ≠ a , then from (2.2.2), we have

$$d(Pa , a) < d(Pa , a),$$

which is a contradiction, so Pa is equal to a . Hence   Sa = a. Similarly, we have Qa = Ra = a.

Uniqueness:

Let us suppose that unique fixed point of P, Q, R and S have other fixed point b such that a ≠ b. Thus

$$d(a,b) = d(Pa,Qb) \leq \alpha \frac{d(Ra,Qb)\left[1 + d(Ra,Pb)\right]}{\left[1 + d(Sa,Rb)\right]}$$

$$+ \beta \left[d(Sa,Pa) + d(Rb,Qb) \right] + \gamma \left[d(Sa,Qb) + d(Rb,Pa) \right] + \delta d \left[(Sa,Rb) \right]$$

$$= (\alpha + 2\gamma + \delta) d(a , b) < d(a , b)$$

This show that a is equal to b.

Sahu, Dhagat and Srivastva M. [10] defined new concept of intimate mappings in 2001. Generalization of the compatible mappings of type (A) was introduced by Kang S. M. and Kim Y.P. [7]. The interesting feature of intimate mappings is that this mapping do not necessarily commute at coincidence points.

A fixed-point theorem by using intimate mappings is proved by us in this section. In this result, we use mappings, which are not continuous. The results of Fisher B. [2], Jachymski J. [3], Kang S.M. and Kim Y.P. [7] and Rhoades B.E. [9] are generalized.

2.4 Lemma: let P, Q, R and S be mappings from a metric space (X, d) into itself satisfying the following conditions:

(2.2.6)   P(X) ⊂R(X) and Q(X) ⊂S(X)

(2.2.7)   d(Px,Qy) ≤ φ(d(Sx,Ry), d(Px,Sx), d(Qy ,Ry), d(Px,Ry), d(Qy,Sx))

for all x, y ϵ X, where $\varphi \epsilon$ F. Then for arbitrary point x_0 in X by (2.2.6), we choose a point x_1 such that $Rx_1 = Px_0$ and for this point x_1 , there exists a point x_2 in X such that $Sx_2 = Qx_1$ and so on . Continuing in this manner, we can define a sequence $\{y_m\}$ in X such that

(2.2.8) $y_{2m} = Px_{2m} = Rx_{2m+1}$, $y_{2m+1} = Qx_{2m+1} = Sx_{2m+2,}$

where m from 0 to ∞. Then

$$\lim_{m\to\infty} d(y_m, y_{m+1}) = 0,$$

where $\{y_m\}$ is the sequence in X defined by (2.2.8) and the sequence $\{y_m\}$ is a cauchy sequence in X.

Proof. Let $d_m = d(y_m, y_{m+1})$, where m from 0 to ∞. Now to prove the sequence $\{d_m\}$ is non-increasing in +ve real numbers, i.e., $d_m \leq d_{m-1}$, where m lies between 1 to ∞ by (2.2.7), we have

$$d(Px_{2m}, Qx_{2m+1}) \leq \phi(d(Sx_{2m}, Rx_{2m+1}), d(Px_{2m}, Sx_{2m}) d(Qx_{2m+1}, Rx_{2m+1}) d(px_{2m}, Rx_{2m+1}), d(Qx_{2m+1}, Sx_{2m}))$$

Using (2.2.8), we have

$$d(y_{2m}, y_{2m+1}) \leq \phi(d(y_{2m-1}, y_{2m}) d(y_{2m}, y_{2m-11}), d(y_{2m+1}, y_{2m}), d(y_{2m}, y_m), d(y_{2m+1}, y_{2m-1}))$$

$$= \phi(d(y_{2m-1}, y_{2m}), d(y_{2m}, y_{2m-1}), d(y_{2m+1}, y_{2m})) 0, [d(y_{2m+1}, y_{2m}) + d(y_{2m}, y_{2m-1})]$$

(2.2.9) $= \varphi(d_{2m-1}, d_{2m-1}, d_{2m}, 0, d_{2m} + d_{2m-1})$

Assume that $d_{m+1} < d_m$ for some m. Then,

$$\alpha < 2, \ d_{m-1} + d_m = \alpha d_m .$$

We know that ϕ is non-increasing for every variable and $\beta < 1$ for some $\alpha < 2$, from (2.2.9.), we have

$$d_{2m} \leq \phi(d_{2m}, d_{2m}, d_{2m}, 0, \alpha d_m) \leq \beta d_{2m} < d_{2m} .$$

Similarily, we have $d_{2m+1} < d_{2m+1}$. Thus , for every $m, d_m \leq \beta d_m < d_m$.

This is a contradiction. Therefore, $\{d_m\}$ is a non-increasing sequence in positive real number.

Again from (2.2.7), we get,

$$d_u = d(y_1, y_2) = d(Px_2, Qx_1)$$

$$\leq \phi(d(Sx_2, Rx_1), d(Px_2, Sx_2), d(Qx_1, Rx_1)) (d(Px_2, Rx_1), d(Qx_1, Sx_2))$$

$$= \phi(d(y_1, y_0), d(y_2, y_1), d(y_1, y_0), d(y_2, y_0), d(y_1, y_1))$$

$$= \phi(d_0, d_1, d_0, d_0 + d_1, 0)$$

$$\leq \phi\left(d_0, d_0, d_0, 2d_0, d_0\right)$$

$$s = \gamma\left(d_0\right).$$

In general, we get, $d_m \leq \gamma^m(d_0)$. This implies that, if $d_0 > 0$, by Lemma 2.1

$$\lim_{m \to \infty} d_m \leq \lim_{m \to \infty} \gamma^m\left(d_0\right) = 0.$$

hence, we have,

(2.2.10) $\lim_{m \to \infty} d_m = 0$,

Since $\{d_m\}$ is non-increasing with $d_0 = 0$. Now, to prove there is a Cauchy sequence $\{y_m\}$ in X. By virtue of (2.2.10), it is a Cauchy sequence in X. We suppose that there is not a Cauchy sequence $\{y_m\}$. Thus, there is an $\varepsilon > 0$ such that for every integer 2u , exist even integers 2r(u) and 2s(u) with $2r(u) > 2s(u) \geq 2u$ such that

(2.2.11) $d(y_{2r(u)}, y_{2s(u)}) > \varepsilon$.

for every even integer 2u, let the least even integer exceeding 2s(u) is 2r(u) with condition (2.2.11) i.e.,

(2.2.12) $d(y_{2s(u)}, y_{2r(u)-2}) \leq \varepsilon$ and $d(y_{2s(u)}, y_{2r(u)}) > \varepsilon$,

then for every even integer 2u, we obtain,

$$\varepsilon \leq d(y_{2s(u)}, y_{2r(u)})$$

$$\leq d(y_{2s(u)}, y_{2r(u)-2}) + d(y_{2s(u)}, y_{2r(u)-1}) + d(y_{2r(u)-1}, y_{2r(u)}).$$

from equation (2.2.10) and (2.2.12), it follows that

(2.2.13) $d(y_{2s(u)}, y_{2r(u)}) \to \varepsilon$ as $u \to \infty$.

So , by triangle inequality, we get,

$$|d(y_{2s(u)}, y_{2r(u)-1}) - d(y_{2s(u)}, y_{2r(u)})| \leq d(y_{2r(u)-1}, y_{2r(u)})$$

and

$$|d(y_{2s(u)}, y_{2r(u)-1}) - d(y_{2s(u)}, y_{2r(u)})| \leq d(y_{2r(u)-1}, y_{2r(u)}) + d(y_{2s(u)}, y_{2s(u)+1}).$$

By using equation (2.2.10) and (2.2.13), as $u \to \infty$

(2.2.14) $d(y_{2s(u)}, y_{2r(u)-1}) \to \epsilon$ and $d(y_{2s(u)+1}, y_{2r(u)-1}) \to \epsilon$,

so, by (2.2.7) and (2.2.8), we obtain,

$$d(y_{2s(u)}, y_{2r(u)}) \leq d(y_{2s(u)}, y_{2s(u)+1}) + d(y_{2s(u)+1}, y_{2r(u)})$$

$$= d(y_{2s(u)}, y_{2s(u)+1}) + d(Px_{2r(u)}, Qx_{2s(u)+1})$$

$$\leq d(y_{2s(u)}, y_{2s(u)+1}) + \phi(d(Sx_{2r(u)}, Rx_{2s(u)+1}),$$

$$d(Px_{2r(u)}, Sx_{2r(u)}), d(Qx_{2s(u)+1}, Rx_{2s(u)+1}),$$

$$d(Px_{2r(u)}, Rx_{2s(u)+1}), d(Qx_{2s(u)+1}, Sx_{2r(u)})$$

$$= d(y_{2s(u)}, y_{2s(u)+1}) + \phi(d(y_{2r(u)-1}, y_{2s(u)}),$$

$$d(y_{2r(u)}, y_{2r(u)-1}), d(y_{2s(u)+1}, y_{2s(u)}),$$

(2.2.15) $$d(y_{2r(u)}, y_{2s(u)}), d(y_{2s(u)+1}, y_{2r(u)-1}),$$

We know ϕ is upper semi continuous, as $u \to \infty$, as in (2.2.15), from (2.2.9), (2.2.12), and (2.2.14), we get,

$$\varepsilon \leq \phi(\varepsilon, 0, 0, \varepsilon, \varepsilon) \leq y(\varepsilon) < \varepsilon,$$

which is a contradiction. Hence, there $\{y_{2m}\}$ is a Cauchy sequence in X. Thus proof is complete.

2.5 Theorem. Let P , Q , R and S be mappings from a metric space (X, d) into itself satisfying (2.2.6), (2.2.7), (2.2.8) and following:

(2.2.16)   the pairs (P , S) is S-intimate and (Q , R) is R-intimate

(2.2.17)   S(X) is complete.

Then P , Q , R and S have a unique common fixed point in X".

Proof: We can see that the sequence $\{y_n\}$ defined by (2.2.8) is Cauchy sequence in X from lemma (2.4). We know that S(X) is complete and there is a Cauchy sequence $\{Sx_{2n}\}$. Then it converges to a point $a = Sz$ for each $z \in X$. Thus, $y_n \to a$ and therefore,

$$Px_{2n}, Sx_{2n}, Qx_{2n+1}, Rx_{2n+1} \to a.$$

From (2.2.7),

$$d(Pz, Qx_{2n+1}) \leq \phi(d(Sz, Rx_{2n+1}), d(Pz, Sz), d(Qx_{2n+1}, Rx_{2n+1}),$$

$$d(Pz, Rx_{2n+1}), d(Qx_{2n+1}, Sz)).$$

Taking $n \to \infty$, we have

$$d(Pz, a) \leq \phi(d(a , a), d(Pz , a), d(a , a), d(Pz , a), d(a , a))$$

$$= \phi(0, d(Pz , a), 0, d(Pz , a), 0)$$

$$< d(Pz , a)$$

Hence, a contradiction. This implies that $Pz = a$. Since $P(X) \subset R(X)$, so there exists $w \in X$ such that $Rw = a$. Hence, from (2.2.7), we obtain,

$$d(a , Qw) = d(Pz , Qw) \leq \phi(d(Sz, Rw), d(Pz, Sz),$$

$$d(Qw , Rw), d(Pz , Rw), d(Qw, Sz))$$

$$= \phi(d(a , a), d(a, a), d(Qw , a) d(a , a), d(Qw , a))$$

$d(a, Qw) \leq \phi(0, 0, d(Qw, a), 0, d(Qw, a)) < d(Qw, a)$

hence, a contradiction implies that $Qw = a$. Since $Pz = Sz = a$ and the pair $\{A, S\}$ is S-intimate. Then we have

$d(Sa, a) \leq d(Pa, a)$

Suppose $Pa \neq a$, then from (2.2.7), we have

$d(Pa, a) = d(Pa, Qw) \leq \phi(d(Sa, Rw), d(Pa, Sa), d(Qw, Rw),$

$\qquad d(Pa, Rw), d(Qw, Sa))$

$\qquad = \phi(d(Sa, a), d(Pa, Sa), d(a, a), d(Pa, a), d(a, Sa))$

$\qquad \leq \phi(d(Pa, a), d(Pa, Sa), 0, d(pa, a), d(Pa, a))$

$\qquad \leq \phi(d(Pa, a), d(pa, a) + d(a, Sa), 0, d(Pa, a), d(Pa, a))$

$\qquad \leq \phi(d(Pa, a), 2d(Pa, a), 0, d(Pa, a), d(Pa, a))$

$\qquad < d(Pa, a)$

Hence, a contradiction, which implies that $Pa = a$. Hence $Sa = a$. Similarly, we get,

$Qa = Ra = a.$

UNIQUENESS:

Now, we shall prove that a is unique. let us consider that P, Q, R and S have common fixed point a and b, $a \neq b$. Therefore, from (2.2.7), we get,

$d(a, b) = d(Pa, Qb) \leq \phi(d(Sa, Rb), d(Pa, Sa), d(Qb, Rb),$

$\qquad d(Pa, Rb), d(Qb, Sa))$

$\qquad = \phi(d(a, b), 0, 0, d(a, b), d(a, b))$

$\qquad \leq \phi(k, 0, k, 0, k, k, k) < k$, where $k = d(a, b)$.

Thus, $a = b$.

The following corollaries follow immediately from theorem 2.5

2.6 Corollary. Let (P, S) be S-intimate and (Q, R) be R-intimate pairs of self mappings of a complete metric space (X, d) satisfying (2.2.6), (2.2.8) and the following:

(2.2.18) $d(Px, Qy) \leq g M(x, y), 0 \leq g < 1, x, y \in X$, where

$\qquad M(x, y) = \max\{d(Sx, Ry), d(Px, Sx), d(Qy, Ry), [d(Px, Ry) + d(Qy, Sx)]/2\}.$

Then P, Q, R and S have a unique common fixed point in X.

Proof: We consider the function $\phi : [0, \infty)^5 \to [0, \infty)$ defined by

$\qquad \phi(x_1, x_2, x_3, x_4, x_5) = g \max \{x_1, x_2, x_3, \frac{1}{2}(x_4 + x_5)\}.$

Since $\phi \in F$, we can apply theorem (2.5) and deduce the Corollary.

2.7 Corollary. Let (P, S) be S-intimate and (Q , R) be R-intimate pairs of self maps of a complete metric space (X, d) satisfying (2.2.4), (2.2.6) and the following:

(2.2.19) $d(Px, Qy) \leq g \max\{d(Px, Sx), d(Qy, Ry), \frac{1}{2} d(Px, Ry),$

$$\frac{1}{2}d(Qy, Sx), d(Sx, Ry)\}$$

for all x, y in X, where $0 \leq g < 1$. Then P, Q, R and S have a unique common fixed point in X.

Proof: We consider the function $\phi : [0, \infty)^5 \to [0, \infty)$ defined by

$$\phi(x_1, x_2, x_3, x_4, x_5) = g \max \{x_1, x_2, x_3, \tfrac{1}{2}(x_4+ x_5\}.$$

Since $\phi \in F$, we can apply theorem (2.5) to get this Corollary.

2.8 Remark:

The result of Jungck G. [5] is generalized by theorem (2.5) by using intimate mappings without continuity at S and R. The result of Fisher B. [2] is also generalized by theorem (2.5). We generalize the results of Jachymski J. [3] , Kang S. M. and Kim Y. P. [7], and Rhoades B.E. [9] for intimate mappings.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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